A Comparative Study of Two Decoupling Control Strategies for a Coupled Tank System

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***Abstract*—This paper presents a comparative study of two controllers namely Internal Model control (IMC) decoupling controller and an inverted decoupling controller applied to a coupled-tank liquid level system. These controllers are designed based on identified First Order Plus Dead Time (FOPDT) model of a coupled tank system. The performance of the two decoupling controllers are studied considering input and output multiplicative uncertainties. From the simulation and experimental studies, it is found that the inverted decoupling approach is more robust compared to IMC.**

***Index Terms*—Decoupling Control, Inverted decoupling Control, Internal Model Control (IMC), Multi Input Multi Output (MIMO), Coupled Tank System (CTS).**

I. Introduction

Automatic Regulation of liquid level is a commonly encountered process control problem [1], [2]. Control of a coupled tank system is one of the many challenging problems in this area due to the reason that the system is a multi-input-multi-output (MIMO) one and overall nonlinear [3]. In practice, implementation of control algorithms for the MIMO process is more complicated than for a Single-Input-Single-Output(SISO) [4], [5] one due to variations in process dynamics that arise because of change in operating points and the coupling effect [2], [6]. The control objective of a CTS is to maintain the desired liquid level in the individual tanks while reducing the coupling effects. Possible control strategies for a CTS are centralized or decentralized. Between them, the latter one is often preferable because of easier implementation and flexible operation. Decoupling control is often used for MIMO systems in order to control individual outputs through individual references while minimizing the coupling effect. The widely used PID controller with a decoupling controller has been designed in [7]. The approach consists of two control strategies realized through two distinct controller parts. The decoupling controller is dedicately used for minimizing the coupling effect and the other controller takes care of the SISO performance of the decoupled plant. Many other decoupling control design approaches for minimizing the interaction have been reported in literature such as detuning approach [2] inverted decoupling [8]. Despite several advanced controllers, SISO PID controllers with decouplers are still preferred in industries because of easy implementation and design [9]. A multivariable model-based control strategy is proposed in [10], wherein authors made a comparative analysis between decentralized control, decoupling control and internal model based control. In [4], [11], [12], dynamic decoupling with stability has been discussed for MIMO systems.A brief review on decoupling techniques is addressed in [13]. In [14], authors reported a decoupling controller design approach within the framework of feedback control structure. A brief description about the practical advantage of inverted decoupling controller is addressed in [15]. Even though several decoupling approaches have been developed in literature as discussed above, comparison of their performances is rarely addressed.

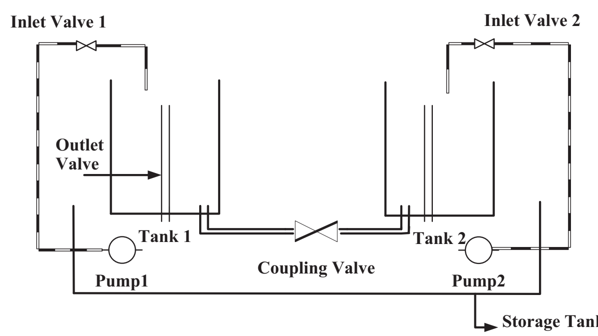

Fig. 1: Schematic Diagram of a Coupled Tank System

In this paper, a comparative analysis is made between the IMC and inverted decoupling approach based on their robust performance. Both the decoupling approaches are implemented on a coupled tank liquid level system to illustrate the efficacy of each method. Finally simulation along with experimental results are presented.

II. Coupled Tank System

A schematic diagram of the CTS consisting of two tanks is shown in Fig.1 . The plant is modeled as a FOPDT from open-

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loop input-output data. The model for the CTS is obtained as:

$$G(s) = \begin{bmatrix} \frac{2.197e^{(-5.5s)}}{1+615s} & \frac{2.3e^{(-9.34s)}}{1+614s} \\ \frac{2.197e^{(-30s)}}{1+601.84s} & \frac{2.82e^{(-30s)}}{1+602s} \end{bmatrix} \quad (1)$$

The above model is used for designing decoupling controllers for the CTS. Next, we briefly describe two decoupling strategies that are compared in this work. Furthermore, in order to compensate the process loop interaction, here two decoupling approaches, namely IMC and inverted decoupling technique are considered. This two decoupling approaches are designed based on the identified FOPDT model of the CTS.

III. IMC DECOUPLING CONTROL

The conventional decoupling structure of a Two Input Two Output process is shown in Fig.2. In IMC decoupler, the desired diagonal system transfer matrix is present in terms of minimum integral square error (ISE) and rapid settling time criterion along with cancelation of RHP zeros of the determinant of the plant (G) to meet the requirements of both decoupling regulation and robust stability. The decoupling controller matrix can be designed by employing the below steps.

Assumptions

Consider the general open-loop transfer matrix of TITO system as :

$$G(s) = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$

The closed loop transfer matrix can be written as

$$H = GC(I + GC)^{-1}$$

The decoupled transfer matrix should be in the form

$$H = \begin{bmatrix} h_{11} & 0 \\ 0 & h_{22} \end{bmatrix}$$

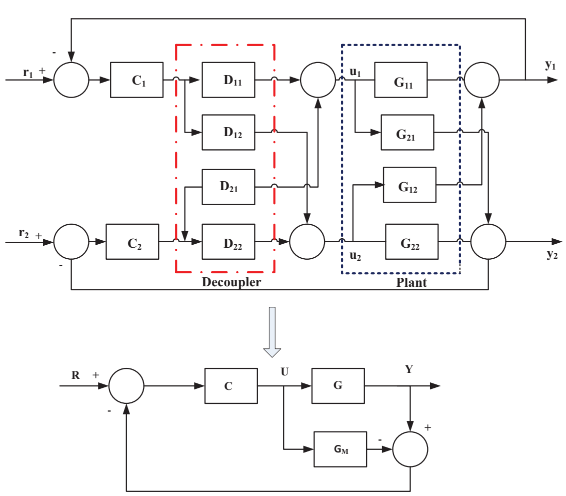


Fig. 2: Decoupling control Structure for TITO System with its IMC structure

TABLE I: Design steps of IMC Decoupling Controller

Step 1:	Check IF the above mentioned assumptions are satisfy for the obtained plant model or not. Form eq (1),it is observed that all the transfer function elements are stable and also $G(s) \neq 0$.
Step 2:	In order to check whether there exists any RHP zeros or LHP zeros or infinite LHP and RHP zeros, determine the determinant of the plant model $G(s)$.
Step 3:	From polezero plot, it is observed that RHP zeros are present at$s = 0.0845 \pm j0.0488$ Since finite number of RHP zeros are present, we apply case 2 [16] for controller design the obtain controller as

where h_{ii} is physically proper and stable transfer function

$$C_{ji}(i, j = 1, 2, ..., m) = K_1.K_2$$

$$K_1 = \frac{D_{ij}e^{-(\theta_i - L_{ij})s}}{(\lambda_i s + 1)^{N_i} \sum_{m=1}^{q_1} (s + z_k^*)}$$

$$K_2 = \frac{1}{1 - \frac{e^{\theta_i s}}{(\lambda_i s+1)^{N_i}} \sum_{m=1}^{q_1} \frac{-s+z_k}{s+z_k^*}}$$

$$D_{ij} = r_{0,ij} \sum_{k=1}^{q_i} (-s + z_k)$$

where z_k are the RHP zeros of $\det(G)$ excluding those cancelled by common RHP zeros of G_{ij} and z_k^* is the complex conjugate of z_k.

$$C_{11} = \frac{D_{11}}{(\lambda_1 s+1)(s^2+0.169s+0.0095)} \cdot F_1$$
$$C_{12} = -\frac{D_{21}}{(\lambda_2 s+1)(s^2+0.169s+0.0095)} \cdot F_2$$
$$C_{21} = -\frac{D_{12}}{(\lambda_1 s+1)(s^2+0.169s+0.0095)} \cdot F_1$$
$$C_{22} = \frac{D_{22}}{(\lambda_2 s+1)(s^2+0.169s+0.0095)} \cdot F_2$$

where

$$F_1 = \frac{1}{1 - \frac{e^{-5.5s}(s^2-0.169s+0.0095)}{(\lambda_1 s+1)(s^2-0.169s+0.0095)}}$$
$$F_1 = \frac{1}{1 - \frac{e^{-30s}(s^2-0.169s+0.0095)}{(\lambda_2 s+1)(s^2-0.169s+0.0095)}}$$

$$D_{11} = \frac{104.325s+0.1646}{130.8322s+1}$$
$$D_{12} = -\frac{96.9587s+0.153}{130.822s+1}$$
$$D_{21} = -\frac{83.4634s+0.1343}{131.324s+1}$$
$$D_{22} = \frac{79.5899s+0.1281}{131.3289s+1}$$

In this approach, the value of adjustable parameter λ, is found out by trial error method. In fact, when values of the adjusted parameter are tuned to be small, at that moment, system output becomes faster. On the contrary, when the values of λ_1 and λ_2, are tuned to be large, it will slow down the system output response. After several iterations, the adjustable parameters

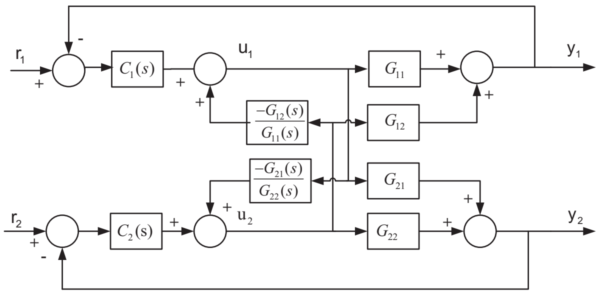


Fig. 3: Inverted decoupling Control

value are chosen as $\lambda_1 = 30$ and $\lambda_2 = 60$.

IV. INVERTED DECOUPLING CONTROLLER DESIGN

The structure of the inverted decoupling control is shown in Fig 3. The decoupler matrix for inverted decoupling can be derived from the control input u_1 and u_2 as shown in Fig. 3

$$\begin{aligned} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{G_{12}}{G_{11}} \\ -\frac{G_{21}}{G_{22}} & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \\ D &= \begin{bmatrix} 1 & -\frac{G_{12}}{G_{11}} \\ -\frac{G_{21}}{G_{22}} & 1 \end{bmatrix}^{-1} \end{aligned} \tag{2}$$

$$D(s) = \begin{bmatrix} 1 & \frac{2.3(1+615s)e^{-4.0349s}}{2.1947(1+614.64s)} \\ \frac{2.63(1+602s)}{2.82(1+601.84s)} & 1 \end{bmatrix} \tag{3}$$

In the inverted decoupler design, parameters of the PI controller are tuned as suggested in [8]

$$k_p = \frac{0.4\tau}{k\theta}, T_i = \tau \tag{4}$$

By using (4), parameters of the PI for the plant (1)is obtained as

$$k_{p_1} = 20.3797, k_{i_1} = 0.0331$$
$$k_{p_2} = 2.8463, k_{i_2} = 0.0047$$

V. ROBUSTNESS ANALYSIS

The input multiplicative uncertainty as shown in Fig 4 is considered for the robustness analysis of decoupling controller design.

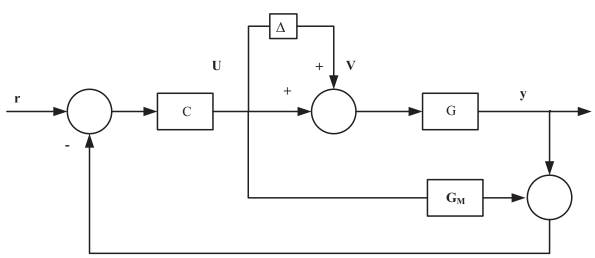


Fig. 4: multiplicative input uncertainty Δ of IMC decoupling approach

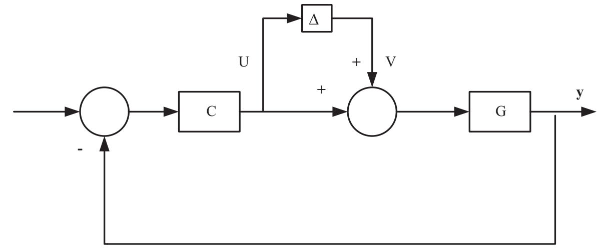


Fig. 5: multiplicative input uncertainty Δ of Inverted decoupling approach

A. *Multiplicative Iutput Uncertainty*

From the $T-\Delta$ structure of Fig 4 for IMC, one can derive the transfer function between U and V is as

$$T = \frac{-CG}{I + (G - G_m)C} \tag{5}$$

In eq (5), $G = G_M$ hence, it can be rewritten as

$$T = -CG$$

similarly for inverted decoupling from Fig.5,one can derive the transfer function between outputs to inputs of δ is as

$$T(s) = -K(I + GK)^{-1}G \tag{6}$$

According to small gain theorem [17], the perturbed system with multiplicative uncertainty holds robust stability if and only if the below-mentioned constraints will satisfy i.e.

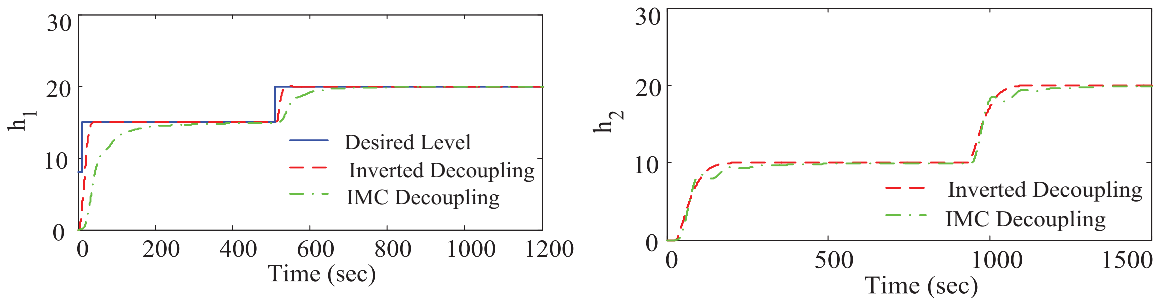
$$\|\Delta\|_\infty \le \frac{1}{\|C\|_\infty} \tag{7}$$

Therefore, for a specified multiplicative uncertainty bound the control system robust stability can be intuitively evaluated by observing the response of magnitude plot of eq (7) if it falls below unity or not.

VI. RESULTS AND DISCUSSION

Fig. 6: Schematic of Experimental setup of coupled tank system

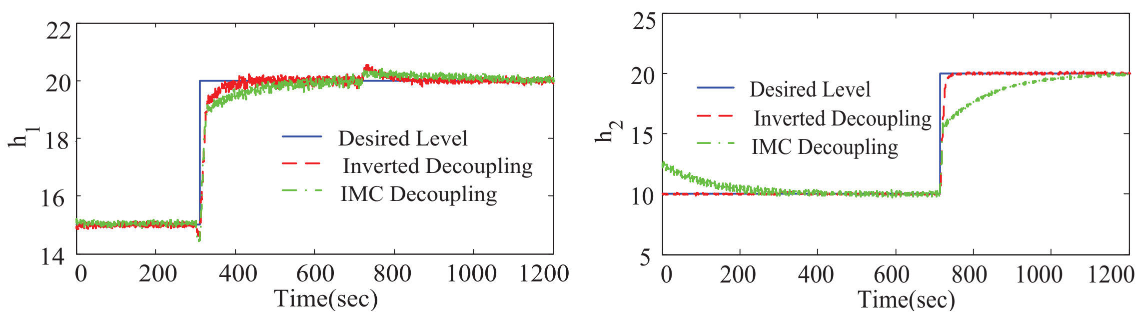
Fig 6 depicts the sketch of the coupled tank system that is used in present work as a benchmark device [18]. The performances of the two decoupling controller design methods are



(a) Variation of level in Tank 1

(b) Variation of level in Tank2

Fig. 7: Variation of level in Tank 1 and Tank 2



(a) Experimental variation of level in tank 1

(b) Experimental variation of level in tank 2

Fig. 8: Experimental response Variation of level in Tank 1 and Tank 2

evaluated and compared experimentally. From the simulation results in Fig.7 it is clearly envisaged that both tanks achieved the desired level smoothly. First, a step change in h_1 demand is made at 300 sec, and then the same in h_2 demand is made in 700 sec. The corresponding results are presented in 8. Fig 9 exhibits the perturbed plant response where a load disturbance, is applied with a magnitude of 2.5 to both the process inputs at 2000 sec. Since the response of spectral radius of both the controllers are fall below the unity shown in 10, so it ensures that both the designed controller preserve the robust stability. However, in IMC decoupling, better robust stability can be obtained to cope up with uncertainty with the proper tune of adjustable parameters λ_1 and λ_2. It is observed from simulations as well as experimental results that, both the IMC and inverted decoupling approaches are capable of removing the coupling effect significantly,whereas in the case of Inverted decoupling approach both the tanks yields smoother response with less steady state error as compared to IMC decoupling controller.

VII. Conclusion

In this paper, a comparative study is made between two decoupling controller namely, IMC and an inverted controller for the real-time liquid level control of a coupled-tank liquid level system. First, a suitable model for the coupled tank system is identified experimentally by employing the system identification. Furthermore, this two decoupling controllers are

TABLE II: Performances assessment of both the decoupling approach

Performance indices	IMC Decoupling control	Inverted Decoupling Control
ISE	2.66 (Tank 1) 4.44 (Tank 2)	0.01007 (Tank 1) 0.00017(Tank 2)
IAE	5.107 6.679	0.1015 0.01448

designed based on the identified model. By observing both the simulation and experimental results, it is clearly observed that the inverted decoupling controller approach provides better control action for regulating the desired level in both the tanks as compared to the IMC decoupling techniques.

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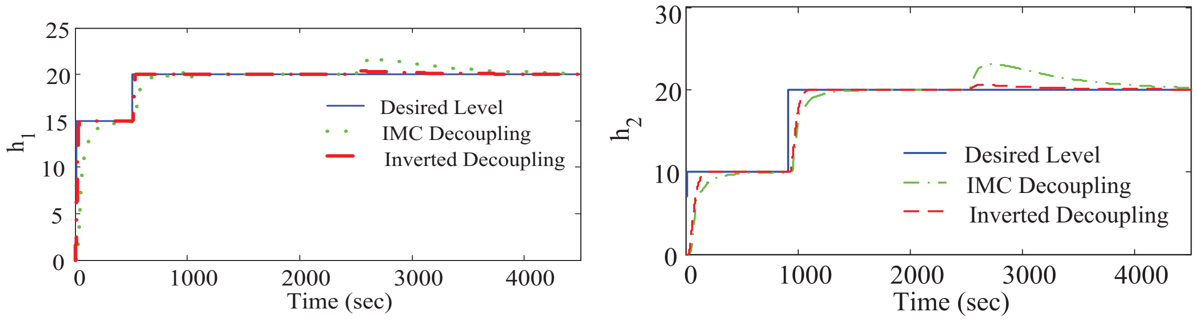
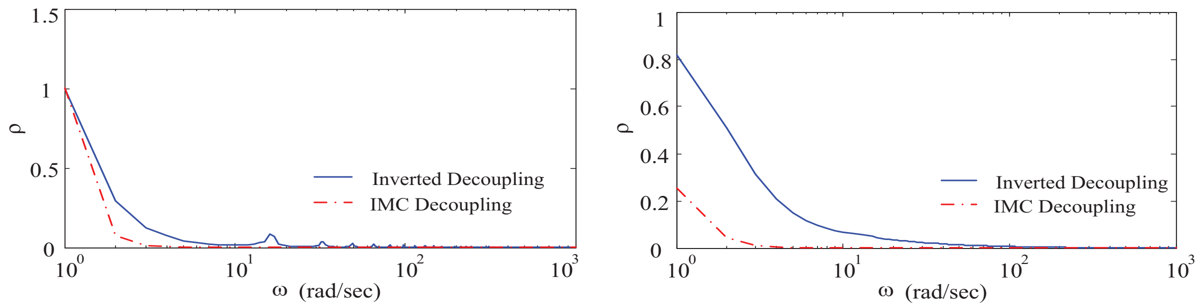


(a) Variation of level in Tank 1 due to disturbance

(b) Variation of level in Tank 2 due to disturbance

Fig. 9: Variation of level in Tank 1 and Tank 2 due to input uncertainty



(a) Spectral radius for multiplicative input uncertainty

(b) Spectral radius for multiplicative output uncertainty

Fig. 10: Response of spectral radius plot

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