Performance Analysis of Dual-Hop DF Relaying Systems in the Combined Presence of CEE and RFI

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***Abstract*—This paper presents the performance analysis of decode-and-forward (DF) relaying system in presence of both channel estimation error (CEE) and radio frequency impairments (RFI). First, the end-to-end signal-to-noise-plus-distortion-and-error ratio (SNDER) expression is derived, followed by an exact closed-form outage probability (OP) expression for Nakagami-m fading channel. As a special case, the OP analysis for Rayleigh fading channel is also provided. From the derived expressions, the relation amongst CEE and RFI is analyzed. For a complete study, the high SNR analysis of the derived equations is also carried out. The analytical results have been verified using the Monte Carlo simulations. Finally, intriguing results are presented with the help of plots shown in the numerical analysis.**

***Index Terms*—Decode-and-forward, channel estimation error (CEE), radio frequency impairments (RFI), outage probability (OP), high signal-to-noise ratio (SNR) analysis.**

I. Introduction

Wireless relaying systems are now becoming a trending topic in the research community due to their immense applications in practical scenarios. They are used to extend the coverage area, improve link reliability and reduce power consumption in order to efficiently combat the issue of fading impairments. Due to the limited availability of power and bandwidth, the idea of relaying is useful for providing a better quality-of-service at the cell boundary. The two most marked methods of wireless relaying are amplify-and-forward (AF) relaying and decode-and-forward (DF) relaying. In AF relaying scheme, the relay node receives the signal, amplifies it and re-transmits it to the destination node, whereas, in the case of DF relaying, the received information is first fully decoded at the relay node and then re-encoded before it is sent to the destination node [1], [2].

Channel state information (CSI) is extremely crucial in order to completely reap the benefits of DF scheme. DF relaying scheme assuming perfect CSI is investigated in [3] and [4]. But, in practice, according to [5], [6] channel estimation performed by pilots is mostly inaccurate due to the limited power of pilot symbols and channel uncertainties. This often leads to erroneous retrieval of information and thus gives rise to the problem of channel estimation error (CEE). The quality of channel estimates inescapably affects the overall performance of a relay assisted communication system and tends to become the performance hindering factor [6]. Further, most of the existing works which take into account imperfect CSI, consider ideal radio frequency hardware. However, this assumption is too optimistic for practical relaying systems, as the transceiver front-end in these systems suffers from several types of radio frequency impairments (RFI) such as, in-phase quadrature-phase imbalance (IQI), high power amplifier (HPA) non-linearities and phase noise (PN) in the oscillator [7]. These impairments create mismatch between the intended transmit signal and actual emitted signal along with distortions of received signal during the reception process [8].

There are two phases involved in the training based practical relaying systems: 1) pilot transmission phase and 2) data transmission phase [9]. RFI has been shown to affect the accuracy of the channel estimation in [10], [11] which invariably leads to pilot contamination and it impacts the performance of training based DF relaying system. The authors in [12], [13] characterized and verified experimentally that the distortion caused by these impairments behave as additive and independent Gaussian noise. This Gaussian behavior can be understood by the central limit theorem, where the distortions from many independent and different sources add up together. Recent work [14], analyzes the end-to-end performance of dual-hop DF relaying system in the presence of two practical problems of RFI and co-channel interference (CCI). The impact of RFI is much more pronounced in the case of high rate systems especially those operating with inexpensive hardware [12]. When we are concerned about the practicality of relaying systems, [15] shows effect of RFI on CEE in the case of fixed-gain AF relaying system.

Based on the above mentioned papers, we find that all the studies related to this topic concentrated on the effect of either RFI or CEE alone. But there has been no study for dual hop DF systems, which demonstrates the combined effect of both these problems. Now, we know that, both RFI and CEE are of grave concern in practical relaying scenarios and neglecting the effect of either of these leads to an incomplete study. CEE with no RFI or RFI with no CEE does not give the real picture when it comes to the performance analysis of a practical DF relay system. The combined study also becomes more important after knowing the fact that RFI not only distorts the data signals but also affects the CEE. Hence, in this paper we analyze the aggregated impact of hardware impairments and imperfect channel estimates on the dual-hop DF relaying. Here, we have focused our attention on exact as well as asymptotic outage probability analysis in Nakagami-m fading scenario.

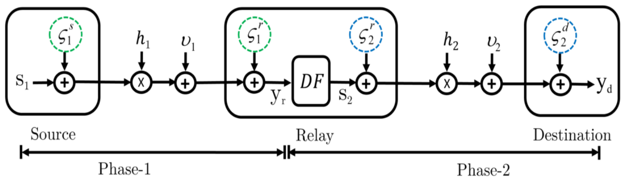


Fig. 1. Dual-hop DF relaying system with hardware impairments and CEE.

II. System and Channel Model

As illustrated in Fig. 1, we consider a dual-hop DF relaying system, where a source transmits the information to the destination with the aid of a relay. Direct link between source and destination is not present due to deep shadowing or heavy blockage. All the nodes are equipped with single antenna and follow half duplex constraints. In contrary to the assumptions of ideal hardware, here all the communicating nodes are considered to have RFI. The received information at the relay, y_r is given by,

$$y_r = h_1 \left(s_1 + \varsigma_1^s\right) + \upsilon_1 + \varsigma_1^r, \tag{1}$$

and the received information at the destination y_d is,

$$y_d = h_2 \left(s_2 + \varsigma_2^r\right) + \upsilon_2 + \varsigma_2^d, \tag{2}$$

where, h_1 and h_2 represent the actual channel fading coefficients of hop-1 and hop-2 respectively. s_1 and s_2 denote the information signal transmitted from the source and the processed information signal transmitted by the relay respectively, with average power $P_i = \mathrm{E}_{s_i}\{|s_i|^2\}$. In addition, $\upsilon_i \sim \mathbb{CN}(0, N_i)$ for $i = 1, 2$ is the complex Gaussian distributed receiver noise at the relay and destination nodes, respectively. The distortion noise[1] present at the source, destination, relay front-end (while receiving) and relay front-end (while transmitting) are denoted by ς_1^s, ς_2^d, ς_1^r and ς_2^r respectively. Their distributions are taken as, $\varsigma_1^s \sim \mathbb{CN}(0,\ (\mu_1^s)^2 P_1)$, $\varsigma_2^d \sim \mathbb{CN}(0,\ (\mu_2^d)^2 P_2 |h_2|^2)$, $\varsigma_1^r \sim \mathbb{CN}(0,\ (\mu_1^r)^2 P_1 |h_1|^2)$, $\varsigma_2^r \sim \mathbb{CN}(0,\ (\mu_2^r)^2 P_2)$. To reduce the complexity, we denote aggregated distortion noise of first hop as $\varsigma_1 \triangleq (h_1 \varsigma_1^s + \varsigma_1^r) \sim \mathbb{CN}(0,\ \mu_1^2 P_1 \left|h_1\right|^2)$ and aggregated distortion noise of second hop as $\varsigma_2 \triangleq (\varsigma_2^r + \varsigma_2^d / h_2) \sim \mathbb{CN}\left(0,\ \mu_2^2 P_2\right)$ having $\mu_1^2 = (\mu_1^s)^2 + (\mu_1^r)^2$ and $\mu_2^2 = (\mu_2^r)^2 + (\mu_2^d)^2$. Here, the design parameters, $\mu_1^s, \mu_2^d, \mu_1^r, \mu_2^r \geq 0$ signify the degree of RFI at the source, destination, relay front-end (while receiving) and relay front-end (while transmitting) respectively. These parameters are also known as error vector magnitudes (EVM), and are defined as the ratio of the magnitude of distortion to magnitude of the average signal. In this analysis, we have assumed that these parameters are constant because they do not cross the dynamic range of the hardware. On violating this, their value increases rapidly ([7] and references therein). The channel fading coefficient $|h_i|$ $(i = 1, 2)$ are modeled as independent but non-identically distributed Nakagami-m random variates. and hence, $\phi_i = |h_i|^2$ $(i = 1, 2)$ follows Gamma distribution. The estimated channel fading coefficients which follows the same distribution as $|h_i|$, is denoted by $|\tilde{h}_i|$ $(i = 1, 2)$. Assuming h_i and $\tilde{h}_i$ to be jointly ergodic processes we have,

$$h_i = \tilde{h}_i + e_{h_i}, \tag{3}$$

where, e_{h_i} denotes the estimation error which is orthogonal to the channel estimate and is assumed to have a zero mean complex Gaussian distribution [16], [17] having variance,

$$\sigma_{i_{ni}}^2 = \Omega_i - \mathrm{E}\{|\tilde{h}_i|^2\} = 1/(T_{i_p} \bar{\Upsilon}_{i_{nip}} + 1), \tag{4}$$

where, $\Omega_i = \mathrm{E}\{|h_i|^2\}$, T_{i_p} is the length of training symbols and $\bar{\Upsilon}_{i_{nip}} = \mathrm{E}\{\Upsilon_{i_{nip}}\} = P_{i_p} \Omega_i / N_i$ is the average SNR of training symbols for the i^{th} hop in the presence of RFI. P_{i_p} denotes the power of the pilot symbols and N_i denotes the noise variance of the i^{th} hop. Since the training symbols are also affected by RFI, we can re-write $\bar{\Upsilon}_{i_{nip}}$ in terms of average SNR of the training symbols of systems having ideal hardware $(\bar{\Upsilon}_{i_{idp}})$ as, $\bar{\Upsilon}_{i_{nip}} = \bar{\Upsilon}_{i_{idp}} / (\bar{\Upsilon}_{i_{idp}} \mu_{i_p}^2 + 1)$, where, μ_{i_p} signify the EVM parameters affecting the training symbols of i^{th} hop. Replacing $\bar{\Upsilon}_{i_{nip}}$ in (4) we obtain,

$$\sigma_{i_{ni}}^2 \simeq (1 + \mu_{i_p}^2 \bar{\Upsilon}_{i_{idp}}) / (1 + (T_{i_p} + \mu_{i_p}^2) \bar{\Upsilon}_{i_{idp}}), \tag{5}$$

where, $\sigma_{i_{ni}}^2$ reflects the quality of channel estimation and is also known as minimum mean square error (MMSE).

III. Performance Analysis

A. Instantaneous End-to-End SNDER

The ultimate aim of a communication system is to ensure that the destination receives the exact information sent by the source. The signal s_2 at front-end of the relay (while transmitting) is the re-encoded version of the signal y_r which itself is a distorted version of the original signal s_1. The instantaneous SNDER is the minimum of the two SNDERs: i) S-R link and ii) R-D link. We get the end-to-end SNDER of DF relaying system with RFI and CEE as,

$$\tilde{\Upsilon}_{e2e} = \min(\tilde{\Upsilon}_1,\ \tilde{\Upsilon}_2), \tag{6}$$

where, $\tilde{\Upsilon}_1 = P_1 \tilde{\phi}_1 / (P_1 \tilde{\phi}_1 \mu_1^2 + P_1 \sigma_{1_{ni}}^2 (1 + \mu_1^2) + N_1)$ and $\tilde{\Upsilon}_2 = P_2 \tilde{\phi}_2 / (P_2 \tilde{\phi}_2 \mu_2^2 + P_2 \sigma_{2_{ni}}^2 (1 + \mu_2^2) + N_2)$ represent the individual SNDERs for hop-1 and hop-2 respectively.

B. Outage Probability

In this section, we derive the exact closed-form expression of OP for a DF relaying system suffering from CEE as well as RFI. The OP is defined as the probability that the instantaneous end-to-end SNDER, $(\tilde{\Upsilon}_{e2e})$ falls below a particular acceptable threshold x. Mathematically, we can write,

$$P_{out}(x) \triangleq \Pr\{\tilde{\Upsilon}_{e2e} \leq x\} = F_{\tilde{\Upsilon}_{e2e}}(x). \tag{7}$$

Theorem 1: Assuming $\tilde{\phi}_1$, $\tilde{\phi}_2$ to be non-negative independent random variables, where, $\tilde{\phi}_i \sim Gamma\left(\alpha_i, \beta_i\right)$

[1]The distortion noises and AWGN receiver noise are different because distortion noise is proportional to the transmitted signal power whereas the AWGN noise has a constant envelope.

with integer shape parameter $\alpha_i \geq 1$ and real valued scale parameter $\beta_i > 0$, the expression for OP comes out to be,

$$F_{\tilde{\Upsilon}_{e2e}}(x) = 1 - \prod_{i=1}^{2}\left(\exp\left(-\frac{\left(\sigma_{i_{ni}}^2\left(1+\mu_i^2\right)+\frac{N_i}{P_i}\right)x}{\beta_i\left(1-\mu_i^2 x\right)}\right) \times \sum_{j=0}^{\alpha_i-1}\frac{1}{j!}\left(\frac{\left(\sigma_{i_{ni}}^2\left(1+\mu_i^2\right)+\frac{N_i}{P_i}\right)x}{\beta_i\left(1-\mu_i^2 x\right)}\right)^j\right), \quad (8)$$

for $x < \frac{1}{\delta}$ where, $\delta \triangleq \max(\mu_1^2, \mu_2^2)$ and $F_{\tilde{\Upsilon}_{e2e}}(x) = 1$ for $x \geq \frac{1}{\delta}$.

Proof: We can see from (7) that $P_{out}(x)$ depends on $\tilde{\Upsilon}_{e2e}$. Now, from (6), we further note that $\tilde{\Upsilon}_{e2e}$ has a functional relationship with the two independent random variables namely, $\tilde{\phi}_1$ and $\tilde{\phi}_2$. Hence,

$$F_{\tilde{\Upsilon}_{e2e}}(x) = 1 - \prod_{i=1}^{2}\Big(1 - F_{\tilde{\phi}_i}(x)\Big), \quad (9)$$

where,

$$F_{\tilde{\phi}_i}(x) = \begin{cases} \Pr\left\{\tilde{\phi}_i \leq \frac{\left(\sigma_{i_{ni}}^2\left(1+{\mu_i}^2\right)+\frac{N_i}{P_i}\right)x}{\left(1-{\mu_i}^2 x\right)}\right\}, & x < 1/\delta \\ 1, & x \geq 1/\delta. \end{cases} \quad (10)$$

Therefore by substituting (10) in (9) we get,

$$F_{\tilde{\Upsilon}_{e2e}}(x) = \begin{cases} 1 - \prod\limits_{i=1}^{2}\left(1 - \Pr\left\{\tilde{\phi}_i \leq \frac{\left(\sigma_{i_{ni}}^2\left(1+{\mu_i}^2\right)+\frac{N_i}{P_i}\right)x}{\left(1-{\mu_i}^2 x\right)}\right\}\right), & x < 1/\delta \\ 1, & x \geq 1/\delta. \end{cases} \quad (11)$$

Using the cumulative distribution function (CDF) of Gamma distributed random variable $\tilde{\phi}_i$ in (11), we get (8).

Theorem 1 provides a tractable and generalized closed-form OP expression for DF relaying system that handles both CEE and RFI. The derived OP expression is a generalization of [8, Eq. (30)], where only hardware impairment is considered ignoring the effects of CEE. We can verify this by substituting $\sigma_{1_{ni}}^2 = \sigma_{2_{ni}}^2 = 0$ in (8). Theorem 1 can easily be extended for a multi-hop DF relaying systems (with Z hops) by simply varying i from 1 to Z instead of varying it from 1 to 2.

Corollary 1: *OP for this system with CEE but in the absence of RFI:* By putting $\mu_i = 0$ in (8), the required expression is obtained as follows,

$$F_{\tilde{\Upsilon}_{e2e}}(x) = 1 - \prod_{i=1}^{2}\left(\exp\left(-\frac{\left(\sigma_{i_{id}}^2+\frac{N_i}{P_i}\right)x}{\beta_i}\right)\sum_{j=0}^{\alpha_i-1}\frac{1}{j!}\left(\frac{\left(\sigma_{i_{id}}^2+\frac{N_i}{P_i}\right)x}{\beta_i}\right)^j\right), \quad (12)$$

for $0 \leq x < \infty$.

The above written corollary provides the closed-form expression for outage performance considering only CEE, where the RF hardware is assumed to be ideal. Here, channel estimation accuracy parameter $\sigma_{i_{id}}^2 = 1/(T_{i_p}\bar{\Upsilon}_{i_{id_p}} + 1)$ is the inverse function of pilot symbol power, which is in contrast to non-ideal hardware case. Now, due to the limited power pilot symbols, $\sigma_{i_{id}}^2$ will never be zero even when the data signal power is very high. This in turn leads to occurrence of outage floor at high average SNR values.

Corollary 2: *OP for this system over Rayleigh fading channel:* After substituting $\alpha_i = 1$, $\beta_i = \Omega_i$, in (8) and further simplification, we get the required expression as,

$$F_{\tilde{\Upsilon}_{e2e}}(x) = 1 - \prod_{i=1}^{2}\exp\left(-\frac{\left(\sigma_{i_{ni}}^2\left(1+\mu_i^2\right)+\frac{N_i}{P_i}\right)x}{\Omega_i\left(1-\mu_i^2 x\right)}\right), \quad (13)$$

for $x < \frac{1}{\delta}$ where, $\delta \triangleq \max\left(\mu_1^2, \mu_2^2\right)$ and $F_{\tilde{\Upsilon}_{e2e}}(x) = 1$ for $x \geq \frac{1}{\delta}$.

This corollary finds immense application in the analysis of practical DF relaying systems with CEE and RFI in rich scattering environment, where Rayleigh fading distribution is the appropriate model. This corollary generalizes [8, Eq. (31)] which has been obtained for Rayleigh fading channel by taking perfect channel estimation. This can also be expressed as a generalization of the classical result presented in [18, Eq. (28)], which finds OP expression of DF relaying system assuming perfect channel estimation and ideal hardware.

C. High SNR Analysis

In order to gain deeper insights on the performance of DF relaying system under the influence of CEE and RFI, we perform the high SNR analysis. Here we have considered the transmitted power $P_1 = P_2 = P$ to be very large. Under this assumption, it is found that the SNDER expression (6) becomes,

$$\tilde{\Upsilon}_{\substack{e2e \\ P_1, P_2 \to \infty}} = \min\left(\frac{\tilde{\phi}_1}{\tilde{\phi}_1\mu_1^2+\sigma_{1_{ni}}^2\left(1+\mu_1^2\right)}, \quad \frac{\tilde{\phi}_2}{\tilde{\phi}_2\mu_2^2+\sigma_{2_{ni}}^2\left(1+\mu_2^2\right)}\right). \quad (14)$$

During this analysis, we also find that $\sigma_{i_{ni}}^2$ reduces to,

$$^{\infty}\sigma_{i_{ni}}^2 = \underset{P_{i_{id_p}} \to \infty}{\sigma_{i_{ni}}^2} \simeq \mu_{i_p}^2\Big/\Big(T_{i_p} + \mu_{i_p}^2\Big) \approx \mu_{i_p}^2\Big/T_{i_p}. \quad (15)$$

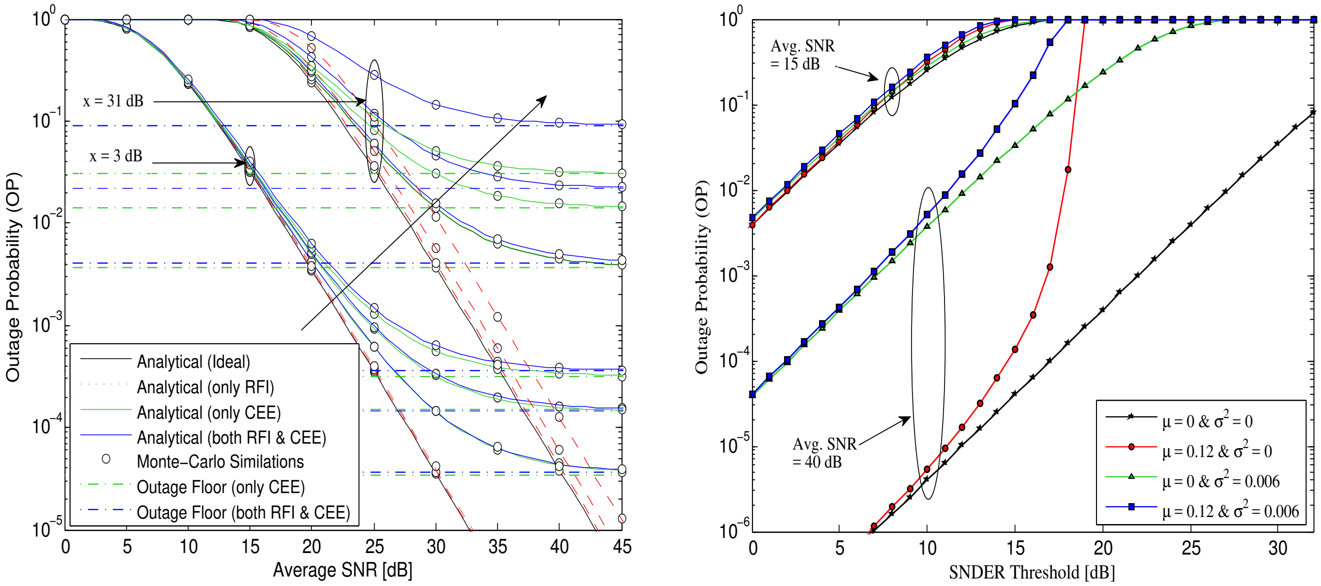
In the case of ideal hardware, the error variance becomes zero at very high power. But, expression (15) shows that the value of error variance is non-zero even at very high transmit power and is directly influenced by the degree of hardware impairment. Another inference which can be drawn from the expression (15) is that, the number of training symbols cannot be decreased below a particular level, as it leads to unavoidable increase in the error variance.

Theorem 2: OP for DF relaying systems having CEE and RFI at high SNR: It can be obtained by putting $P_1 = P_2 = P \to \infty$ in (8) as,

$$F_{\tilde{\Upsilon}_{e2e}}^{\infty}(x) = 1 - \prod_{i=1}^{2}\left(\exp\left(-\frac{{}^{\infty}\sigma_{i_{ni}}^2\left(1+\mu_i^2\right)x}{\beta_i\left(1-\mu_i^2 x\right)}\right)\sum_{j=0}^{\alpha_i-1}\frac{1}{j!}\left(\frac{{}^{\infty}\sigma_{i_{ni}}^2\left(1+\mu_i^2\right)x}{\beta_i\left(1-\mu_i^2 x\right)}\right)^j\right) \quad (16)$$

for $x < \frac{1}{\delta}$ where $\delta \triangleq \max(\mu_1^2, \mu_2^2)$ and $F_{\tilde{\Upsilon}_{e2e}}^{\infty}(x) = 1$ for $x \geq \frac{1}{\delta}$.

Proof: Similar to Theorem 1, by using (14) instead of (6).



(a)

(b)

Fig. 2. (a) Comparison of OP vs. average SNR (in dB) for different cases: (i) varying $\mu \in \{0,\, 0.04,\, 0.08, 0.12\}$ and $\sigma^2 = 0$ (ii) varying $\sigma^2 \in \{0,\, 0.002,\, 0.004, 0.006\}$ and $\mu = 0$ and (iii) varying μ and varying σ^2, $\left(\mu,\ \sigma^2\right) \in \{(0,\, 0)\,,\, (0.04,\, 0.002)\,,\, (0.08,\, 0.004)\,,\, (0.12,\, 0.006)\}$; (b) OP vs. SNDER threshold (in dB) at average SNR values of 15 dB and 40 dB.

Theorem 2 gives a generalized OP expression for dual-hop DF relaying at high SNR with Nakagami-m_i fading channel. From this theorem, we can see that the outage floor obtained at high SNR depends not only on CEE but also on RFI. The studies in paper [5], [6] did not consider RFI in their analysis. Hence, this paper brings out the indirect contribution of RFI on the outage floor through its effect on CEE.

Corollary 3: *High SNR analysis in case of Rayleigh Fading:* By putting $\alpha_i = 1, \beta_i = \Omega_i$ in (16) and simplifying, the OP for high SNR in the case of Rayleigh fading channel is found to be,

$$F^{\infty}_{\tilde{\Upsilon}_{e2e}}(x) = 1 - \prod_{i=1}^{2} \exp\left(-\frac{{}^{\infty}\sigma^2_{i_{ni}}\left(1+\mu_i^2\right)x}{\Omega_i\left(1-\mu_i^2 x\right)}\right) \tag{17}$$

for $x < \frac{1}{\delta}$ where $\delta \triangleq \max(\mu_1^2,\, \mu_2^2)$ and $F^{\infty}_{\tilde{\Upsilon}_{e2e}}(x) = 1$ for $x \geq \frac{1}{\delta}$.

Here, the combined effect of CEE and RFI are responsible for creating an outage floor in the Rayleigh fading scenario.

IV. Numerical and Simulation Results

In this section, set of numerical results showing the performance of DF relaying model in the presence of CEE and RFI are presented. To validate the theoretical results, Monte-Carlo simulation is performed. Here, we assumed that $\mu_1 = \mu_2 = \mu$ and $\sigma^2_{1_{ni}} = \sigma^2_{2_{ni}} = \sigma^2$. In Fig. 2(a) and 2(b), the values of shape and scale parameters are taken as $\alpha_1 = \alpha_2 = \alpha = 2$ and $\beta_1 = \beta_2 = \beta = 1$ respectively.

Fig. 2(a) depicts the OP vs. average SNR values in accordance to Theorem 1. It is to be noted that a relay assisted communication system uses two time slots, therefore, we write $x = 2^{2R} - 1$ where, R represents transmission rate and x represents SNDER threshold. Corresponding to two different SNDER threshold values $x = 3$ dB and $x = 31$ dB representing transmission rates of 1 and 2.5 bits/sec/Hz respectively, we get two sets of plots. It is clearly evident that the curves with CEE as well as those with both CEE and RFI have outage floors at high SNR region. An important notable observation is that, the curve with the combined effect of CEE and RFI has a higher outage floor than the one with only CEE. We know that RFI alone cannot produce any outage floor, yet it increases the effect of CEE in terms of elevating the outage floor and hence has a significant impact on CEE. This can be attributed to the fact that at high power regime, ${}^{\infty}\sigma^2_{i_{ni}} \approx \mu^2_{i_p}/T_{i_p}$ as found in (15). Another point worth noting is that the elevation of outage floor is much greater in case of high rate systems. This shows that high rate systems are more vulnerable to the combined presence of RFI and CEE.

Fig. 2(b) shows OP against SNDER threshold for two different average SNR values of 15 dB and 40 dB. Initially the curve with CEE (only) dominates the curve with RFI (only). But, as the SNDER threshold increases, the RFI starts to dominate over CEE after a particular SNDER threshold for both the average SNR values. Further, we observe that there is an SNDER ceiling for the OP curves, i.e, for a particular average SNR value, there is a certain SNDER threshold beyond which the OP value will always remain 1.

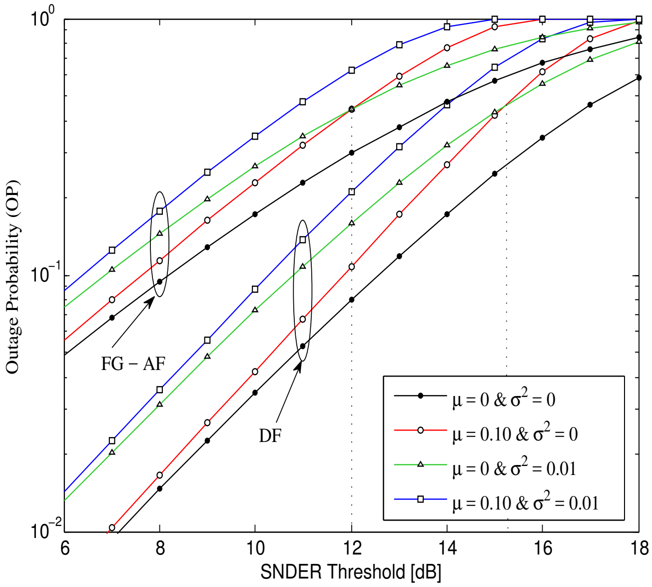


Fig. 3. Outage Probabilities for (a) fixed-gain AF relaying and (b) DF relaying for average SNR of 20 dB.

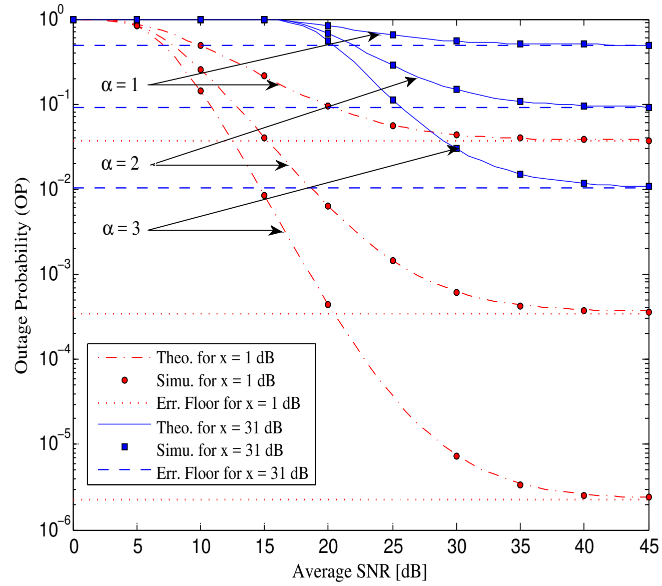


Fig. 4. OP vs. average SNR for different values of shape parameter α taking $\mu = 0.12$ and $\sigma^2 = 0.006$.

Fig. 3 displays and compares the performance of two relaying methods: (a) fixed-gain (FG) AF relaying [15] and (b) DF relaying. For both the scenarios, the average SNR is taken to be 20 dB, $\mu = 0.10$ and $\sigma^2 = 0.01$. The first thing that the comparison reveals is that, DF relaying performs much better than FG AF relaying in the combined presence of RFI and CEE. Another observation is that, in case of DF relaying, CEE dominates the RFI for a larger range of transmission rates than it does for FG AF relaying. It is evident from the crossing points of CEE and RFI curves in both sets of plots. With the above mentioned parameter values, we can see that for FG AF relaying, only CEE curve dominates the only RFI curve till the transmission rate of 1.85 bits/sec/Hz (corresponding to $x = 12$ dB), whereas, for DF relaying, only CEE curve dominates the only RFI curve till the transmission rate of 2.00 bits/sec/Hz (corresponding to $x = 15.2$ dB).

In Fig. 4, OP vs. average SNR is plotted for different values of the shape parameter α. It is evident that, as the channel condition improves with increasing α, the OP performance improves. But the rate at which it improves at higher transmission rate is much slower than the rate at which it improves at lower transmission rate. Hence, we can infer that high rate systems are much more susceptible to the combined effects of CEE and RFI than low rate systems.

V. Conclusions

In this paper we evaluate the performance of dual-hop DF relaying systems in the presence of both CEE and RFI. The exact and asymptotic closed-form expression for the OP is derived. Simplified asymptotic results in the high SNR regime show that, RFI alone has no role in the production of an outage floor, yet it enhances the effect of CEE in terms of elevating the outage floor further. We observe that the effect of CEE is much more pronounced in the case of DF relaying system at low-to-medium average SNR. It is also found that the combined effect of both CEE and RFI on high rate DF relaying systems is greater than that on low rate DF relaying systems.

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