Maximum Likelihood Estimator for Velocity Estimation in HetNets Based on Handoff Count

Ravi Tiwari and Siddharth Deshmukh
Department of Electronics and Communication Engineering
National Institute of Technology, Rourkela, Odisha 769008
Email: ravitiwari.tiwariravi@gmail.com, deshmukhs@nitrkl.ac.in

***Abstract*—In this paper, we propose a maximum likelihood based estimation technique for accurately estimating the velocity of mobile users in Heterogeneous networks (HetNets). In HetNets, base station (BS) density around a particular user is more compared to the traditional cellular network, resulting in frequent handoffs for a better quality of service. However, if the mobility management is not efficient, there is always a high probability of handover failures, unnecessary handoffs and call drops. The accurate estimation of the velocity of mobile users is one of the most challenging task in mobility management. The proposed velocity estimation strategy is based on handoff count which occurs during a predefined time span. Here we model densely deployed BSs using random waypoint process (RWP) and analyse the statistics of handover count as a function of velocity, BS density, and time span. Using these statistics we first derive the Cramer-Rao lower bound (CRLB) and later we determine a maximum likelihood estimator (MLE), which is an asymptotic unbiased estimator. We validate our approach by simulation which show the tight closeness of MLE asymptotic variance with CRLB. In addition, our result illustrates that velocity estimation error decreases with increase in BS density and time span of handover count measurements.**

***Keywords*—Cramer-Rao lower bound (CRLB), Maximum likelihood estimator (MLE), Heterogeneous networks (HetNets), Long term evolution (LTE), Mobile velocity estimation, Base station density, Handover count.**

I. INTRODUCTION

Over the past few decades, the communication network technology has undergone remarkable development. To meet the users demands, like seamless connectivity, high data rate, etc. The network operators are aiming to provide different inter-network services. Deploying small cells inside macro cell seems the most promising solution to support the requirement of high data rate, such cellular networks are termed as heterogeneous networks (HetNets). HetNets typically comprise of a variety of the base stations (BSs) supported by diverse radio access networks with hierarchical power levels. In HetNets, the chances of handoff (process of transferring ongoing call from one BS to another) and call drops are frequent because the BS density around the typical user is more as compared to traditional macro cell networks [1], [2]. Thus, mobility management is a crucial task in HetNets.

Mobility management provides a high quality of service by reducing the number of handoff failure and unnecessary handoff. In comparison to other parameters, user velocity, and BS density plays an important role in a successful handoff process. Handoff decision process becomes more difficult with the increase in the BS density, resulting in handoff failure. In addition, increase in user velocity results in unnecessary handovers and more frequent handoff failure. These challenges motivate the need of user-BS specific handoff optimization, which requires an accurate velocity strategy [3], [4]. Velocity estimation may also be used for mobility load balancing and energy management [5]. Smart devices are equipped with global positioning system (GPS), Wi-Fi, and other sensors which can be readily used for estimation . However, power supply in mobile devices is quite limited. In addition, these services are not ubiquitous; for example GPS signal are weak in the dense urban area while wi-fi signals are not available in rural areas. Thus, these technology features are insufficient for velocity estimation [6].

In Release-8 of long term evaluation (LTE) specification, mobility state detection has been standardized based on handover count. The handover count based user velocity estimation and approximate probability mass function (PMF) count for handover count is introduced in literature [7]. Based on our limited literature survey, there is no practical ML velocity estimator based on handover count. ML estimator is a most popular approach for obtaining practical estimator, which is also an alternative to minimum variance unbiased (MVU) estimator. It is desirable in a situation where MVU estimator does not exist, or can not be found even if it does. In this paper, we introduce novel ML estimator for velocity estimation in HetNets based on handover count. Cramer-Rao lower bound (CRLB) is used to characterize its accuracy when base station density (λ) is known. Since the service provider has the information of the number of BSs in a particular geographical area, the BSs density in that area can be calculated and broad-casted as part of system information in next generation networks. The BSs density may also be signaled in a user-specific manner to next generation user equipments which are capable of velocity estimation.

Rest of the paper is organized as follows. In section II, we described system model used for deploying small cell inside the macro cell by using stochastic geometry. Subsequently, we calculated the PMF for handover count using Gaussian distribution in section III. In section IV, we find the lower bound variance for velocity estimation. In section V, the novel ML estimator is derived, which is asymptotically unbiased and efficient. The accuracy of the proposed estimator is proved by the numerical results in section VI. Section VII carries the conclusion of the paper.

II. System model

Consider a HetNet scenario, where small cell BSs are nested inside macro cell coverage area. The respective coverage areas of small cell BSs is modeled as Poisson-Voronoi tessellation, as shown in Fig.1. We assume that the BS are randomly deployed using stochastic geometry. For simplification, we further assume in the analysis that user travels in straight path trajectory (for example, through X axis). Here we define handover count as the measure of number of handovers in a predefined time span (T). In other words, it is equal to a number of intersections between the user trajectory and base station boundaries [8], [9].

Since the location of small cell BSs and its coverage area considered as stochastic values, the number of handoff or handover count is also a random value. Here we assume that statistics of handover count does not changes with the change in the direction of mobile user. Further, computing exact density function of handover count is computationally very complex and mathematically intractable. With the help of simulation, we plot the PMF for handover count as a function of velocity and BS density. The plot of PMF against handover count for different BS density and user velocity for the fixed time slot $T = 10s$ are shown in Fig. 2 and Fig. 3.

From Fig. 2 it is clear that, when the BS density is less, the accuracy of velocity estimation based on handover count will be lower as the PMF for different velocity is overlapped. For higher BS density, the PMF is significantly separated leading to better velocity estimation, as shown in Fig. 3. It is to be noted that with an increase in user velocity, the variance of handover count increases, and thus decreases the accuracy of velocity estimation. Finally, we also observe that with the increase in BS density and velocity, the PMF of handover count resembles Gaussian distribution. These observations can also be verified in previous research work [7].

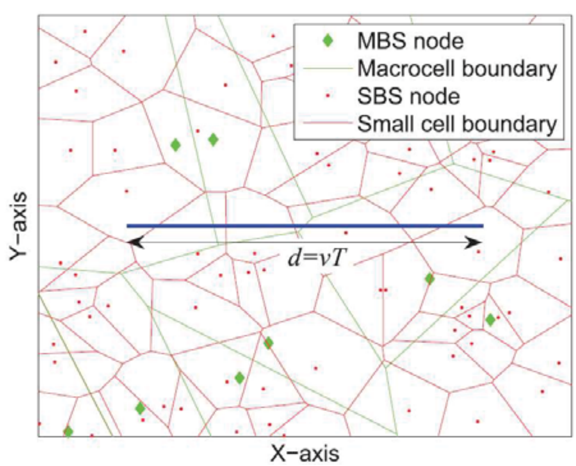

Fig. 1: Coverage of heterogeneous cellular network

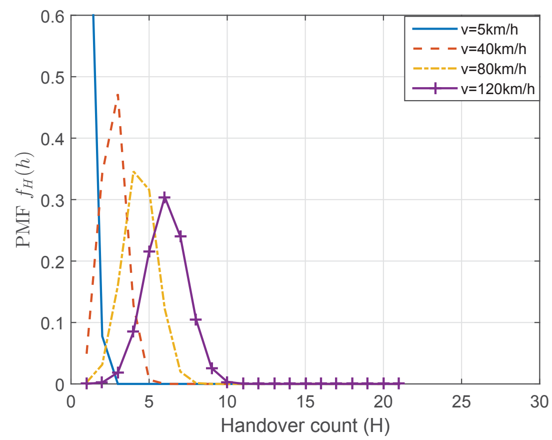

Fig. 2: PMF of handoff count for $\lambda = 100BS/km^2$

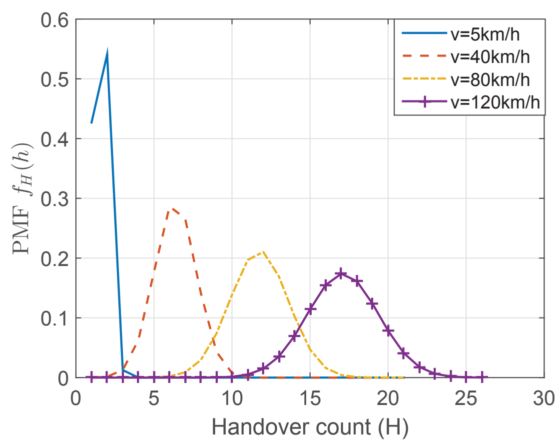

Fig. 3: PMF of handoff count for $\lambda = 1000BS/km^2$

III. PMF for handover count

As discussed in the introduction section, the objective of this work is to estimate the velocity of a user based on the number of handover in a predefined time span. However, in the literature, there is no exact expression available for PMF of handover count. Hence we approximate PMF for handoff count using Gaussian distribution this approach is validated in [8]. In approximation method, the parameters of a Gaussian distribution are calculated by using curve fitting tools in MATLAB. The expression for Gaussian probability density function (PDF) can be stated as,

$$p\,(x;\mu) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{1}$$

where, μ and σ^2 are mean and variance of random variable x respectively. The discrete and approximate version of Gaussian PMF $p(h;v)$ for handover count can be expressed as [7],

$$p\,(h;v) = \frac{1}{\sqrt{2\pi\sigma^2\,(v)}} e^{-\frac{(h-\mu(v))^2}{2\sigma^2(v)}}, \quad for\ h \in \{0,1,2,\ldots\} \tag{2}$$

The values of $\mu(v)$ and $\sigma^2(v)$ are calculated based on minimum mean square error (MSE) between approximated PMF $p(h;v)$ and simulated PMF for handover count. The approximate values of $\mu(v)$ and $\sigma^2(v)$ are given as.

$$\mu\left(v\right)=\frac{4vT\sqrt{\lambda}}{\pi} \tag{3}$$

$$\sigma^2\left(v\right)=0.07+0.41vT\sqrt{\lambda} \tag{4}$$

Here we have assumed that the distance traveled by the mobile user is $d = vT$ during fixed time slot. The approximation of variance for various distance is plotted against BS density as shown in Fig. 4. It can be noticed that with an increase in distance d, the variance of handover count also increases implying lower estimation accuracy.

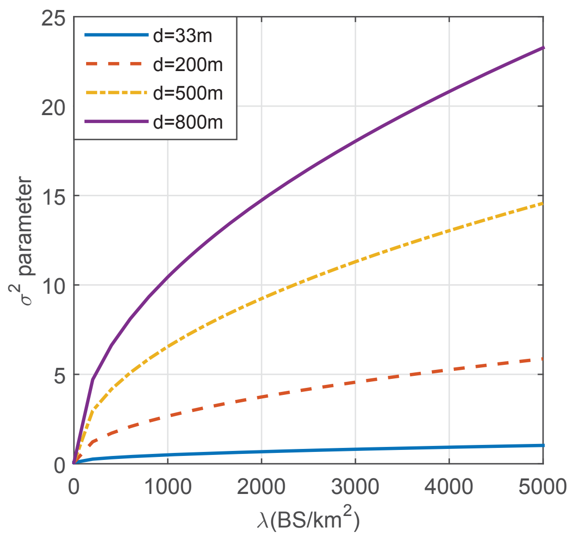

Fig. 4: Approximate variance $\sigma^2(v)$ for PMF of handover count versus BS density

IV. CRLB FOR VELOCITY ESTIMATION

The Cramer-Rao lower bound (CRLB), is the lower bound in the variance for an unbiased estimator. If an estimator on an average able to give the true value of an unknown parameter, then it is termed as an unbiased estimator. An efficient estimator are those whose variance can achieve CRLB. However, if it is not feasible to calculate an efficient estimator than the estimator which gives the lowest variance is called MVU estimator. Consider an approximated PMF for handover count as given in equation (2). The generalized expression for Fisher information $I(v)$ can be calculated by following procedure in [10] and For velocity estimation a similar expression can be stated as,

$$I\left(v\right)=\frac{1}{\sigma^2\left(v\right)}\left(\frac{\partial\mu\left(v\right)}{\partial v}\right)^2+\frac{1}{2\sigma^2(v)^2}\left(\frac{\partial\sigma^2\left(v\right)}{\partial v}\right)^2 \tag{5}$$

Substituting value of $\mu(v)$ and $\sigma^2(v)$ from equations (3) and (4) in equation (5) we get,

$$I\left(v\right)=\left(\frac{4T\sqrt{\lambda}}{\pi}\right)^2\frac{1}{\sigma^2\left(v\right)}+\left(0.41T\sqrt{\lambda}\right)^2\frac{1}{2\sigma^2(v)^2} \tag{6}$$

So CRLB for velocity estimation denoted as $CRLB(\hat{v})$ can be written as,

$$CRLB\left(\hat{v}\right)=\frac{1}{I\left(v\right)}=\frac{1}{\left(\frac{4T\sqrt{\lambda}}{\pi}\right)^2\frac{1}{\sigma^2(v)}+\left(0.41T\sqrt{\lambda}\right)^2\frac{1}{2\sigma^2(v)^2}} \tag{7}$$

A simplified form of CRLB for velocity estimator also be expressed as,

$$CRLB\left(\hat{v}\right)=\frac{1}{\left(\frac{\mu(v)}{v\sigma(v)}+\frac{1}{2}\left(\frac{0.41T\sqrt{\lambda}}{\sigma^2(v)}\right)^2\right)^2} \tag{8}$$

V. MLE FOR VELOCITY ESTIMATION

As the name suggest, a ML estimator is based on maximum likelihood principle. It is a popular approach for obtaining practical estimator and can be implemented for complicated estimation problems [10]. The performance of estimator relies on the property that ML estimator is asymptotically efficient. Once again we consider an approximated PMF value for handoff count using Gaussian distribution. In this approximation method, the parameters of Gaussian PDF are calculated using the curve fitting tool in MATLAB. Consider equation (2) for Gaussian PMF for handover count and taking logarithm both sides, we get,

$$\ln p\left(h;v\right)=-\frac{1}{2}\ln\left(2\pi\sigma^2\left(v\right)\right)\left(-\frac{\left(h-\mu\left(v\right)\right)^2}{2\sigma^2}\right) \tag{9}$$

After differentiating above equation we get,

$$\begin{aligned}\frac{\partial \ln\, p\left(h;v\right)}{\partial v}=&-\frac{1}{2\sigma^2\left(v\right)}\frac{\partial\sigma^2\left(v\right)}{\partial v}+\frac{\partial\mu\left(v\right)}{\partial v}\frac{\left(h-\mu\left(v\right)\right)}{\sigma^2\left(v\right)}\\&-\frac{1}{2}\left(h-\mu\left(v\right)\right)^2\left(-\frac{1}{\left(\sigma^2\left(v\right)\right)^2}\frac{\partial\sigma^2\left(v\right)}{\partial v}\right)\end{aligned} \tag{10}$$

For MLE, we equate the derivative of log-likelihood function to zero i.e.,

$$\frac{\partial \ln\, p\left(h;v\right)}{\partial v}=0 \tag{11}$$

By substituting the values of $\mu(v)$ and $\sigma^2(v)$ from equations (3) and (4) in equation (10) and solving for velocity v we get,

$$\begin{aligned}&v^2+\frac{v}{T\sqrt{\lambda}}\left(\frac{0.41\pi^2}{16}+\frac{2\times0.07}{0.41}\right)\\&-\frac{1}{\left(T\sqrt{\lambda}\right)^2}\frac{\pi^2}{16}h^2+\frac{0.07\pi}{2\times0.41}h-\frac{0.07\pi^2}{16}=0\end{aligned} \tag{12}$$

The above equation is a quadratic equation of the form $ax^2 + bx + c = 0$, such that,

$$a = 1$$
$$b = \frac{1}{T\sqrt{\lambda}}\left(\frac{0.41\pi^2}{16} + \frac{2\times 0.07}{0.41}\right)$$
$$c = -\frac{1}{\left(T\sqrt{\lambda}\right)^2}\left(\frac{\pi^2}{16}h^2 + \frac{0.07\pi}{2\times 0.41}h - \frac{0.07\pi^2}{16}\right)$$

Now solving for the roots of $\hat{v}$, by using the following equation

$$\hat{v} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solving for $\hat{v}$, we get

$$\hat{v} = -\frac{1}{2T\sqrt{\lambda}}\left(\frac{0.41\pi^2}{16} + \frac{2\times 0.07}{0.41}\right) \pm \frac{\pi}{4T\sqrt{\lambda}}\sqrt{\begin{array}{c} h^2 + \frac{8\times 0.07h}{0.41\pi} + \\ \frac{4}{\pi^2}\left(\frac{4\times 0.07^2}{0.41^2} + \frac{0.41^2\pi^4}{16^2}\right)\end{array}} \tag{13}$$

Finally the ML estimator for velocity estimation based on handover count, which takes N handover count sample as the input can be expressed as,

$$\hat{v} = \frac{\pi}{4T\sqrt{\lambda}}\Bigg(-\frac{0.41\pi}{8} - \frac{4\times 0.07}{0.41\pi} + \sqrt{\begin{array}{c}\frac{1}{N}\sum_{n=0}^{N-1} h_n^2 + \frac{8\times 0.07}{0.41\pi N}\sum_{n=0}^{N-1} h_n \\ + \frac{16\times 0.07^2}{0.41^2\pi^2} + \frac{0.41^2\pi^2}{4\times 16}\end{array}}\Bigg) \tag{14}$$

To determine the biasness of the estimator, we take expectation of estimator i.e. $E(\hat{v})$,

$$E\left[\hat{v}\right] = \frac{\pi}{4T\sqrt{\lambda}}\Bigg(-\frac{0.41\pi}{8} - \frac{4\times 0.07}{0.41\pi} + \sqrt{\begin{array}{c}\frac{1}{N}\sum_{n=0}^{N-1} E\left[h_n^2\right] + \frac{8\times 0.07}{0.41\pi N}\sum_{n=0}^{N-1} E\left[h_n\right] \\ + \frac{16\times 0.07^2}{0.41^2\pi^2} + \frac{0.41^2\pi^2}{4\times 16}\end{array}}\Bigg) \tag{15}$$

On further simplification and substituting $E(h) = \mu(v)$ and $E(h^2) = \mu^2(v) + \sigma^2(v)$ we get,

$$E\left[\hat{v}\right] = \frac{\pi}{4T\sqrt{\lambda}}\Bigg(-\frac{0.41\pi}{8} - \frac{4\times 0.07}{0.41\pi} + \frac{4}{\pi}\sqrt{\begin{array}{c}\left(vT\sqrt{\lambda}\right)^2 + vT\sqrt{\lambda}\left(\frac{0.41\pi^2}{16} + \frac{2\times 0.07}{0.41}\right) \\ + \frac{\pi^2}{16}\left(0.07 + \frac{0.41^2\pi^2}{4\times 16} + \frac{16\times 0.07^2}{0.41^2\pi^2}\right)\end{array}}\Bigg) \tag{16}$$

Thus, ML estimator expressed in equation (14) is an asymptotically unbiased estimator as $E\left[\hat{v}\right] \to v$ for $N \to \infty$. The proposed estimator for velocity estimation is a nonlinear function of h, by linearizing it, we get the variance of ML estimator as,

$$var_{MLE}\left(\hat{v}\right) \approx \left(\frac{v\sigma\left(v\right)}{\mu\left(v\right)}\right)^2 \tag{17}$$

ML estimator expressed in equation (14) is an asymptotically efficient as for $N \to \infty$ and high BS density (λ), variance approaches to CRLB.

VI. Numerical results

In this section, we plot the CRLB and proposed ML estimator variance for different BS density with respect to the user velocity and time span used for handoff count measurement.

A. *Results for CRLB*

The CRLB plot for velocity estimation with the variation of user velocity for various BS density $(\lambda = 100, 500, 1000, 5000)$ is shown in Fig. 5. It can be observed that CRLB increases with increase in velocity and decreases with increase in BS density. It can also be verified from Fig. 2 and Fig. 3 where, with $\lambda = 100 BSs/km^2$, the peak of PMF are overlapped for different values of velocity, this results in higher estimation error. For the case where BS density is more say $\lambda = 1000 BSs/km^2$, the peaks of PMFs for different velocity are distinct, resulting in lower CRLB. Fig. 6 show the variation of CRLB with respect to time span used for handover count. When time span $T = 30s$, velocity $v = 50km/h$ and $\lambda = 500 BSs/km^2$ the standard deviation is $8km/h$ and for the similar case, when time span $T = 60s$ then standard deviation found to be $4km/h$. It indicates that accuracy of estimator increases with increase in BS density. It can also be clear that CRLB decreases with increase in time span, so the accuracy of estimator increases by taking more time span for handover count.

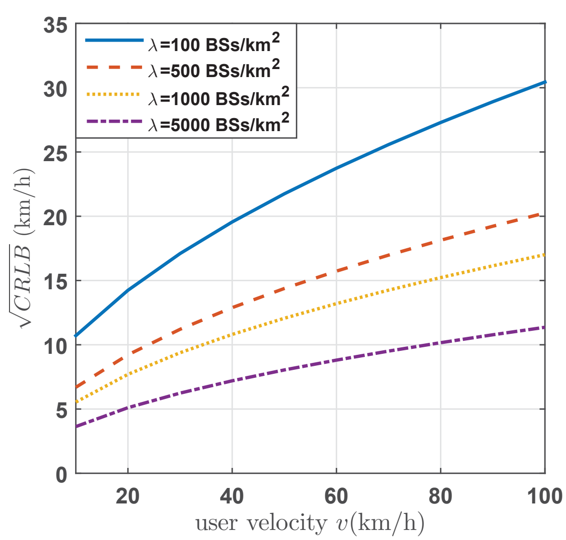

Fig. 5: CRLB versus user velocity for various BS density, $(T = 10s)$

B. *Variance of the ML based velocity estimator*

The asymptotic variance of the ML estimator is given in equation (17). The performance metric is square root of variance which is equivalent to root mean square error (RMSE). Fig. 7 shows the variation of RMSE with respect to

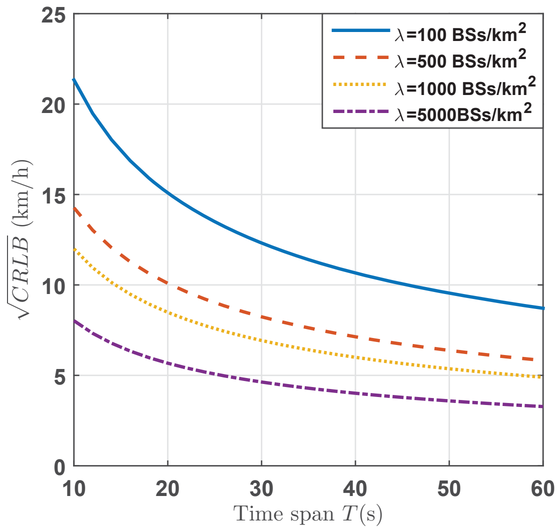

Fig. 6: CRLB versus time span for various BS density, ($v = 50km/h$)

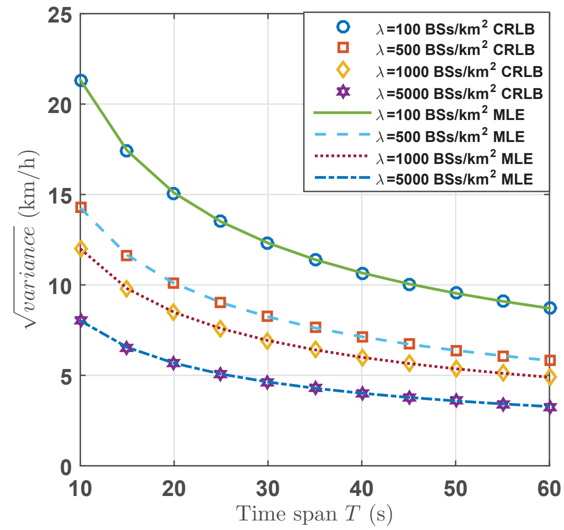

Fig. 8: RMSE versus time span for various BS density, ($v = 50km/h$)

user velocity for various BS density. It is observed from the plot that the variance of ML estimator is tightly matching with CRLB. We can also observe that as the BS density increases, the variance of ML estimator matches more accurately with CRLB. For example with user velocity of $80km/h$ and BS density $\lambda = 500$, the RMSE of ML estimator is just $18km/h$ which is same as CRLB.

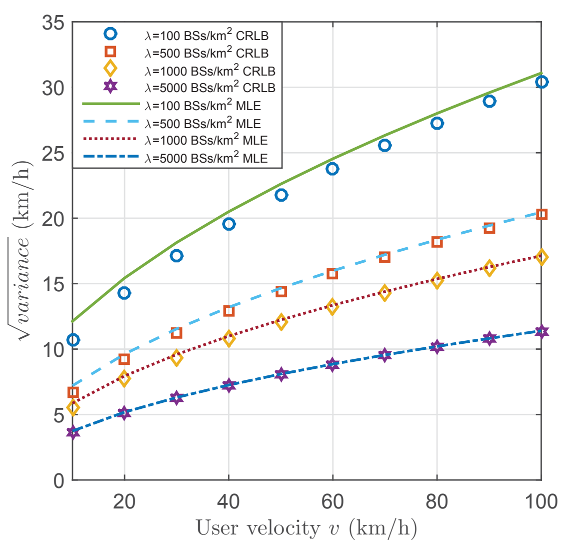

Fig. 7: RMSE versus velocity for various BS density, ($T = 10s$)

We have used smaller measurement time span $T = 10s$ so that estimator can provide fast results. The time span for handover count depends on service provider strategy. Therefore the variation of RMSE for ML estimator with time span is also investigated and is shown in Fig. 8. The variance of ML estimator decreases with increase in the time span of handover count measurement. Here we can also observe that longer time span increases the accuracy of ML velocity estimation. However, increase in time span slow downs the response of the estimator. So there exist a the trade-off between accuracy and rapidness in velocity estimation .

VII. Conclusion

In this paper, we have proposed ML estimator for velocity estimation based on handover count. Since computation of exact PMF of handover count is mathematically intractable, we approximate PMF of handover count using Gaussian distribution. Next, we derived the CRLB for velocity estimation and compared it with the variance of proposed ML estimator. The results show the tight closeness of ML estimator asymptotic variance with CRLB. Further, we have observed that the variance of ML velocity estimator decreases with increases in time span for specified handover count. Thus, there exists the trade-off between the accuracy and rapidness in velocity measurement. In addition, the variance of velocity estimator decreases with increase in BS density, which facilitate more accurate velocity estimation in the hyper-dense network.

References

[1] H. S. Dhillon, R. K. Ganti, F. Baccelli, and J. G. Andrews, "Modeling and analysis of k-tier downlink heterogeneous cellular networks," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 3, pp. 550–560, 2012.

[2] J. Niu, T. Su, G. Y. Li, D. Lee, and Y. Fu, "Joint transmission mode selection and scheduling in lte downlink mimo systems," *IEEE Wireless Communications Letters*, vol. 3, no. 2, pp. 173–176, 2014.

[3] D. Lopez-Perez, I. Guvenc, and X. Chu, "Mobility management challenges in 3gpp heterogeneous networks," *IEEE Communications Magazine*, vol. 50, no. 12, pp. 70–78, 2012.

[4] X. Yan, Y. A. Şekercioğlu, and S. Narayanan, "A survey of vertical handover decision algorithms in fourth generation heterogeneous wireless networks," *Computer networks*, vol. 54, no. 11, pp. 1848–1863, 2010.

[5] X. Lin, R. K. Ganti, P. J. Fleming, and J. G. Andrews, “Towards understanding the fundamentals of mobility in cellular networks,” *IEEE Transactions on Wireless Communications*, vol. 12, no. 4, pp. 1686–1698, 2013.

[6] C.-H. Chen, C.-A. Lee, and C.-C. Lo, “Vehicle localization and velocity estimation based on mobile phone sensing,” *IEEE Access*, vol. 4, pp. 803–817, 2016.

[7] A. Merwaday and . Gven, “Handover count based velocity estimation and mobility state detection in dense hetnets,” *IEEE Transactions on Wireless Communications*, vol. 15, no. 7, pp. 4673–4688, July 2016.

[8] A. Merwaday and I. Guvenc, “Handover count based ue velocity estimation in hyper-dense heterogeneous wireless networks,” in *2015 IEEE Globecom Workshops (GC Wkshps)*. IEEE, 2015, pp. 1–6.

[9] J. Moller, *Lectures on random Voronoi tessellations*. Springer Science & Business Media, 2012, vol. 87.

[10] S. M. Kay, “Fundamentals of statistical signal processing, volume i: estimation theory,” 1993.