Ambiguity-Region Analysis for Double Threshold Energy Detection in Cooperative Spectrum Sensing

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***Abstract*—In this paper we investigate cooperative spectrum sensing technique in which double threshold based energy detection is used at local cognitive radio (CR) sensors. Cooperative spectrum sensing improves reliability in detection of underutilized spectrum by fusing the decisions of local CR sensors. Local CR sensors use two thresholds, λ_1 and λ_2 (with $\lambda_2 > \lambda_1$). If the sensed energy is greater than λ_2 or less than λ_1 then with complete certainty local sensors decide for the presence or absence of primary user (PU) respectively. The difference between λ_2 and λ_1 defines the ambiguity region, i.e., $\Delta_{th} = \lambda_2 - \lambda_1$ in which local CRs are unable to decide the presence or absence of PU. In this work local sensors report the decision along with situation in which sensed energy falls in ambiguity region. In our analysis we use majority rule at the fusion center and compute an analytical expression of average probability of sensing error as a function of width of ambiguity region, i.e., Δ_{th}. Using the derived analytical expression and by simulation we show that there exists an optimal value of Δ_{th} for which the probability of sensing error is minimum.**

Index terms— Cooperative sensing; ambiguity region; false alarm; double threshold; spectrum hole.

I. Introduction

Radio frequency spectrum is identified as one of the most valuable resource for current generation wireless technologies. Due to the rapid advancement in communication technologies and exponential growth in bandwidth hungry applications the need for efficient utilization of the spectrum resources has gained a considerable attention among the researchers. The existing spectrum assignment policy aims in improving reliability in communication; however, the allocation policy leaves a great portion of spectrum severely under-utilized. Thus, there is an urge for a more intelligent and flexible communication technology that can exploit the spectrum resource in a more efficient way. In order to address the above challenge cognitive radio (CR) technology [1], [2], has been found to be an attractive solution. By definition, a cognitive radio is an intelligent device which according to the surrounding radio environment and other user requirements, adapts its transmission power, frequency, modulation technique etc.

The CR technology enables the unlicensed *secondary user* (SU) to coexist with the licensed *primary user* (PU) without causing significant interference or very little interference. The secondary user uses a portion of the spectrum, which is unused by the primary user, called spectrum hole, at a particular duration of time. Spectrum hole detection is performed by secondary user and is termed as *spectrum sensing*, [7]. Based on the signal processing approach spectrum sensing can be broadly classified into three categories: energy detection, matched filter detection and feature detection[9]. Since energy detection approach cannot differentiate the type of signal from the primary user, it is the most simple form of spectrum sensing technique. The energy detector collects a number of samples from the radio environment, computes the energy, compares it with a threshold and decides for the presence or absence of the primary user signal. To further improve the performance of spectrum sensing, cooperative spectrum sensing technique have been studied in [8], [11]. Here a number of CR sensors with different spatial locations are used which individually sense the spectrum and reports to the fusion center, through a reporting channel. Fusion center combines the decision using *Soft Combination* or *Hard Combination* technique [3]. In soft combination technique the quantized value of the calculated energy, i.e., the test statistic is sent to the fusion center, where as in hard combination technique the local decisions of the CR sensors are sent. In this work hard combining technique is used as it is more cost efficient and less time consuming. Fusion center finally makes the decision based on rules like *AND*, *OR* or *Majority* rule.

Considering CR sensors it has been observed that around the threshold of an energy detector the probability density functions of the test statistics which decides the presence or absence of primary user signal has a very small difference. So no decisions can be made with a good certainty when the test statistic falls around the threshold. The double threshold energy detection technique [4], [5], is introduced in place of the conventional single threshold, which implements two thresholds λ_1 and λ_2. The sensed energies lying between upper and lower threshold ($\Delta_{th} = \lambda_2 - \lambda_1$) are considered unreliable and are not considered in cooperation. In [10] it has been shown that the double threshold scheme with cooperative sensing shows better results than the conventional single threshold scheme. Further, [6] implemented dynamic double threshold considering the noise uncertainty. One of the basic question which has remained unanswered in the above works is the width of arbitrary double threshold region (ambiguity region). Further there is a need to compute the width of ambiguity region for maximizing the cooperative CR performance.

In this paper we reinvestigate the theory behind the double threshold scheme. The local sensors report the decision along with situation in which sensed energy falls in ambiguity region to the fusion center. In our analysis we use majority rule at the fusion center and compute an analytical expression of average probability of sensing error as a function of width of ambiguity region, i.e., Δ_{th}. Using the derived analytical expression and by simulations we show that there exists an optimal value of Δ_{th} for which the probability of sensing error is minimum.

The remaining part of the paper is organized as follows. Section II gives description of the system model; Section III analyses the ambiguity-region and finds the cooperative probabilities as a function of the width of ambiguity-region; Section IV derives the analytical formula for average probability of sensing error. Section V presents the description of the simulation done and displays the results. In section VI we finally present conclusion and the scopes of future works on this topic.

II. System Model

Consider a cooperative cognitive radio (CR) sensing scenario in which N CR sensors sense a given narrow band spectrum. The local CR sensors are similar detectors and are arbitrarily located at different spatial locations such that they experience approximately uncorrelated channel from the primary user. Every local CR sensors i, where $i \in \{1, ..., N\}$, collects L number of samples and are denoted as $x_i(n)$, where $n \in \{0, ..., L-1\}$, in a given time frame. The corresponding test statistics which will be compared with the threshold for decision making can be expressed as,

$$T_i(x) = \sum_{n=0}^{L-1} \mid x_i(n) \mid^2$$

where the samples $x_i(n)$ can take values in two categories,

$$\mathcal{H}_0 : x_i(n) = w_i(n)$$
$$\mathcal{H}_1 : x_i(n) = h_i(n) \cdot s(n) + w_i(n)$$

Here $\mathcal{H}_0$ and $\mathcal{H}_1$ denotes null and alternate hypothesis for absence and presence of primary user signal respectively. $h_i(n)$ denotes Rayleigh distributed channel fading co-efficient; $s(n)$ denotes the primary user signal; $w_i(n)$ denotes AWGN noise with variance σ_n^2. Thus expected signal to noise ratio (SNR) can be expressed as

$$\eta = \frac{E\left[\mid h_i(n) \mid^2\right] P}{\sigma_n^2}$$

where P is the power of the primary user signal. It is assumed, for simplicity, that expected SNR η and noise variance σ_n^2 are same at all the CR sensors' frontend. So that test statistic $T_i(x)$ at all the CR frontends are identically distributed.

Next, we assume that the number of samples, L is large enough such that distribution of test statistics $T_i(x)$, represented as $\mathcal{P}(T_i(x))$ is Gaussian distributed. Thus, distribution of $T_i(x)$ under two hypothesis can be expressed as,

$$T_i(x) \overset{\mathcal{H}_0}{\sim} \mathcal{N}\left(L\sigma_n^2, 2L\sigma_n^4\right)$$

$$T_i(x) \overset{\mathcal{H}_1}{\sim} \mathcal{N}\left(L\sigma_n^2(\eta+1), 2L\sigma_n^4(2\eta+1)\right)$$

The conventional energy detector based single threshold λ can be calculated by fixing the probability of false alarm $(P(\mathcal{H}_1 \mid \mathcal{H}_0))$, denoted as P_{fa_i}. Applying the Neyman-Pearson Hypothesis testing criterion [12], the threshold can be expressed as

$$\lambda = \sqrt{2L\sigma_n^4} Q^{-1}(P_{fa_i}) + L\sigma_n^2$$

For double threshold detection, an ambiguity region of width Δ_{th}, around the threshold can be defined as shown in figure 1.

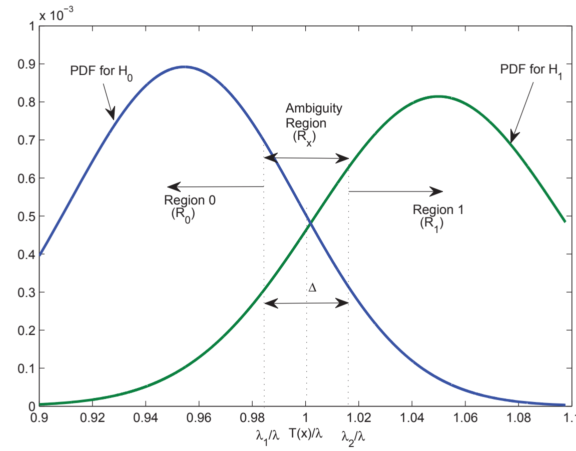

Fig. 1. Ambiguity region in null and alternate hypothesis

Figure 1 shows the generalized scenario, where normalized ambiguity region width Δ is expressed as

$$\Delta = \frac{\Delta_{th}}{\lambda}$$

The two threshold λ_1 and λ_2 can now be expressed as,

$$\lambda_1 = \lambda\left(1 - \frac{\Delta}{2}\right)$$

$$\lambda_2 = \lambda\left(1 + \frac{\Delta}{2}\right)$$

Finally, based on the occurance of the test statistic $T_i(x)$ in the regions as shown in figure 1, the corresponding double threshold based energy detection rule is expressed as:

$$T_i(x) \geq \lambda_2 : \mathcal{H}_1(\text{presence of primary user}, R_1)$$
$$T_i(x) \leq \lambda_1 : \mathcal{H}_0(\text{absence of primary user}, R_0)$$
$$\lambda_1 < T_i(x) < \lambda_2 : \text{no decision}(R_x)$$

After detection process, the *hard decisions*[3], i.e., whether primary user is absent or present, from the local CR sensors is reported to a central fusion center. Here we assume that local sensors also inform the fusion center about their undecided state when the test statistics fall in the ambiguity region. The fusion center neglects such local sensors in the final decision making and only fuses the data from those local CR sensors which without uncertainty decide for $\mathcal{H}_1$ or $\mathcal{H}_0$. The fusion rule can be either AND, OR or Majority rule; in this paper we limit our analysis to majority rule only.

III. Ambiguity-Region Analysis

Since the test statistic of any local CR sensor can fall in ambiguity region, we assume that at a particular sensing phase only K out of N sensors are effective as rest $N-K$ sensors report "no decision" to the fusion center. Next for each local CR sensors the probability of false alarm and probability of detection are respectively given by,

$$P_{fa} = \int_{\lambda_2}^{\infty} \mathcal{P}\left(T_i(x) \mid \mathcal{H}_0\right) dx = Q\left(\frac{\lambda_2 - L\sigma_n^2}{\sqrt{2L\sigma_n^4}}\right) \quad (1)$$

$$P_d = \int_{\lambda_2}^{\infty} \mathcal{P}\left(T_i(x) \mid \mathcal{H}_1\right) dx = Q\left(\frac{\lambda_2 - L\sigma_n^2(\eta+1)}{\sqrt{2L\sigma_n^4(2\eta+1)}}\right) \quad (2)$$

Similarly, probability of misdetection and probability of correct decision of absence of primary user respectively are given by,

$$P_{md} = \int_{-\infty}^{\lambda_1} \mathcal{P}\left(T_i(x) \mid \mathcal{H}_1\right) dx = 1 - Q\left(\frac{\lambda_1 - L\sigma_n^2(\eta+1)}{\sqrt{2L\sigma_n^4(2\eta+1)}}\right) \quad (3)$$

$$P_c = \int_{-\infty}^{\lambda_1} \mathcal{P}\left(T_i(x) \mid \mathcal{H}_0\right) dx = 1 - Q\left(\frac{\lambda_1 - L\sigma_n^2}{\sqrt{2L\sigma_n^4}}\right) \quad (4)$$

The probability that a sensor makes no decision, i.e., the test statistic $T_i(x)$ of the sensor lies in the ambiguity region R_x, assuming $\mathcal{H}_0$ is true is given by

$$\begin{aligned} P_{L0} &= \int_{\lambda_1}^{\lambda_2} \mathcal{P}\left(T_i(x) \mid \mathcal{H}_0\right) dx \\ &= Q\left(\frac{\lambda_1 - L\sigma_n^2}{\sqrt{2L\sigma_n^4}}\right) - Q\left(\frac{\lambda_2 - L\sigma_n^2}{\sqrt{2L\sigma_n^4}}\right) \end{aligned} \quad (5)$$

Similarly assuming $\mathcal{H}_1$ is true, the probability that a sensor makes no decision is given by

$$\begin{aligned} P_{L1} &= \int_{\lambda_1}^{\lambda_2} \mathcal{P}\left(T_i(x) \mid \mathcal{H}_1\right) dx \\ &= Q\left(\frac{\lambda_1 - L\sigma_n^2(\eta+1)}{\sqrt{2L\sigma_n^4(2\eta+1)}}\right) - Q\left(\frac{\lambda_2 - L\sigma_n^2(\eta+1)}{\sqrt{2L\sigma_n^4(2\eta+1)}}\right) \end{aligned} \quad (6)$$

Assuming only K out of N number of CR sensors report a non-ambiguous decision to the fusion center (i.e., K number of sensors are effective), a decision of $\mathcal{H}_1$ is made when

- Exactly $(N-K)$ number of CR sensors' test statistic $T_i(x)$ lies in ambiguity region R_x as shown in figure 1.
- Atleast $\lceil \frac{K}{2} \rceil$ number of CR sensors' test statistic $T_i(x)$ lies in region R_1.
- Rest of the other CR sensors' test statistic $T_i(x)$ lies in region R_0.

TABLE I
Expressions for P'_{fa} for $N=4$

K	$P'_{fa} = P(\mathcal{H}_1 \mid \mathcal{H}_0)$
0	$P_{L0}^4 \times 1$
1	$\binom{4}{1}(P_{L0})^3 \times P_{fa}$
2	$\binom{4}{2}(P_{L0})^2 \times \sum_{k=0}^{1} \binom{2}{k} P_{fa}^{2-k} P_c^k$
3	$\binom{4}{3}(P_{L0}) \times \sum_{k=0}^{1} \binom{3}{k} P_{fa}^{3-k} P_c^k$
4	$(P_{L0})^0 \times \sum_{k=0}^{2} \binom{4}{k} P_{fa}^{4-k} P_c^k$

TABLE II
Expressions for P'_{md} for $N=4$

K	$P'_{md} = P(\mathcal{H}_0 \mid \mathcal{H}_1)$
0	$P_{L1}^4 \times 0$
1	$\binom{4}{1}(P_{L1})^3 \times P_{md}$
2	$\binom{4}{2}(P_{L1})^2 \times P_{md}^2$
3	$\binom{4}{3}(P_{L1}) \times \sum_{k=0}^{1} \binom{3}{k} P_{md}^{3-k} P_d^k$
4	$(P_{L1})^0 \times \sum_{k=0}^{1} \binom{4}{k} P_{md}^{4-k} P_d^k$

Hence the cooperative probability of false alarm $P'_{fa}(K)$ when K number of CR sensors are effective is given by

$$\begin{aligned} P'_{fa}(K) &= \binom{N}{N-K}(P_{L0})^{N-K} \times \sum_{k=K}^{\lceil \frac{K}{2} \rceil} \binom{K}{k} P_{fa}^k P_c^{K-k} \\ &= \binom{N}{K}(P_{L0})^{N-K} \times \sum_{k=0}^{K-\lceil \frac{K}{2} \rceil} \binom{K}{k} P_{fa}^{K-k} P_c^k \end{aligned} \quad (7)$$

Similarly, assuming K out of N number of CR sensors are effective, a decision of $\mathcal{H}_0$ is made when

- Exactly $(N-K)$ number of CR sensors' test statistic $T_i(x)$ lies in ambiguity region R_x .
- Atleast $\left(\lfloor \frac{K}{2} \rfloor + 1\right)$ number of CR sensors' test statistic $T_i(x)$ lies in region R_0.
- Rest of the other CR sensors' test statistic $T_i(x)$ lies in region R_1.

Hence the cooperative probability of misdetection $P'_{md}(K)$ when K number of CR sensors are effective is given by

$$\begin{aligned} P'_{md}(K) &= \binom{N}{N-K}(P_{L1})^{N-K} \times \sum_{k=K}^{\lfloor \frac{K}{2} \rfloor+1} \binom{K}{k} P_{md}^k P_d^{K-k} \\ &= \binom{N}{K}(P_{L1})^{N-K} \times \sum_{k=0}^{K-\lfloor \frac{K}{2} \rfloor-1} \binom{K}{k} P_{md}^{K-k} P_d^k \end{aligned} \quad (8)$$

Here K takes values from $\{0, 1, ..., N\}$. P_{fa}, P_d, P_{md} and P_c are given respectively by (1)-(4) and P_{L0} and P_{L1} are given by (5) and (6) respectively.

An example for the expressions of all the cooperative metrics assuming $N=4$ are shown in tables I and II.

IV. Sensing Error Rate vs. Ambiguity-Region

All the cooperative probabilities expression derived in the previous section are function of the number of effective sensors making decisions K, which takes values from $\{0, 1, ..., N\}$. Thus, the average probability of sensing error can be expressed as,

$$\begin{aligned} P_{e_{avg}} &= \alpha P_{av}(\mathcal{H}_0 \mid \mathcal{H}_1) + (1-\alpha) P_{av}(\mathcal{H}_1 \mid \mathcal{H}_0) \\ &= \alpha P_{md_{av}} + (1-\alpha) P_{fa_{av}} \end{aligned} \tag{9}$$

where $P_{md_{av}} = P_{av}(\mathcal{H}_0 \mid \mathcal{H}_1)$ is defined as average probability of misdetection; $P_{fa_{av}} = P_{av}(\mathcal{H}_1 \mid \mathcal{H}_0)$ is defined as average probability of false alarm; and α denotes the probability that the primary user is present, i.e., the probability of occupancy of the channel by the primary user, $\alpha = P(\mathcal{H}_1)$.

Further, since the cases of different values of K are mutually exclusive and the corresponding cooperative probability of false alarm and cooperative probability of misdetection for each of the cases are given by $P'_{fa}(K)$ and $P'_{md}(K)$ respectively, the average probability of misdetection and average probability of false alarm can be expressed as the sum of the probabilities for each cases:

$$P_{md_{av}} = \sum_{K=0}^{N} P'_{md}(K) \tag{10}$$

$$P_{fa_{av}} = \sum_{K=0}^{N} P'_{fa}(K) \tag{11}$$

Substituting the expression of $P_{md_{av}}$ and $P_{fa_{av}}$ in (9) the final expression for sensing error can be expressed as:

$$\begin{aligned} P_{e_{avg}} =& \alpha P_{md_{av}} + (1-\alpha) P_{fa_{av}} \\ =& \alpha \sum_{K=0}^{N} P'_{md}(K) + (1-\alpha) \sum_{K=0}^{N} P'_{fa}(K) \end{aligned} \tag{12}$$

It can be observed from equation (12) that the average probability of sensing error is a function of Δ_{th} for some constant values of SNR η, P_{fa_i} and number of sensing samples L. Thus the probability of sensing error can be minimized when an optimal value of Δ_{th} is used at every local sensors.

V. Simulation and Result

A cooperative cognitive radio system is simulated taking $N = 4$ number of CR sensors. The sensing channel, i.e., the channel between PU and CR sensors is assumed to be Rayleigh distributed fast fading channel. The PU signal is taken as BPSK modulated with probability of occupancy of channel as $P(\mathcal{H}_1) = \alpha = 0.5$. The channel coefficients observed by the CR sensors are assumed to be uncorrelated. The probability of false alarm for each of the CR sensors, P_{fa_i} is set to obtain the conventional single threshold λ. Then a variable width of ambiguity region, Δ_{th} is taken for the CR sensors. The sensors report their decision along with the situation in which the sensed energy falls in the ambiguity region through a reporting channel to the fusion center using 2 bit hard decision as shown in the table below.

Decision	Hard decision bits
$\mathcal{H}_0$	00
$\mathcal{H}_1$	11
No decission	01 or 10

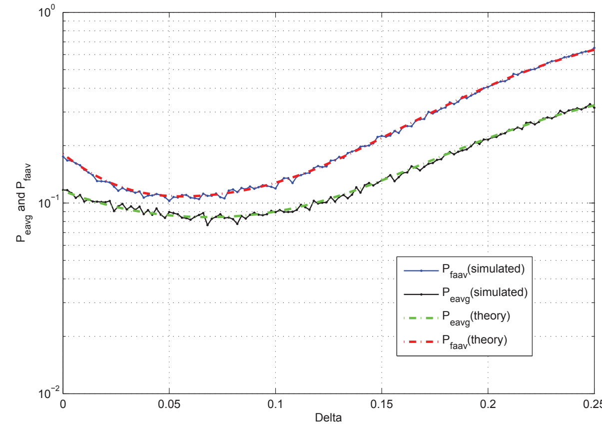

Fig. 2. $P_{e_{avg}}$ and $P_{fa_{av}}$ vs Δ for $L = 500$, $\eta = -10$dB and $P_{fa_i} = 0.2$

The reporting channel, for simplicity is assumed to be perfect. The fusion center then fuses the decisions from the CR sensors using the Majority rule to obtain the final decision.

Now, for different values of SNR η, at the CR sensor front end, number of sensing samples L and probability of false alarm for each of the CR sensors P_{fa_i}, average probability of false alarm $P_{fa_{av}}$ and the average probability of sensing error $P_{e_{avg}}$ are plotted as a function of Δ. The theoretical values are obtained from equation (11) and (12) respectively. Figure 2 shows the plot of $P_{e_{avg}}$ and $P_{fa_{av}}$ vs Δ for number of sensing samples $L = 500$, SNR at CR sensor front end $\eta = -10$dB and probability of false alarm $P_{fa_i} = 0.2$. Figure 3 shows the plot of $P_{e_{avg}}$ and $P_{fa_{av}}$ vs Δ for number of sensing samples $L = 1000$, $\eta = -12$dB and $P_{fa_i} = 0.15$. Figure 4 shows the plot of $P_{e_{avg}}$ and $P_{fa_{av}}$ vs Δ for number of sensing samples $L = 2000$, $\eta = -10$dB and $P_{fa_i} = 0.05$.

The following inferences can be made from the simulated plots:

- The theoretical and simulated plots overlap each other assuring correctness of the theory.
- There exists an optimum value of Δ for which the sensing error is minimum.
- The values of $P_{e_{avg}}$ and $P_{fa_{av}}$ at $\Delta = 0$ are the values for conventional single threshold scheme.
- The optimum value of Δ is a function of the number of sensing samples L, SNR η and initial probability of false alarm P_{fa_i}.

From the plots of figure 2, 3 and 4 it can be observed that an optimum value of Δ and hence Δ_{th} can be obtained by putting some constraints on $P_{fa_{av}}$. For example, let us assume a constraint on $P_{fa_{av}}$ as

$$P_{fa_{av}} \leq P_{fa_0}$$

where P_{fa_0} is the cooperative probability of false alarm for $\Delta = 0$, i.e., the cooperative probability of false alarm when double threshold scheme is not used. Or in other words, a restriction is put on the average probability of false alarm that

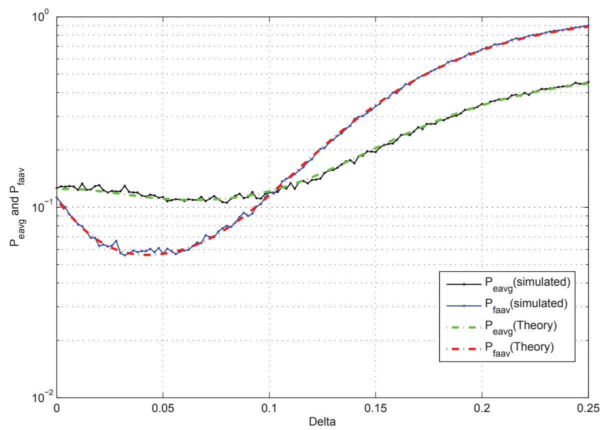

Fig. 3. $P_{e_{avg}}$ and $P_{fa_{av}}$ vs Δ for $L = 1000$, $\eta = -12$dB and $P_{fa_i} = 0.15$

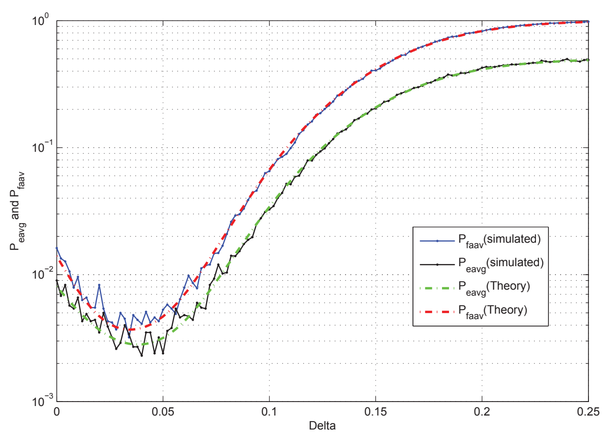

Fig. 4. $P_{e_{avg}}$ and $P_{fa_{av}}$ vs Δ for $L = 2000$, $\eta = -10$dB and $P_{fa_i} = 0.05$

it cannot exceed the value of cooperative probability of false alarm for single threshold scheme, which is given by

$$P_{fa_0} = \sum_{k=0}^{N-\lceil \frac{N}{2} \rceil} \binom{N}{k} P_{fa_i}^{N-k} \left(1 - P_{fa_i}\right)^k$$

Hence the final optimization problem is simplified to

$$\begin{aligned} &\underset{\Delta}{\text{minimize}}\ P_{e_{avg}} \\ &\text{subject to}\ P_{fa_{av}} \leq P_{fa_0} \end{aligned} \tag{13}$$

By solving this optimization problem an optimum value of Δ_{th} can be obtained for a particular value of number of sensing samples L, SNR η and initial probability of false alarm P_{fa_i}.

VI. Conclusion

Hence in this paper the double threshold energy detection technique in cooperative spectrum sensing scenario is reinvestigated. An analytical expression for probability of sensing error is found out using majority fusion rule at the fusion center as an example. The analytical expression and simulation results confirms that there exists an optimal value of Δ_{th} for which the probability of sensing error is minimum. Finally a simple optimization problem (13), is formulated subject to a nonlinear constraint on average probability of false alarm $P_{fa_{av}}$ solving which the expression for the optimum width of the ambiguity region can be obtained.

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