Fuzzy implementation of a Bayesian equaliser in the presence of intersymbol and co-channel interference

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Abstract: The authors investigate the problem of channel equalisation in the presence of co-channel interference (CCI), intersymbol interference and additive white gaussian noise. The optimal Bayesian decision feedback equaliser decision function for this problem is derived and an elegant fuzzy implementation of the optimal solution is proposed. This fuzzy implemented equaliser is able to provide performance close to the optimal equaliser with a substantial reduction in computational complexity. The equaliser consists of a fuzzy equaliser with an input processing block for co-channel compensation. This preprocessor can be used under severe-to-moderate CCI and can be removed under low CCI conditions. Simulation studies demonstrate the performance of the fuzzy equaliser developed.

1 Introduction

The demand for cellular mobile communication has been increasing rapidly in the last decade. With the limitation on the available signal spectrum, one of the ways to incorporate more users is to reduce cell size, increasing frequency reuse. With this, there is rise in interference from the users of one cell to the users in another cell using the same carrier frequency and is termed co-channel interference (CCI). Communication systems also suffer from the effects of intersymbol interference (ISI) due to nonideal channel characteristics and additive white gaussian noise (AWGN) [1]. The problems of CCI, ISI and AWGN are encountered in digital cellular radio [2], dual-polarised microwave radio [3] and twisted-pair subscriber loops [3, 4] to name a few. The equaliser present in the receiver should be capable of compensating these effects with limited computational complexity [5].

The equaliser that can provide the minimum bit error rate (BER) under one or more of above conditions is called the maximum likelihood sequence estimator (MLSE) [6]. The MLSE, although very effective in combatting these problems, suffers from a large computation requirement. The optimum solution for the symbol-spaced equalisers can be derived from maximum *a posteriori* probability and is termed Bayesian equalisers. The Bayesian equaliser can be implemented using a feedforward or feedback structure [7]. When feedback is employed the equaliser is termed the Bayesian decision feedback equaliser (DFE). A conventional DFE [5] with linear filters tries to approximate the performance of the Bayesian DFE. These equalisers can perform satisfactorily for channels corrupted by ISI and AWGN. But in the presence of CCI they suffer from performance degradation by treating CCI as additive noise and exploiting spectral characteristics for equalisation. The CCI can be treated as cyclostationary in nature and linear fractionally spaced DFE can be used [2, 8] to overcome the problem with limited success. The satisfactory performance of these equalisers is limited to the condition of low AWGN with moderate CCI or *vice versa*. Equalisation in general is a nonlinear problem and hence nonlinear equalisers using artificial neural networks (ANN) [9, 10] and radial basis function (RBF) networks [11, 12] have been shown to provide superior performance to linear equalisers for channels corrupted with ISI and AWGN. In case of channels with CCI similar attempts have also been made to successfully design nonlinear equalisers using RBF [13], ANN [14, 15] and polynomial perceptron [16]. However, in most of these studies either the co-channel power or the channel noise power has to be low for satisfactory performance.

In a recent study Chen *et al.* [17] proposed a Bayesian DFE that incorporates CCI compensation (Bayesian CCI_DFE), providing the optimum solution for the symbol-spaced architecture. However, an MLSE incorporating CCI compensation can provide better performance but would be computationally very expensive and difficult to design owing to the difficulty in estimating the co-channel without a training signal. The equaliser proposed in [17] outperforms MLSE that treats CCI as noise, under severe to moderate CCI. This equaliser is still computationally complex and the complexity grows if there is more than one co-channel. In recent years, fuzzy systems have been successfully used for equalisation [18–21]. In a earlier study [22], we proposed fuzzy implemetation of Bayesian equalisers. This equaliser uses scalar channel states instead of vector channel states used by RBF equalisers. The use of scalar channel states reduces the computational complexity in implementation. Additionally the use of scalar channel states provides a scheme for subset states selection reducing the computational complexity even

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further. The advantages of using scalar channel states motivated us to use these structures for equalisation of channel corrupted with CCI [23]. In this paper we propose a technique to implement the Bayesian CCI_DFE with fuzzy systems which use scalar channel states and scalar co-channel states. This equaliser provides near optimal performance but its computation complexity is comparable to the Bayesian DFE that treats CCI as noise (Bayesian DFE). The equaliser is also able to equalise channels with more than one co-channel without a substantial rise in computational complexity.

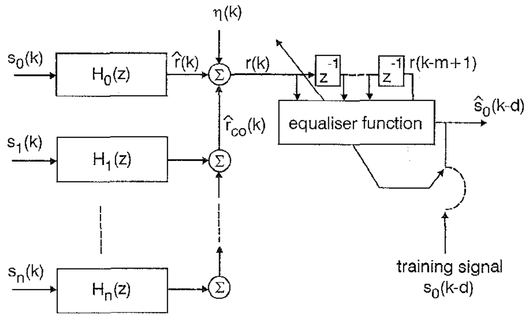

Fig. 1 *Discrete time model of communication system with co-channel interference*

2 System model

The discrete-time model of the communication system discussed in this paper is presented in Fig. 1. This model is widely used to represent a communication system corrupted with CCI, ISI and AWGN [3]. Here $H_0(z)$ is the channel transfer function which is corrupted with n interfering co-channels $H_i(z)$ with $1 \leq i \leq n$. The impulse response of the channels and the co-channels can be represented as

$$H_i(z) = \sum_{j=0}^{p_i-1} a_{i,j} z^{-j} \qquad 0 \leq i \leq n \tag{1}$$

Here p_i and $a_{i,j}$ are the length and tap weights of ith channel impulse response. We assume a binary communication system which would make the analysis simple, though it can be extended to any communication system in general. The transmitted symbols $s_i(k)$, $0 \leq i \leq n$ for the channel and co-channels are drawn from a set of independent, identically distributed (IID) dataset comprising of $\{\pm 1\}$ and are mutually independent. They satisfy the condition

$$E[s_i(k)] = 0 \tag{2}$$

$$E\left[s_i(k_1)s_j(k_2)\right] = \delta(i-j)\delta(k_1-k_2) \tag{3}$$

where $E[.]$ denotes the expectation operator and

$$\delta(k) = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases} \tag{4}$$

The channel output scalars can be represented as

$$r(k) = \hat{r}(k) + \hat{r}_{co}(k) + \eta(k) \tag{5}$$

Here $\hat{r}(k)$ is the desired received signal, $\hat{r}_{co}(k)$ is the interfering signal and $\eta(k)$ is the noise component. The noise $\eta(k)$ is assumed to be Gaussian with variance $E[\eta^2(k)] = \sigma_\eta^2$ and is uncorrelated with the data. The desired and the interfering signal can be represented as

$$\hat{r}(k) = \sum_{j=0}^{p_0-1} a_{0,j} s_0(k-j) \tag{6}$$

$$\hat{r}_{co}(k) = \sum_{i=1}^{n} \sum_{j=0}^{p_i-1} a_{i,j} s_i(k-j) \tag{7}$$

With this the signal-to-noise ratio (SNR), signal-to-interference ratio (SIR) and signal-to-interference noise ratio (SINR) as defined as

$$SNR = \frac{\sigma_s^2}{\sigma_\eta^2} \tag{8}$$

$$SIR = \frac{\sigma_s^2}{\sigma_{co}^2} \tag{9}$$

$$SINR = \frac{\sigma_s^2}{\sigma_\eta^2 + \sigma_{co}^2} \tag{10}$$

The task of the equaliser depicted in Fig. 1 is to estimate the transmitted sequence $s_0(k-d)$ based on the channel observation vector $\mathbf{r}(k) = [r(k), r(k-1), \ldots r(k-m+1)]^T$. Here m is the order of the equaliser and d is associated detection delay. During the initialisation period called training, a copy of the transmitted sequence $s_0(k)$ is available locally at the receiver for the equaliser to update its parameters. During the actual data transmission the equaliser updates its parameters in a decision directed mode. However, the equaliser does not have access to the transmission sequence $s_i(k)$, $1 \leq i \leq n$ of the co-channels.

In a DFE implementation the past decisions of the equaliser are fed back to the equaliser as shown in Fig. 2. This equaliser uses the information contained in the observed channel output vector $\mathbf{r}(k)$ and the past detected symbol vector

$$\hat{\mathbf{s}}_f(k) = \left[\hat{s}_0(k-d-1), \hat{s}_0(k-d-2), \ldots, \hat{s}_0(k-d-q)\right]^T \tag{11}$$

to produce the estimate $\hat{s}_0(k-d)$. Here q is the equaliser feedback order. Without loss of generality, we can select [7] $d = p_0 - 1$ to cover the entire channel dispersion with $m = d + 1 = p_0$ and $q = p_0 + m - d - 2 = p_0 - 1$.

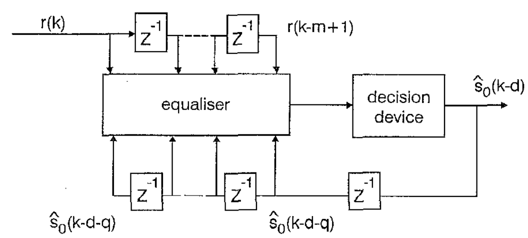

Fig. 2 *Schematic of DFE*

3 Bayesian DFE in presence of CCI

The optimal decision function for a Bayesian equaliser in the presence of ISI and AWGN can be derived by Bayes probability theory [24] and can be expressed as [7]

$$f(\mathbf{r}(k)) = \sum_{i=1}^{n_s^+} (2\pi\sigma_\eta^2)^{-m/2} \exp\left(\frac{-\|\mathbf{r}(k) - \mathbf{c}_i^+\|^2}{2\sigma_\eta^2}\right) - \sum_{j=1}^{n_s^-} (2\pi\sigma_\eta^2)^{-m/2} \exp\left(\frac{-\|\mathbf{r}(k) - \mathbf{c}_j^-\|^2}{2\sigma_\eta^2}\right) \tag{12}$$

Here $\mathbf{r}(k)$ represents the equaliser input vector, σ_η^2 represents the channel noise variance, $\mathbf{c}_i^+ \in R^m$ and $\mathbf{c}_j^- \in R^m$ are the noise-free received signal vectors corresponding to $s_0(k-d) = +1$ and $s_0(k-d) = -1$, respectively, and are called channel states. For convenience, we term $\mathbf{c}_i^+$ and $\mathbf{c}_j^-$ as positive and negative channel states, respectively. The terms n_s^+ and n_s^- are the number of positive and negative channel states, respectively, and they are equal. With the assumption of binary transmission the sign of the decision function in eqn. 12 is sufficient to provide the decision and scaling terms can be ignored. With this, the decision function can be represented as

$$f(\mathbf{r}(k)) = \sum_{i=1}^{n_s} w_i \exp\left(\frac{-\|\mathbf{r}(k) - \mathbf{c}_i\|^2}{2\sigma_\eta^2}\right) \tag{13}$$

where n_s is the number of channel states, equal to 2^{p_0+m-1} with $n_s^+ = n_s^- = n_s/2$, w_i are the weights associated with each of the centres. $w_i = +1$ if $\mathbf{c}_i \in n_s^+$ and $w_i = -1$ if $\mathbf{c}_i \in n_s^-$. The estimate of the symbol from the memoryless detector is

$$\hat{s}(k-d) = \begin{cases} 1 & f(\mathbf{r}(k)) \geq 0 \\ -1 & f(\mathbf{r}(k)) < 0 \end{cases} \tag{14}$$

However, when decision feedback is employed the feedback vector $\hat{\mathbf{s}}_f(k)$ can assume one of $n_f = 2^q$ states, and the equaliser forms the decision based on n_s/n_f channel states for each of the feedback states. Thus the n_s channel states in eqn. 13 can be grouped into n_f subsets based on the feedback states, with each of the feedback states containing n_{sf} states.

$$\bigcup_{i=1}^{n_s} \mathbf{c}_i = \bigcup_{j=1}^{n_f} \bigcup_{l=1}^{n_{sf}} \mathbf{c}_l^j \tag{15}$$

Here $\cup$ represents the union operation with j corresponding to the feedback state and l corresponding to the channel state in each of the feedback states. The vector $\mathbf{c}_l^j$ is the channel state l corresponding to the feedback state j. With this the equaliser decision function can be represented as

$$f_{DFE}\left(\mathbf{r}(k)|\hat{\mathbf{s}}_f(k) = s^j\right) = \sum_{i=1}^{n_{sf}} w_i \exp\left(\frac{-\|\mathbf{r}(k) - \mathbf{c}_i^j\|^2}{2\sigma_\eta^2}\right) \tag{16}$$

To derive the decision function of Bayesian CCI_DFE we assume that there is only one interfering co-channel. If there are more the same analysis can be extended. In the presence of CCI the interfering signal $\mathbf{r}_{co}(k) = [r_{co}(k), r_{co}(k-1), \ldots, r_{co}(k-m+1)]^T$ will have $n_{s,co} = 2^{p_1+m-1}$ co-channel states $\mathbf{c}_{co,l}$, where $1 \leq l \leq n_{s,co}$ corresponding to each of the channel states. The presence of the co-channel states will modify the decision function as

$$f_{DFE,CCI}\left(\mathbf{r}(k)|\hat{\mathbf{s}}_f(k) = s^j\right) = \sum_{i=1}^{n_{sf}} \sum_{l=1}^{n_{s,co}} w_i \exp\left(\frac{-\|\mathbf{r}(k) - \mathbf{c}_i^j - \mathbf{c}_{co,l}\|^2}{2\sigma_\eta^2}\right) \tag{17}$$

This forms the optimum solution for a symbol-spaced DFE decision function when the channel is corrupted with CCI, ISI and AWGN.

3.1 Normalised Bayesian DFE with scalar channel states

The Bayesian equaliser presented in eqn. 13 can be expressed in another form with the scalar channel states. Each of the n_s channel states $\mathbf{c}_i \in R^m$ of the equaliser decision function has m components taken from a set of $M_0 = 2^{p_0}$ scalar channel states. Expanding the vectors in the square norm of eqn. 13 as a product of scalar exponential

$$f(\mathbf{r}(k)) = \sum_{i=1}^{n_s} w_i \left\{ \prod_{l=0}^{m-1} \exp\left(-\frac{|r(k-l) - c_{il}|^2}{2\sigma_\eta^2}\right) \right\} \tag{18}$$

here c_{il} is the $(l+1)$th component of channel state vector $\mathbf{c}_i$ corresponding to the $(l+1)$th component of the input vector $\mathbf{r}(k)$. This Bayesian equaliser decision function can also be represented in a normalised form in line with the normalised RBF [25]

$$f(\mathbf{r}(k)) = \frac{\sum_{i=1}^{n_s} w_i \left\{ \prod_{l=0}^{m-1} \exp\left(-\frac{|r(k-l)-c_{il}|^2}{2\sigma_\eta^2}\right) \right\}}{\sum_{i=1}^{n_s} \left\{ \prod_{l=0}^{m-1} \exp\left(-\frac{|r(k-l)-c_{il}|^2}{2\sigma_\eta^2}\right) \right\}} \tag{19}$$

The Bayesian equaliser described by eqn. 13 can be implemented with an RBF network and the equaliser described by eqn. 19 can be implemented with normalised RBF [26] or fuzzy systems [22]. The advantage of the implementation of the decision function in eqn. 19 lies in its computational simplicity [22].

A DFE form of the NBEST can be represented as

$$f_{DFE}(\mathbf{r}(k)|\hat{\mathbf{s}}_f(k) = s^j) = \frac{\sum_{i=1}^{n_{sf}} w_i \left\{ \prod_{l=0}^{m-1} \exp\left(-\frac{|r(k-l)-c_{il}^{jo}|^2}{2\sigma_\eta^2}\right) \right\}}{\sum_{i=1}^{n_{sf}} \left\{ \prod_{l=0}^{m-1} \exp\left(-\frac{|r(k-l)-c_{il}^{jo}|^2}{2\sigma_\eta^2}\right) \right\}} \tag{20}$$

Here c_{il}^{jo} corresponds to the $l+1$ component of the vector channel state $\mathbf{c}_i$ and o has been added for convenience to represent the exact scalar state involved in vector state calculation, corresponding to the feedback state j, where $1 \leq j \leq n_f$. In the presence of CCI the Bayesian DFE with scalar channel states can be presented as

$$f_{DFE,CCI}(\mathbf{r}(k)|\hat{\mathbf{s}}_f(k) = s^j) = \frac{\sum_{i=1}^{n_{sf}} \sum_{\alpha=1}^{n_{s,co}} w_i \left\{ \prod_{l=0}^{m-1} \exp\left(-\frac{|r(k-l)-c_{il}^{jo}-c_{co,l}^{\alpha}|^2}{2\sigma_\eta^2}\right) \right\}}{\sum_{i=1}^{n_{sf}} \sum_{\alpha=1}^{n_{s,co}} \left\{ \prod_{l=0}^{m-1} \exp\left(-\frac{|r(k-l)-c_{il}^{jo}-c_{co,l}^{\alpha}|^2}{2\sigma_\eta^2}\right) \right\}} \tag{21}$$

Here $c_{co,l}^{\alpha}$ corresponds to the $(l+1)$ component of vector co-channel state $\mathbf{c}_{co,\alpha}$. The scalar α corresponds to the specific co-channel state being considered. Like the channel states the scalar co-channel states are taken from a set of $M_1 = 2^{p_1}$ scalar co-channel states. The equaliser presented in eqn. 17 and its normalised form (eqn. 21) provide the same decision function but the equaliser in eqn. 21 can be implemented with lower computational complexity as it can take advantage of the regular array of the channel states and the time shifting property of the equaliser input.

A comparison of the computational complexity of the equalisers is presented in Table 1. It is evident that the normalised form of Bayesian equaliser with scalar states uses a smaller number of additions, divisions and exponential functions with a slight increase in multiplication operations with respect to the Bayesian equaliser decision function using vector channel states.

Table 1: Computational complexity comparison for alternate implementation of Bayesian DFE with CCI, ISI and AWGN

Bayesian CCI_DFE (eqn. 17)	Computation aspects	NBEST DFE with CCI (eqn. 21)
$2mn_{sf}n_{s,co}$	addition	$n_{sf}n_{s,co} + M_0M_1$
$mn_{sf}n_{s,co}$	multiplication	$mn_{sf}n_{s,co} + M_0M_1$
$n_{sf}n_{s,co} + 1$	division	$M_0M_1 + 1$
$n_{sf}n_{s,co}$	exponential	M_0M_1

$M_0 = 2^{p_0}$, $n_{sf} = 2^{p_0+m-1}/2^d$, $M_1 = 2^{p_1}$ and $n_{s,co} = 2^{p_1+m-1}$

4 Fuzzy implementation of Bayesian CCI_DFE

We have derived the Bayesian DFE decision function for channels with CCI, ISI and AWGN in Section 3. The Bayesian equaliser implemented with scalar channel states can provide the same decision function with a substantial reduction in computational complexity. The equalisers in eqns. 17 and 21 use $n_{sf}n_{s,co}$ channel states in decision function. However, in the presence of two or more co-channels the number of channel states in the decision function increases by a factor equal to the number of co-channel states for each co-channel. The NBEST can be implemented with fuzzy systems [22]. For this implementation the Bayesian equaliser in eqn. 20 can be represented as

$$f(\mathbf{r}(k)|\hat{\mathbf{s}}_f(k) = s^j) = \frac{\sum_{i=1}^{n_{sf}} w_i \left\{\prod_{l=0}^{m-1} \psi_{il}^{jo}\right\}}{\sum_{i=1}^{n_{sf}} \left\{\prod_{l=0}^{m-1} \psi_{il}^{jo}\right\}} \tag{22}$$

where

$$\psi_{il}^{jo} = \exp\left(-\frac{|r(k-l) - c_{il}^{jo}|^2}{2\sigma_\eta^2}\right) \tag{23}$$

The DFE presented in eqns. 22 and 23 represents a fuzzy system with gaussian membership function (eqn. 23), product inference, Singleton's fuzzifier and centre of gravity defuzzifier [27]. Modification of the membership function to

$$\psi_{il}^{jo} = \bigwedge_{\alpha=0}^{M_1-1} \left\{\exp\left(-\frac{|r(k-l) - c_{il}^{jo} - c_{co,\alpha}|^2}{2\sigma^2}\right)\right\} \tag{24}$$

can provide a powerful tool to compensate CCI. Here $\bigwedge$ refers to the maximum of the exponential function for different values of α ranging from 0 to $M_1 - 1$. In this membership function equation σ is the spread parameter to be optimally selected. This membership function equation finds the maximum of the membership function with scalar co-channel states corresponding to each of the scalar channel states and is used to find the equaliser decision function given by eqn. 22. In implementation this membership function can be evaluated with the following function with a substantial reduction in computational complexity:

$$\psi_{il}^{jo} = \exp\left(-\frac{\left\{\bigvee_{\alpha=0}^{M_1-1} |r(k-l) - c_{il}^{jo} - c_{co,\alpha}|\right\}^2}{2\sigma^2}\right) \tag{25}$$

Here $\bigvee$ performs minimum operation to find the smallest absolute distance for values of α ranging from 0 to $M_1 - 1$. The schematic of the co-channel equaliser with fuzzy implementation is shown in Fig. 3. Here the input scalar is fed to the membership function generator which are centred at the scalar channel states. The output of the membership function generator is delayed and this forms the membership function for previous received signal samples. The product block has n_{sf} sub-blocks and each of these sub-blocks receive membership functions from one of the centres corresponding to each input scalar. These membership functions are suitably combined to provide the modified channel state output. The membership function generators consist of M_0 membership function sub-blocks. Each of the sub-blocks has M_1 centres. The closest centre to the corresponding input scalar provides the membership function to the product block. The product blocks corresponding to the positive and the negative channel

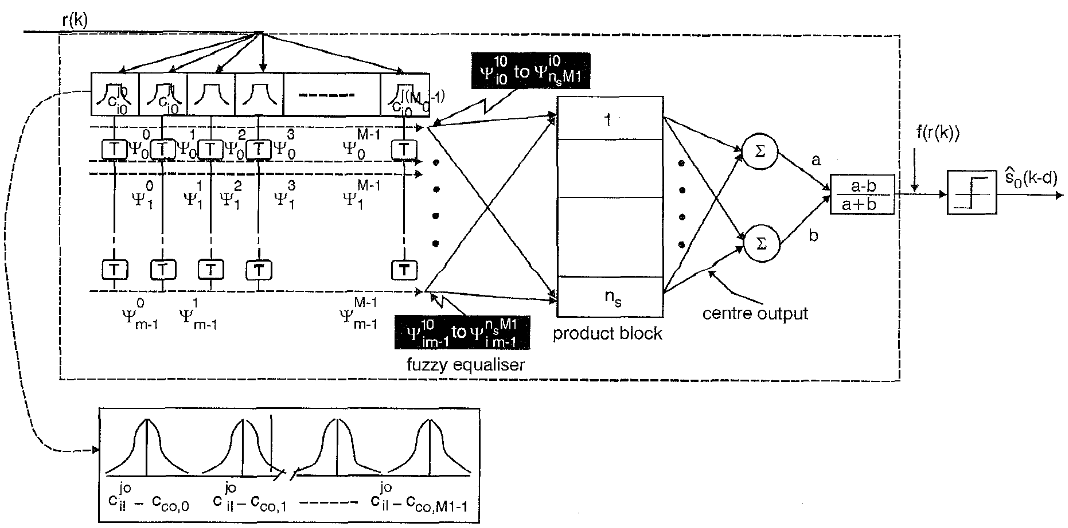

Fig. 3 *Schematic of fuzzy co-channel equaliser*

states are suitably combined to provide the equaliser output.

To study the effectiveness of fuzzy equaliser in CCI compensation consider a channel with impulse response

$$H_{ch0}(z) = 0.5 + z^{-1} \tag{26}$$

corrupted with CCI from the channel

$$H_{ch1}(z) = \lambda(1 + 0.2z^{-1}) \tag{27}$$

The SIR can be varied by varying the factor λ. We observe the decision boundaries for the equalisers without decision feedback for SNR of 15dB under SIR of 5 and 10dB which correspond to SINR = 4.586dB and SINR = 8.8067dB, respectively. The decision boundary provided by the Bayesian CCI equaliser and the fuzzy CCI equalisers is plotted in Fig. 4. From the decision boundaries it is observed that CCI compensation is essential at 5dB SIR but not essential at 10dB SIR in this problem. However, fuzzy co-channel equaliser provides near optimal decision boundary with only eight channel states in comparison to Bayesian CCI equaliser using 64 channel states.

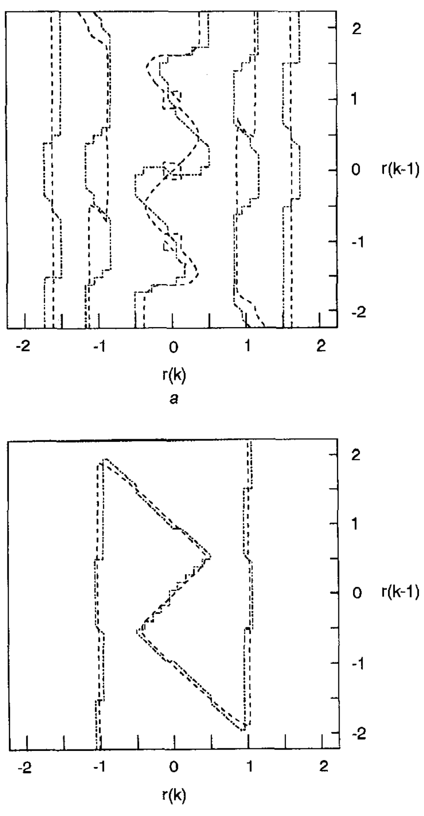

Fig. 4 *Comparison of decision boundary formed by fuzzy CCI equaliser and Bayesian CCI equaliser with channel $H_{ch0}(z) = 0.5 + z^{-1}$ and co-channel $H_{ch1}(z) = 1 + 0.2z^{-1}$*
a SIR = 5dB
b SIR = 10dB
– – – – fuzzy CCI equaliser
– · – · optimum (Bayesian) CCI equaliser

4.1 Adaptive implementation

The fuzzy CCI_DFE discussed can be trained in two steps. The first step in training involves estimation of the scalar channel and scalar co-channel states and the second step involves learning weights with the LMS algorithm.

4.1.1 Step 1. Determination of channel and co-channel states: The scalar channel and scalar co-channel states of the equaliser can be estimated by a clustering algorithm. The scalar channel states can be determined from the noisy received scalars with the help of a training signal via a supervised clustering algorithm. The noise and co-channel state being zero mean, their effect will cancel in the process of channel state estimation. The SINR can also be estimated during this period. The scalar channel states estimated, along with the training signal sequence producing them, can be arranged to form the vector channel states [17]. Once the channel states have been determined the channel residue $r_{res}(k) = r(k) - c_s^j$ (c_s^j is the scalar channel state) arising from the CCI and AWGN can be used to estimate co-channel state and noise variance with an unsupervised clustering algorithm such as k-means or improved k-means [17].

4.1.2 Step 2. Weight training: On completion of the channel and co-channel scalar state estimation, the equaliser can be constructed (Fig. 3). The initial weights w_i of the equaliser can be assigned as +1 if $\mathbf{c}_i \in n_s^+$ else they can be assigned as –1. The LMS algorithm can be used to fine tune the equaliser weights so as to reduce the error at the equaliser output due to error in the channel states estimate.

4.2 Advantages of fuzzy implementation of Bayesian CCI_DFE

The advantages of fuzzy CCI_DFE over Bayesian CCI_DFE (eqn. 17) are as follows:

• The fuzzy CCI_DFE can provide the near optimal decision function with substantial reduction in computational complexity. The computational complexity of this equaliser is compared with Bayesian CCI_DFE and Bayesian DFE in Table 2.

Table 2: Computational complexity comparison of Bayesian CCI_DFE, Fuzzy CCI_DFE and Bayesian DFE

Computation aspects	Bayesian CCI_DFE (eqn. 17)	Fuzzy CCI_DFE (eqns. 22, 25)	Bayesian DFE (eqn. 16)
Addition	$2mn_{sf}n_{s,co}$	$n_{sf} + M_0M_1$	$2mn_{sf}$
Multiplication	$mn_{sf}n_{s,co}$	$mn_{sf} + M_0$	mn_{sf}
Division	$n_{sf}n_{s,co} + 1$	$M_0 + 1$	n_{sf}
Exponential	$n_{sf}n_{s,co}$	M_0	n_{sf}

$M_0 = 2^{p_0}$, $n_{sf} = 2^{p_0+m-1}/2^d$, $M_1 = 2^{p_1}$ and $n_{s,co} = 2^{p_1+m-1}$

• The fuzzy CCI_DFE (eqn. 22) uses the fuzzy equaliser with a modified membership function preprocessor. This makes it very flexible. The co-channel compensation module can be introduced when the SIR drops below acceptable limits.

• The scalar channel and co-channel states provide a suitable method to find the condition under which co-channel compensation is not essential. This condition can be represented as

$$\bigvee_{\substack{i=0\\ j=i}}^{\substack{i=M_0-2\\ j=M_0-1}} (c_i - c_j) \geq 2 \bigwedge_{k=0}^{M_1} c_{co,k} \tag{28}$$

In this inequality, the left-hand side represents the smallest distance between any two scalar channel states and the right side represents the maximum scalar co-channel state corresponding to any channel state. If this condition is not true then co-channel compensation may be required.

- Training the fuzzy CCI equaliser is much simpler as it uses scalar unsupervised clustering for the co-channel state estimate. Whereas Bayesian CCI_DFE will require unsupervised vector clustering for co-channel states estimation, unsupervised vector clustering requires a longer training sequence and its performance cannot be guaranteed.

5 Simulation results

For the purpose of validation of the fuzzy CCI_DFE developed in the preceding sections extensive simulations were carried out. We considered the following channel and co-channel models for simulation:

$$H_{ch}(z) = 0.3482 + 0.8704z^{-1} + 0.3482z^{-2} \quad (29)$$

$$H_{co1}(z) = \lambda(0.5 + 0.81z^{-1} + 0.31z^{-2}) \quad (30)$$

In the simulations SIR was set to 5 and 10dB. The BER was considered as the parameter for evaluation of equaliser performance. The transmitted signal constellation was set to $\{\pm 1\}$ keeping the transmitted power unity. In all the simulation studies the DFE parameters were set to $m = 3$, $d = 2$ and $q = 2$, and the detected symbols were fed back to the equaliser input.

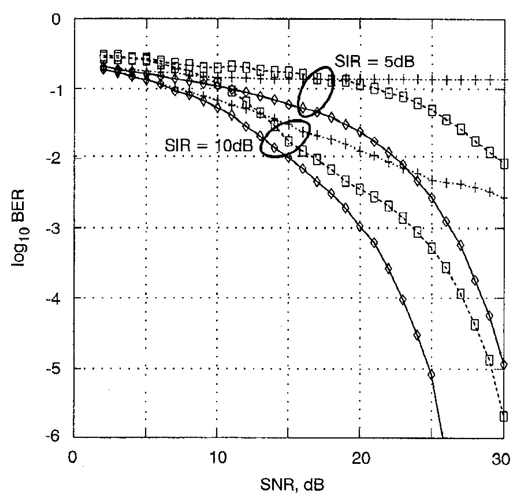

Fig. 5 *BER performance for different equalisers under SIR = 5 , 10dB with channel $H_{ch0}(z) = 0.3482 + 0.8704z^{-1} + 0.3482z^{-2}$, co-channel $H_{ch1}(z) = 0.5 + 0.81z^{-1} + 0.31z^{-2}$ with estimated channel and co-channel states*
—◇— Bayesian CCI_DFE
····+···· Bayesian DFE
- -□- - fuzzy CCI_DFE

The BER performance of the Bayesian CCI_DFE (eqn. 17), Bayesian DFE (treating CCI as noise) (eqn. 16), fuzzy CCI_DFE (eqn. 22 with eqn. 25) were compared. The Bayesian CCI_DFE uses 256 of the available 1024 channel states whereas the other equalisers use only eight out of 32 channel states to estimate each of the samples. The channel states were estimated with supervised clustering algorithm and the co-channel states were estimated with unsupervised k-means clustering. The performance of the fuzzy CCI_DFE and the Bayesian DFE are compared with the optimal Bayesian CCI_DFE. Perfect knowledge of channel states, co-channel states and noise statistics were assumed for the Bayesian DFE and the Bayesian CCI_DFE. For SIR of 10dB the spread parameter σ was set to $\sigma_\eta^2 + \sigma_{co}^2$ (estimated from the supervised clustering algorithm) whereas for 5dB SIR the fuzzy CCI_DFE spread σ was set to estimated σ_η^2 (estimated from the unsupervised clustering algorithm). The scalar channel and co-channel states for the fuzzy equalisers were estimated with 500 training samples averaged over 50 experiments. The equaliser weights were trained with 500 training samples averaged over 20 experiments. The BER performance of the equalisers is presented in Fig. 5. From the BER performance of the equalisers, it is seen that under severe CCI conditions (5, 10dB) the fuzzy CCI_DFE performs close to the optimal BER, whereas the the Bayesian DFE which treats CCI as noise fails, even though both structures use the same number of channel states. From further simulations for SIR = 15dB (results not presented here) it was observed that the performance of all the equalisers is almost the same. Hence at this low SIR, compensation of the CCI is not essential.

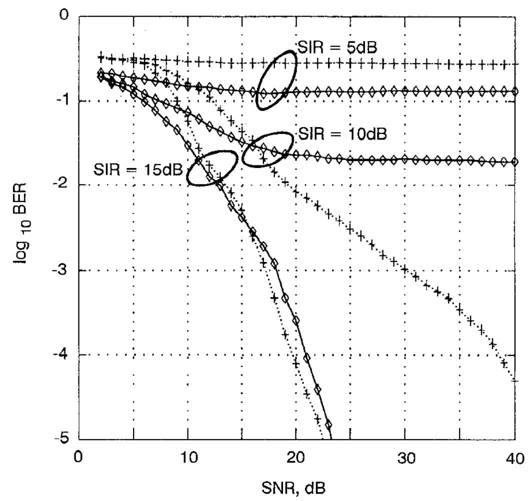

Fig. 6 *Fuzzy CCI_DFE BER performance with two co-channels, CCI = 5, 10, 15dB with channel $H_{ch0}(z) = 0.3482 + 0.8704z^{-1} + 0.3482z^{-2}$, co-channel 1 $H_{ch1}(z) = \lambda(0.5 + 0.81z^{-1} + 0.31z^{-2})$ and co-channel 2 $H_{ch2}(z) = \lambda_1(0.5 + 1z^{-1})$*
—◇— Bayesian DFE
····+···· fuzzy CCI_DFE

A further experiment considered the performance of the fuzzy equaliser in a channel corrupted with two co-channel interferers. The same system model as used in previous examples was considered. Here the second co-channel impulse response was assumed to be $H_{co2}(z) = \lambda_1(1.0 + 0.2z^{-1})$. The co-channel weights λ and λ_1 were adjusted to divide the co-channel power equally between the two co-channels. The optimal Bayesian CCI_DFE for this problem would involve evaluation of 4096 out of 16348 channel states and was not simulated owing to its impracticability of implementation. We compare the BER performance of the fuzzy CCI_DFE and the Bayesian DFE under CCI of 5, 10 and 15dB. The actual number of scalar co-channel states in this problem is $2^{p_1} * 2^{p_2} = 32$. In the simulation studies only eight co-channel states were estimated. The BER performance of the fuzzy CCI_DFE and Bayesian DFE is presented in Fig. 6. From the simulation results it is seen that the fuzzy CCI_DFE fails under severe CCI (SIR = 5dB) with multiple co-channels. But under moderate CCI (SIR = 10dB) it is able to perform better than the Bayesian DFE using the same number of channel states. The Bayesian DFE fails to provide a

BER of better than $10^{-1.7}$ even under infinite SNR, but the fuzzy CCI_DFE BER performance shows improvement with an increase in SNR. However, under 15dB SIR the effect of co-channel compensation is minimal and the fuzzy CCI_DFE performs only marginally better.

To investigate the effect of the number of estimated co-channel states on fuzzy equaliser BER performance, the number of scalar states in the unsupervised clustering algorithm was varied and the equaliser BER performance was evaluated. The performance of the fuzzy CCI_DFE for 4, 8, 16 and 32 co-channel states, for SIR = 10dB is presented in Fig. 7. It is seen that assuming a very small number of co-channel states degrades the equaliser performance substantially. With the assumption of 8, 16 or 32 co-channel states, the performance tradeoff is small. However, the performance of the equaliser with fewer number of co-channel states is nearer to the Bayesian DFE as seen from Fig. 7 in conjunction with Fig. 6.

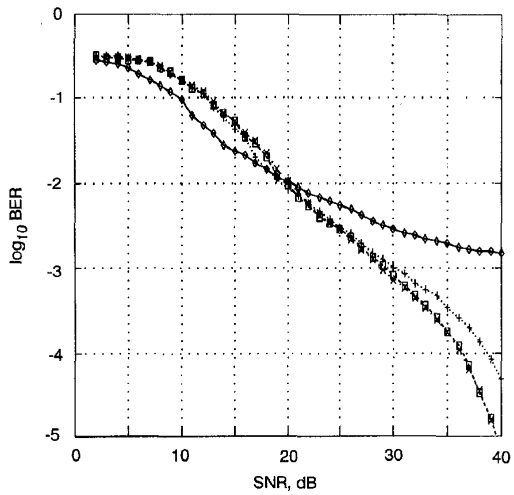

Fig.7 *Effect of number of co-channel clusters on fuzzy CCI_DFE performance for CCI = 10dB with channel IR $H_{ch0}(z) = 0.3482 + 0.8704z^{-1} + 0.3482z^{-2}$, co-channel 1 IR $H_{ch1}(z) = 0.5 + 0.81z^{-1} + 0.31z^{-2}$ and co-channel 2 IR $H_{ch2}(z) = 0.5 + 1z^{-1}$*

—◇— co-channel states = 4
····+···· co-channel states = 8
– –□– – co-channel states = 16
····×···· co-channel states = 32

In the earlier Section we stated the condition which under which CCI compensation is not essential (eqn. 28). The scalar co-channels are estimated by unsupervised clustering and in low SNR conditions the estimation of the scalar co-channel states is not accurate. From the simulation studies we found the following rule to determine the necessity of implementing CCI compensation.

• The scalar co-channel states can be determined with an assumption of $p_1 = 1$ and $p_1 = 3$ ($p_1 > 3$ does not provide much performance improvement). This would provide $M_1 = 2$ and $M_1 = 8$ scalar co-channel states, respectively.

• If the scalar co-channel for $M_1 = 2$ is less than half the distance between the closest scalar channel states, co-channel compensation is not necessary. Otherwise the scalar co-channel states estimated with $p_1 = 3$ should be used to modify the membership function generation so as to incorporate CCI compensation.

6 Conclusion

We have implemented a new elegant fuzzy CCI_DFE which performs close to the optimal Bayesian CCI_DFE with substantial reduction in computational complexity. This equaliser can be easily modified in the presence of CCI providing a low computational complexity equaliser for high SIR, and a more complex structure for low SIR conditions. Simulation studies have demonstrated the equaliser's performance. In certain applications where the DFE structure cannot be used, the fuzzy CCI equaliser can provide greater computational reduction than the Bayesian equaliser. This equaliser can provide further performance to computational complexity tradeoff with the introduction of combination of minimum inference and maximum defuzzification rules [22].

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