Model based Predictive Control of the Four Tank System

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Abstract. This paper deals with the implementation of Model Predictive Control (MPC) in a Four Tank System (FTS). The nonlinear model of FTS has been developed from the mechanism modelling. The FTS is a typical application with nonlinear, coupling and time delay characteristics which can be utilized to examine different control algorithms. The aim of the process is to keep the liquid level in the tanks at the reference values. This problem is solved using different control methods such as proportional-integral-derivative (PID), internal model control (IMC), MPC, and Fuzzy Modified Model Reference Adaptive Control (FMMRAC). The MPC allows closed-loop solution to the optimization problem to be obtained off-line. A general MPC control is applied to the FTS and different performance indices as well as error indices are calculated. The responses of these controllers are corroborated and are compared with other control algorithms through simulation. The simulation results show that good tracking performance is attained.

Keywords: Proportional-integral-derivative controller, Model Predictive Control, Four Tank System, FMMRAC

1 Introduction

The four tank system (FTS) is a typical application with nonlinear, coupling and time delay characteristics. Most of the process industries have the problem of controlling the level of liquid in tanks as well as flow between tanks. Most of the times, the liquids are used in chemical plants as well as for mixing treatment (Bequette 2003; Smith et al.) in the tanks. But, the level of liquid in the tanks must be maintained and the flow between tanks must be adjusted in process industries.

Several researchers have investigated the difficulty of controlling the level of liquid of a single or multiple tanks. The dynamic model of the quadruple tank system for laboratory process has been developed by Johanson (2000). Both minimum and non-minimum phase systems for tank system are discussed in this work. Intelligent controls including fuzzy logic (FL) control (Aydogmus Z 2009; Prusty et al. 2014; Prusty et al. 2015), neural network (NN) control (Kamalasadan et al. 2007; Tani et al. 1996), and genetic algorithms (GA) (Mohideen et al. 2013) have also been employed to the tank system. Adaptive control methods such as Model Reference Adaptive Control (MRAC), Gain Scheduling and Self Tuning Regulator (STR) have been used to enhance the transient response of the controller (Mohideen et al. 2013). Mohideen et al. have proposed Modified MRAC scheme for controlling the liquid level in FTS with good transient performance as well as steady-state performance. A constrained predictive control algorithm employed to a coupled tank setup has discussed in (Poulsen et al. 2001). The comparative analysis of conventional and intelligent control utilized in the process has been described by Zumberge and Passino (1998).

The controller must have the capability to adjust the various changes in plant dynamics. An adaptive method for the enhancement of the performance in case of a motor drive system has been proposed by Liu and Hsu (2007). The methods like NN, FL, GA and their hybrid combinations have been implemented to control the level of liquid in tanks. Also, other evolutionary algorithms such as Ant Colony optimization, Particle Swarm Optimization (PSO), and Bacterial Foraging methods have been applied to control the various parameters of the process.

Predictive control techniques are used to solve the control issues of the process. The Model Predictive Control (MPC) is an advanced optimization based control method for process industries since 1980 (Seborg et al. 2010). Power system balancing models are also controlled by using MPC. One of the most important advantage of MPC is that the current timeslot are to be optimized by considering the future time slots. MPC has the capability of future event predictions and can utilize the control action accordingly. A typical application of the process control is the tank level control system. System status and system parameters are the main causes of control accuracy of the liquid level system. Centralized and decoupling of quadruple tank system using MPC method has discussed by Srinivasarao and Subbaiah (2013). The generalized predictive control (GPC) which reduces the computational time has been described by Muthukumar et al. (2013). In this paper, MPC controller is implemented to control the level of liquid in lower two tanks of the system.

This paper is arranged as follows: Section II describes about the dynamic model of the FTS. In section III, MPC controller algorithm is discussed. Section IV shows the simulation results as well as descriptions about the responses of the different controllers. Finally, the conclusion is given in Section V.

2 General Formulation of FTS

The schematic of the FTS has proposed by Johansson [3] as shown in Fig. 1. The FTS has the property of multivariable interaction due to the influences of the outputs by the pumps. An adjustable multivariable zero can be occurred due to the change of the valve settings of the system. The multivariable zero can be present at the left half plane or right half plane. Fluid mechanics theory is used to analyze the dynamics of FTS and the system model is established based on the nonlinear mechanism. The goal of the process is to control the level of liquid in the lower two tanks. The FTS has two inputs and two outputs. Input voltages to the pump (v_1 and v_2) are considered as the input to the process and voltages from the level measurement devices (y_1 and y_2) are considered as the outputs. There is a reservoir tank under the tanks to store the water coming from tank 1 and tank 2. In Fig. 1, h_i is the liquid level in tank i where i =1, ..., 4.

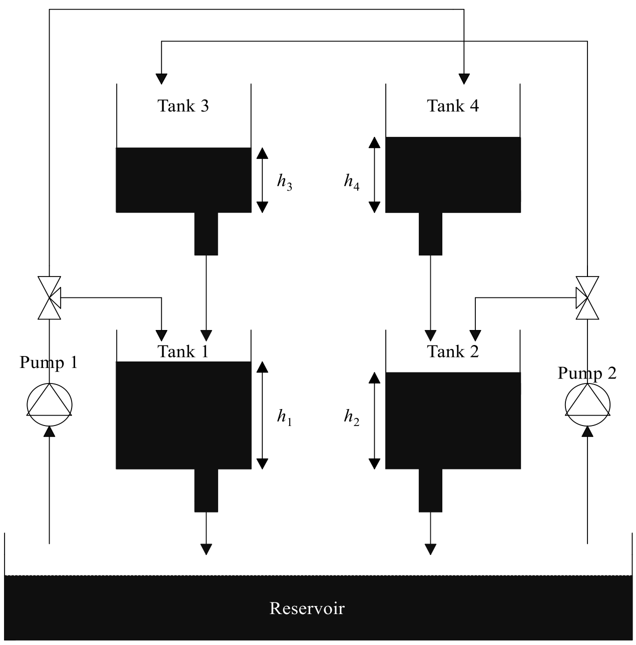


Fig. 1 Schematic representation of FTS

For developing the mathematical model for FTS, the density of liquid in the inlet, in the outlet and in the tank is assumed to be constant. The nonlinear model of the FTS is obtained using Mass balance equation and Bernoulli's law which are shown in Eq.s (1) - (4) as

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_a k_1}{A_1} v_1 \tag{1}$$

$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_b k_2}{A_2} v_2 \tag{2}$$

$$\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_b) k_2}{A_3} v_2 \tag{3}$$

$$\frac{dh_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_a) k_1}{A_4} v_1 \tag{4}$$

where

A_i = Area of cross-section of Tank i, i =1, ..., 4
a_i = Area of cross-section of outlet hole,
h_i = Level of liquid in tanks

The voltage applied to pump i is v_i and the corresponding flow is $k_i v_i$. The parameters (γ_a, γ_b) are determined from the valve settings of the system. It can be shown that a multivariable right half plane zero will be present when $(\gamma_a + \gamma_b) < 1$ for the nonlinear system. The flow to tank 1 is $\gamma_a k_1 v_1$ and the flow to tank 4 is $(1-\gamma_a) k_1 v_1$. Similarly, the flow to tank 2 is $\gamma_b k_2 v_2$ and the flow to tank 3 is $(1-\gamma_b) k_2 v_2$. The acceleration due to gravity is designated by g. The process parameter values are given in Table I [3].

Table 1. Parameter values of the FTS model.

Parameters	Value
a_1, a_3	0.071cm^2
a_2, a_4	0.057cm^2
A_1, A_3	28 cm^2
A_2, A_4	32 cm^2

In this work, the model and control of the FTS are studied at minimum-phase characteristics. The variables $H_i = h_i - \bar{h}_i$ and $u_i = v_i - \bar{v}_i$ are the deviation variables where $\bar{h}_i$ and $\bar{v}_i$ are the steady-state values of h_i and v_i, respectively. The linearized model equations for the FTS are

$$\frac{dH}{dt} = \begin{bmatrix} -\frac{C_1}{A_1} & 0 & \frac{C_3}{A_1} & 0 \\ 0 & -\frac{C_2}{A_2} & 0 & \frac{C_4}{A_2} \\ 0 & 0 & -\frac{C_3}{A_3} & 0 \\ 0 & 0 & 0 & -\frac{C_4}{A_4} \end{bmatrix} H + \begin{bmatrix} \frac{\gamma_a k_1}{A_1} & 0 \\ 0 & \frac{\gamma_b k_2}{A_2} \\ 0 & \frac{(1-\gamma_b)k_2}{A_3} \\ \frac{(1-\gamma_a)k_1}{A_4} & 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} H \qquad (5)$$

where

$$C_i = a_i \sqrt{\frac{g}{2\bar{h}_i}}, \qquad i = 1, ..., 4$$

The inputs are pseudorandom binary sequences (PRBS) with low amplitudes, so that the dynamics are captured by a linear model. The model outputs match with the responses of the real process. The four tanks in the FTS are of Acrylic type. It has also four numbers of smart level transmitters (DPT) to sense the level of each tank. Two numbers of control valves are mounted in the mechanical rigid frame to control the flow rate of the water. The storage tank has the capacity of 75 liters. Centrifugal pumps are provided to circulate the water from the storage tank. Four numbers of rotameters are connected in the inlet of the process tank to visualize the flow rate which is (10 - 100) liters per hour (LPH). For simulating the FTS, its mathematical model [1] is necessary and has developed using Mass balance equation and Bernoulli's law which are shown in Eq.s (1) – (4). The system is designed according to the mathematical model.

3 Model Predictive Control

The MPC is an advanced method of process control that has been in use in various process industries in chemical plants and oil refineries since 1980s. MPC controllers depend on dynamic models of the process (linear empirical models obtained by using system identification methods). MPC refers to a class of computer control algorithms that utilize an explicit process model to predict the future response of a plant. At each control interval, an MPC algorithm attempts to optimize future plant behaviour by computing a sequence of future manipulated variable adjustments. The first input in the optimal sequence is then sent into the plant and the entire calculation is repeated at subsequent control intervals. MPC technology can now be found in a wide variety of application areas including chemicals, food processing, automotive and aerospace applications.

The basic block diagram of MPC controller is shown in Fig. 2. The MPC is required a model of the process. A model of the process is used to predict the future evolution of the process to optimize the control signal. A model also describes the input to output behaviour of the process. There are two types of MPC calculations such as set-point calculations and control calculations which are performed at each sampling instants. MPC has a greater influence on industrial practice than Internal Model control (IMC) and Smith Predictor because it is more acceptable for constrained MIMO control problems.

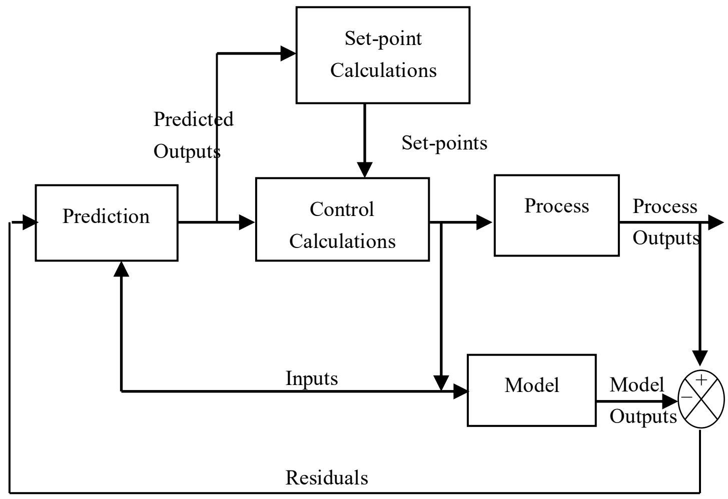


Fig. 2 Block diagram of MPC Controller

The objective of MPC control calculation is to determine a sequence of control moves (manipulated input changes) so that the predicted response moves to the set point. In the MPC controller, the number of predictions, *P* is referred to as prediction horizon while the number of control moves, *M* is called as control horizon. The MPC has a distinguishing feature that is its receding horizon approach. Here, a sequence of *M* control moves is calculated at each sampling instant, only the first move is actually implemented. Then, a new sequence is calculated at the next sampling instant, after new measurements become available, only the first input move is implemented. This procedure is repeated at each sampling instant. Discrete-time models are more convenient for predictions. Hence discrete step response model is used in the MPC control calculations.

3.1. Prediction for SISO Step Response Models

The one-step-ahead prediction is written as

$$\hat{y}(k+1) = \sum_{i=1}^{N-1} S_i \Delta u(k-i+1) + S_N u(k-N+1) \tag{6}$$

Similarly, the step response model of a SISO process for a *j-step-ahead prediction,* where *j* is an arbitrary positive integer is written as

$$\hat{y}(k+j) = \sum_{i=1}^{j} S_1 \Delta u(k-i+j) + \hat{y}^{\circ}(k+j) \tag{7}$$

where

$$\hat{y}^{\circ}(k+j) = \sum_{i=j+1}^{N-1} S_i \Delta u(k-i+j) + S_N u(k-N+j) \tag{8}$$

This term $\hat{y}^{o}(k+j)$ contains only the past control action, so, it is referred to as the predicted unforced response.

The MPC calculations are based on state-space models due to unified framework for both linear and nonlinear control problems. The control objective is to compute a set of control moves (input changes) that produce the corrected predictions as close to a reference trajectory as possible. A reference trajectory is used to form a gradual change to the desired reference.

3.2. MPC Performance Index

The *rM*-dimensional vector $\Delta \boldsymbol{U}(k)$ is calculated so as to minimize

1. The predicted errors over the prediction horizon, *P*
2. The size of the control moves over the control horizon, *M*

A quadratic performance index which is used for MPC controller is expressed as

$$\min_{\Delta \boldsymbol{U}(k)} \boldsymbol{J} = \hat{\boldsymbol{E}}(k+1)^T \boldsymbol{Q} \hat{\boldsymbol{E}}(k+1) + \Delta \boldsymbol{U}(k)^T \boldsymbol{R} \Delta \boldsymbol{U}(k) \tag{9}$$

where $\boldsymbol{Q}$ is a positive-definite weighting matrix and $\boldsymbol{R}$ is a positive semi-definite matrix. Both are diagonal matrices with positive diagonal elements. The MPC control law that minimizes the quadratic objective function is calculated mathematically as

$$\frac{\partial \boldsymbol{J}}{\partial \Delta \boldsymbol{U}(k)} = 0 \tag{10}$$

After simplification of Eq. (9) and Eq. (10), we have

$$\Delta \boldsymbol{U}(k) = (\boldsymbol{S}^T \boldsymbol{Q} \boldsymbol{S} + \boldsymbol{R})^{-1}\ \boldsymbol{S}^T \boldsymbol{Q} \hat{\boldsymbol{E}}^{\boldsymbol{o}}(k+1) \tag{11}$$

The control law is then rewritten as

$$\Delta \boldsymbol{U}(k) = \boldsymbol{K_C}\ \hat{\boldsymbol{E}}^{\boldsymbol{o}}(k+1) \tag{12}$$

where

$$\boldsymbol{K_C} = (\boldsymbol{S}^T \boldsymbol{Q}\, \boldsymbol{S} + \boldsymbol{R})^{-1}\ \boldsymbol{S}^T \boldsymbol{Q} \tag{13}$$

Controller gain matrix, $\boldsymbol{K_C}$ is a multivariable, proportional control law based on the predicted error. For evaluating the value of $\boldsymbol{K_c}$ which is an *rM* x *mP* matrix, the dynamic matrix, $\boldsymbol{S}$ as well as the weighting matrices, $\boldsymbol{Q}$ and $\boldsymbol{R}$, are assumed to be constant. All physical systems have constraints. In MPC, the following constraints are normally defined to minimize the inequalities. Constraints in the outputs: $y_{\min} \le y \le y_{\max}$

Constraints in the inputs:

$$\Delta u_{\min} \le \Delta u \le \Delta u_{\max}$$

$$u_{\min} \le u \le u_{\max}$$

where $\Delta u(k) = u(k) - u(k-1)$

The MPC controller takes all these constraints into consideration when calculating the future controls.

Algorithm for MPC

1. The process output, $y(k)$ is measured and is used to estimate the disturbance.
2. The predicted unforced error, $\hat{\boldsymbol{E}}^{\boldsymbol{o}}$ $(k + 1)$ is updated.

   $$\hat{\boldsymbol{E}}^{\boldsymbol{o}}(k+1) = \boldsymbol{Y}_r(k+1) - \hat{\boldsymbol{Y}}^{\boldsymbol{o}}(k+1) - \phi\left[y(k) - \hat{y}(k)\right]$$

3. Solve for control moves

   $$\Delta \boldsymbol{U}(k) = K_C\ \hat{\boldsymbol{E}}^{\boldsymbol{o}}(k+1)$$

4. The first input step $\Delta \boldsymbol{U}(k)$ is only implemented.
5. Counter is updated. $k = k + 1$

4 Simulation Results

The FTS is simulated using LabVIEW software. In order to carry out the performance analysis of different controllers, a step input of amplitude 2 is applied to the system. The model and control of the FTS are analyzed at minimum-phase characteristics. The operating point values of the parameters of the FTS model are shown at Table 2. For the minimum-phase characteristics, Z ₋, time constants are shown in Table 3.

Table 2. Parameter values of the Four Tank System

Operating Parameters	Value	Z-
$\bar{h}_1, \bar{h}_2$	cm	12.4, 12.7
$\bar{h}_3, \bar{h}_4$	cm	1.8, 1.4
$\bar{v}_1, \bar{v}_2$	V	3.0, 3.0
k_1, k_2	cm^3/Vsec	3.33, 3.35
γ_a, γ_b		0.7, 0.6

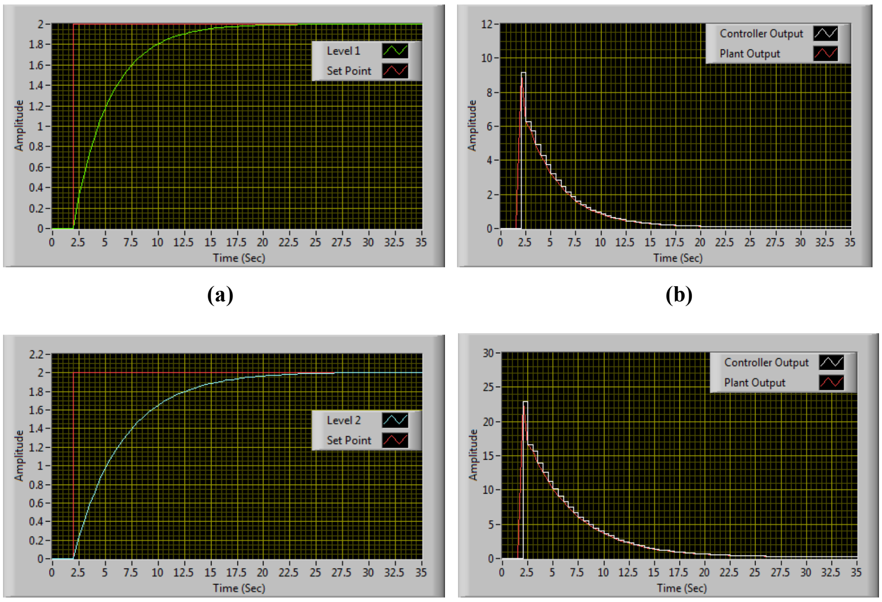
Table 3. Time constants for minimum-phase system, Z ₋

Time Constants	Z ₋
T_1, T_2	62.7, 90.3
T_3, T_4	23.9, 30.0

The physical modeling gives the two transfer function matrices as

$$G_{\_}(s) = \begin{bmatrix} \frac{2.61}{62.7s+1} & \frac{1.5}{(62.7s+1)(23.9s+1)} \\ \frac{1.41}{(90.3s+1)(30s+1)} & \frac{2.84}{90.3s+1} \end{bmatrix} \quad (14)$$

The response is plotted for the levels in the lower two tanks of the FTS. For the minimum-phase characteristics, the response of output level y_1 with input control signal, u_1 is shown in Fig. 3(a). It comprises the responses of the set point and response of output level of tank 1 using MPC controller. The corresponding control input signal, u_1 is presented in Fig. 3(b). Similarly, Fig. 3(c) shows the response of output level, y_2 with input control signal, u_2. Fig. 3(d) shows it's corresponding input control signal, u_2 for the FTS. From the figure, it can be seen that the output level attains the desired set point value. The time of set point change is given at t = 2 sec. The control signal is changed sharply at t = 2 sec. After that the signal changes step by step and the plant output tracks the control signal accordingly.

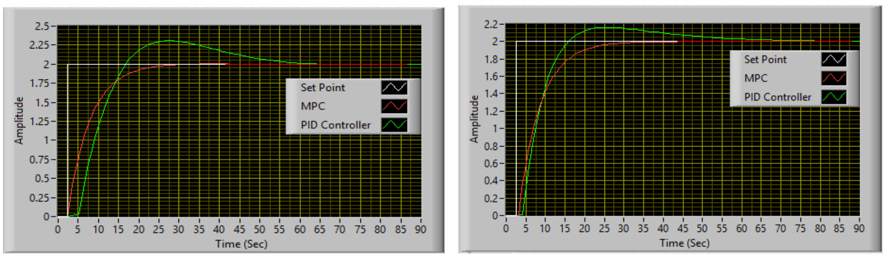


(a) **(b)**

(c) **(d)**

Fig. 3 Minimum- phase system: (a) Response of output level y1 with input u1, (b) Response of output level y2 with input u2, (c) Control signal, u1 and (d) Control signal, u2.

The step response of different controllers for minimum phase characteristics is shown in Fig. 4.



(a) **(b)**

Fig. 4 Response of controllers for minimum-phase system: (a) output level, y1, (b) output level, y2

The step response of output level, y_1 is shown in Fig. 4 (a). It comprises the responses of the set point, PID controller and MPC controller. The response of the

PID controller has overshoot which is clearly shown from the figure. From the figure, it is observed that the MPC controller has no overshoot, but the PID controller has overshoot of 8.65 % for the output level, y_1 and has overshoot of 15.48 % for the output level, y_2.

The performance criterion for minimum-phase system characteristics are calculated for the two controllers are shown in Table 4. It makes a comparison of four performance indices namely, rise time (t_r), settling time (t_s), percentage overshoot and mean square error (MSE). Here, the objective is to track the output response to the desired reference model as close as possible. From the Table, it can be seen that the MPC controller has given the better performance than PID controller in terms of rise time, settling time, % overshoot and MSE. For the level output of tank, the MPC controller has 45.9 % less than that of PID controller in terms of settling time. Similarly, the MPC controller has 54.21 % less than that of PID controller in terms of settling time for the level in tank 2. The MSE of PID controller is greater than that of the MPC controller. In case of minimum-phase characteristics, the PID controller has 89.23 % and 87.83 % greater MSE than that of MPC controller for output level, y_1 and output level, y_2 respectfully.

Table 4. Comparison of performance indices of different controllers for minimum-phase characteristics

Type		PID Controller	MPC Controller
Level Output of Tank 1	Rise Time, tr (sec)	6.085	6.01
	Settling Time, ts (sec)	61.09	33.07
	% Overshoot	8.65	0
	MSE	2.014	0.217
Level Output of Tank 2	Rise Time, tr (sec)	8.81	4.04
	Settling Time, ts (sec)	58.38	26.73
	% Overshoot	15.48	0
	MSE	2.32	0.28

Conclusion

This paper presents the control of liquid level in Four Tank System (FTS) using MPC method. The FTS is analyzed and modeled using Mass balance and Bernoulli's law. The linearized model of the FTS has derived. The design procedure for MPC controller for FTS has been described. The step response of the process for minimum phase systems is studied and analyzed. Numerical simulation indicates that the MPC has more advantages than the PID controller. MPC controller exhibits stable response without overshoots for phase characteristics compared to PID controller. The MPC controller has low settling time, good robustness and also low MSE. Future work may address about the non-minimum phase characteristics of the FTS system as well as the rejection of disturbance which occurs due to parameter uncertainty.

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