APPARENT SHEAR STRESS IN AN UNSYMMETRICAL COMPOUND CHANNEL FLOW

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ABSTRACT

When the river has two equal adjacent flood plains on both sides of it, is known symmetric compound channel otherwise it is called as unsymmetrical compound channel. In an unsymmetrical compound open channel flow, much of the hydraulic resistance may be ascribed by channel and flood plain geometry, roughness and flow characteristics with other factors. An expression to predict this boundary shear distribution in its unequal flood plains has been presented. From this expression, the apparent shear at vertical interface can easily be perceived. However, the precise prediction of apparent shear at junctions becomes arduous when the river is unsymmetrical instead of a symmetric one. The inequality of apparent shear force prevailing at the junctions of an unsymmetrical compound channel has been quantified in terms of apparent shear stress ratio. The consequence of the analysis for predicting the apparent shear stress ratio is analyzed through some natural river data sets. Moreover, the current approach is also capable of quantifying the intensity of momentum transfer at both the vertical interfaces and for computation of individual flood plain discharge by reasonable accuracy.

Keywords: *Apparent shear, boundary shear, shear stress ratio, momentum transfer, flood plain discharge*

1. INTRODUCTION

The natural rivers extend laterally and form a large area of flow in the time of flood generally known as compound river section. So the main river channel has some discharge throughout the year while the adjoining flood plain area remains dry yet perform a vital role during flood. If the compound section has two equal side flood plains, it is known as symmetric compound section otherwise it is unsymmetrical one. In much engineering works, prediction of discharge capacity, velocity distribution and boundary shear distribution are indispensible to be carried out. The lateral exchange of momentum generally occurs at the shear layer at the junction between faster moving main channel and slower moving flood plain. For symmetric compound channels, the equal discharge distribution occurs in both flood plains. But for unsymmetrical cases, the adjacent flood plains carry unequal discharge distribution as there is unequal momentum exchange occurs at both junctions. Research on this has been made generally for symmetrical compound channels and very limited work has been done on unsymmetrical compound channels.

Many investigators developed relation for distribution of boundary shear stress in compound channel in terms of percentage shear force distribution on flood plain ($\%S_{fp}$) such as Knight and Demetriou (1983), Knight and Hamed (1984) and Khatua and Patra (2007). Improving these models Khatua et al. (2011) developed a relationship of $\%S_{fp}$ with percentage area of flood plain $(\%A_{fp})$ taking more experimental data sets. Then Mohanty et al. (2013) observed that these above relationship fail to work for some rivers of higher width ratio and developed a new relationship for compound channels of width ratios 12. Where width ratio $\alpha = B\ /\ b$, B = Total width of the compound channel and b = bottom width of the main channel. These approaches are providing the total shear force on flood plain perimeter. So dividing this total force into two equal parts, the individual shear on each floodplain perimeter can be found out for symmetrical compound channel only. Because of the inequality in flood plain perimeter, the unequal interaction takes place at the junction of unsymmetrical compound channel. The present work mainly focuses on evaluation of momentum transfer on both the right and left junctions.

2. THEORETICAL ANALYSIS

For steady uniform flow, if we consider equilibrium of a subsection, the gravitational force balances the sum of the apparent shear force produced at vertical interface and the bed shear force along the wetted perimeter. The apparent shear stress develops on vertical interfaces at the junction between the main channel and the floodplains. The cross section of an unsymmetrical compound channel having unequal floodplains has been shown in Figure 1.

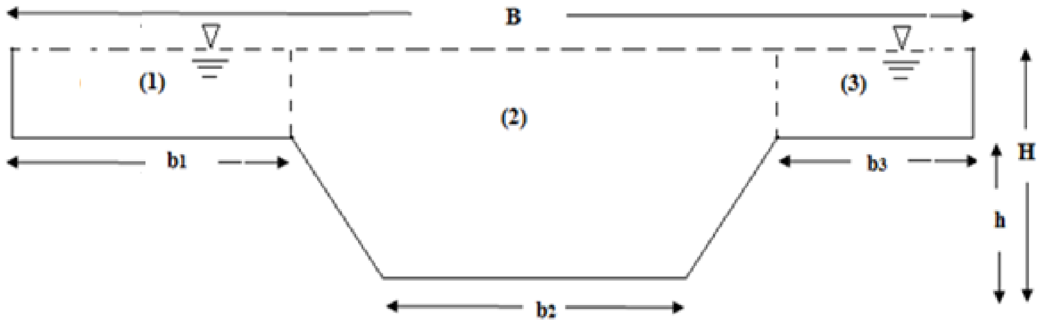


Figure 1. Cross section of an unsymmetrical compound channel

Here, considering the flow body with a longitudinal length of 1 m, the force balance relationships can be expressed as follows:

- For the left floodplain (subarea 1)

$$\rho g A_{fpl} S_0 - \tau_{fpl} P_{fpl} + \tau_{a12}(H-h) = 0 \tag{1a}$$

- For the main channel (subarea 2)

$$\rho g A_{mc} S_0 - \tau_{mc} P_{mc} - \tau_{a12}(H-h) - \tau_{a23}(H-h) = 0 \tag{1b}$$

- For the right floodplain (subarea 3)

$$\rho g A_{fpr} S_0 - \tau_{fpr} P_{fpr} + \tau_{a23}(H-h) = 0 \tag{1c}$$

where ρ = water density; g= acceleration due to gravity; S_0 =longitudinal bed slope; A_i, P_i, and τ_i= cross-sectional area, the wetted perimeter and the boundary shear stress in subarea i, respectively; the subscripts mc , fpl and fpr refer to main channel, left flood plain and right flood plain of the compound channel; $H-h$= flow depths on vertical interfaces; B= total width of the compound channel; τ_{a12} and τ_{a23}= mean apparent shear stress at vertical interface.
From equations 1(a) and 1(c), the ratio of the apparent shear force (k) on the left interface and right interface can be described by

$$k = \frac{\tau_{a12}(H-h)}{\tau_{a23}(H-h)} = \frac{\tau_{a12}}{\tau_{a23}} \tag{1d}$$

Further , k can be expressed as

$$k = \frac{\tau_{fpl} P_{fpl} - \rho g A_{fpl} S_0}{\tau_{fpr} P_{fpr} - \rho g A_{fpr} S_0} \tag{1e}$$

Dividing the total weight component along longitudinal direction $(\rho g A S_0)$, in both numerator and denominator in equation (1e) and taking percentage, one can get

$$k = \frac{\%A_{fpl} - \%S_{fpl}}{\%A_{fpr} - \%S_{fpr}} \tag{1f}$$

where A is the total cross sectional area of compound channel.
where, $\%A_{fpl}$, $\%A_{fpr}$ and $\%S_{fpl}$, $\%S_{fpr}$ are the percentage of corresponding flow area and percentage of shear force in left and right flood plain respectively. For $\%S_{fpl}$ and $\%S_{fpr}$, the equation (2i) has been derived which is described later.

2.1 Determination of boundary shear distribution

For compound channels having different width ratios, various investigators such as Knight and Demetriou (1983), Knight and Hamed (1984) and Khatua and Patra (2007) developed expressions for percentage shear force on flood plains $(\%S_{fp})$. The sequential developments of the expressions for boundary shear ($\%S_{fp}$) for different compound sections by different investigators are briefly outlined below.

(1) Knight and Demetriou (1983) developed an equation which is valid for compound channels with α up to 4 and represented $\%S_{fp}$ as

$$\%S_{fp} = 48(\alpha - 0.8)^{0.289}(2\beta)^{m} \quad (2a)$$

where, β is the relative flow depth defined by the ratio of the flow depth over flood plain and main channel.

(2) Knight and Hamed (1984) modified the previous model for channels having non-homogeneous roughness as

$$\%S_{fp} = 48(\alpha - 0.8)^{0.289}(2\beta)^{m}\{1 + 1.02\sqrt{\beta}log\gamma\} \quad (2b)$$

The exponent m can be evaluated from the relation

$$m = {}^{1}/_{[0.75e^{0.38\alpha}]} \quad (2c)$$

(3) Khatua and Patra (2007) presented another model for α valid up to 5.25 and given by

$$\%S_{fp} = 1.23(\beta)^{0.1833}(38Ln\alpha + 3.6262)\{1 + 1.02\sqrt{\beta}log\gamma\} \quad (2d)$$

(4) Khatua et al. (2011) modified the previous expression for α up to 6.67

$$\%S_{fp} = 4.1045(\%A_{fp})^{0.6917} \quad (2e)$$

(5) Mohanty and Khatua (2014) developed a new relationship for $\alpha \leq 12$

$$\%S_{fp} = 3.3254(\%A_{fp})^{0.7467}\{1 + 1.02\sqrt{\beta}log\gamma\} \quad (2f)$$

As the above empirical relationships have been developed for specific ranges of width ratio so, the latest expression developed by Mohanty and Khatua (2014) can be used in present analysis as the width ratio ranging between 2 to 12. As the percentage of shear force on flood plain $(\%S_{fp})$ is a power function of the percentage of corresponding flood plain area $(\%A_{fp})$, so for a compound channel with unsymmetrical flood plains, the percentage of shear force in the left and right flood plains can be written as

$$\%S_{fpl} = 3.3254(\%A_{fpl})^{0.7467}\{1 + 1.02\sqrt{\beta}log\gamma\} \quad (2g)$$

$$\%S_{fpr} = 3.3254(\%A_{fpr})^{0.7467}\{1 + 1.02\sqrt{\beta}log\gamma\} \quad (2h)$$

So the value of apparent shear stress ratio (k), which requires $\%S_{fpl}$ and $\%S_{fpr}$ in equation (1f) can be expressed as

$$k = \frac{\%A_{fpl} - [3.3254(\%A_{fpl})^{0.7467}\{1+1.02\sqrt{\beta}log\gamma\}]}{\%A_{fpr} - [3.3254(\%A_{fpr})^{0.7467}\{1+1.02\sqrt{\beta}log\gamma\}]} \quad (2i)$$

2.2 Application of MDCM

Khatua et al. (2011) have developed an equation to quantify the momentum transfer in terms of an appropriate length of interface for symmetric compound channels. They stated the expressions of X_{mc} and X_{fp} can be expressed as

$$X_{mc} = P_{mc}\left[\frac{100}{(100-\%S_{fp})}\frac{A_{mc}}{A} - 1\right] \quad \text{and} \quad X_{fp} = P_{fp}\left[\frac{100}{\%S_{fp}}\left(\frac{A_{mc}}{A} - 1\right) + 1\right] \quad (3a)$$

After finding out the interaction length X_{mc} and X_{fp}, the total discharge in a compound channel is evaluated by putting these two values in the Manning's equation.

$$Q = \frac{\sqrt{S_0}}{n_{mc}}A_{mc}^{5/3}(P_{mc} + X_{mc})^{-2/3} + \frac{\sqrt{S_0}}{n_{fp}}A_{fp}^{5/3}(P_{fp} - X_{fp})^{-2/3} \quad (3b)$$

where, n_{mc} and n_{fp} are the manning's roughness coefficients for main channel and flood plains. The individual flow in main channel (Q_{mc}) and flood plain (Q_{fp}) can be written as

$$Q_{mc} = \frac{\sqrt{S_0}}{n_{mc}}A_{mc}^{5/3}(P_{mc} + X_{mc})^{-2/3} \quad \text{and} \quad Q_{fp} = \frac{\sqrt{S_0}}{n_{fp}}A_{fp}^{5/3}(P_{fp} - X_{fp})^{-2/3} \quad (3c)$$

They also expressed the total apparent shear force $\{2\tau_a(H - h)\}$ at both junctions in terms of boundary shear stress and interaction length as

$$2\tau_a(H - h) = \tau_{mc}X_{mc} = -\tau_{fp}X_{fp} \quad (3d)$$

where, τ_a is the mean apparent shear stress at vertical interface of a symmetrical compound channel. But for channels with unsymmetrical flood plains, the interaction lengths at left and right junctions (X_{fpl} and X_{fpr}) are different. A step is now carried out to quantify the interaction lengths X_{fpl} and X_{fpr} by following the next steps. The induced apparent shear forces $\{\tau_{a12}(H-h)\ or\ \tau_{a23}(H-h)\}$ are given by

$$\tau_{a12}(H-h) = \tau_{fpl}X_{fpl} \text{ and } \tau_{a23}(H-h) = \tau_{fpr}X_{fpr} \tag{3e}$$

So, the ratio of apparent shear force (k) in equation (1d) can be rewritten as

$$k = \frac{\tau_{a12}(H-h)}{\tau_{a23}(H-h)} = \frac{\tau_{fpl}X_{fpl}}{\tau_{fpr}X_{fpr}} \tag{3f}$$

Dividing the total weight component along longitudinal direction $(\rho g A S_0)$ and $P_{fpl}P_{fpr}$, in both numerator and denominator in equation (3f) and taking percentage, one can get k

$$\text{Then, } \frac{X_{fpl}}{X_{fpr}} = k\frac{\%S_{fpr}P_{fpl}}{\%S_{fpl}P_{fpr}} \tag{3g}$$

The total interaction length for flood plain perimeter (X_{fp}) is the sum of X_{fpl} and X_{fpr}. So the second part of equation (3a) can be written as

$$X_{fpl} + X_{fpr} = P_{fp}\left[\frac{100}{\%S_{fp}}\left(\frac{A_{mc}}{A} - 1\right) + 1\right] \tag{3h}$$

So, solving equation 2(i), 3(g) and 3(h), the values of X_{fpl} and X_{fpr} can be calculated. Then, the apparent shear force distribution in each subsection of an unsymmetrical compound section can be obtained by following the formulae given in equation (4) as

For left flood plain area

$$\tau_{fpl}X_{fpl} = X_{fpl}\frac{\%S_{fpl}\rho g A S_0}{P_{fpl}100} \tag{4a}$$

For right flood plain area

$$\tau_{fpr}X_{fpr} = X_{fpr}\frac{\%S_{fpr}\rho g A S_0}{P_{fpr}100} \tag{4b}$$

The total apparent shear force can be obtained by adding the sub sectional discharges demonstrated in equation (4a) and (4b).

3. EXPERIMENTAL AND FIELD DATA

The analytical model so developed involves the geometrical, hydraulic and roughness parameters that are likely to be important as input parameters for its solution. Although the available experimental compound channel data sets are symmetric in cross section however in practical field, the natural rivers exhibit inequality in their flood plains. So, for applying the current approach to estimate the apparent shear force in every sub compartments of a symmetrical cross section, experimental data sets from Flood channel facility, UK channels (FCF-A) and Knight and Hamed (1984) have been utilised here. The FCF series 1, FCF series 2 and channels are smooth channels having equal surface roughness on both main channel and flood plain perimeters. Then, the method is employed to rough compound channels of Knight and Hamed (1984) having differential roughness on main channel and flood plains.

Finally for unsymmetrical cross section, a natural river data set i.e., river Severn has been considered (Knight 1989). The cross section of the river is given in Figure 2. Three flow depths are considered from the depth averaged velocity distribution plots given in McGahey et al. (2006)..

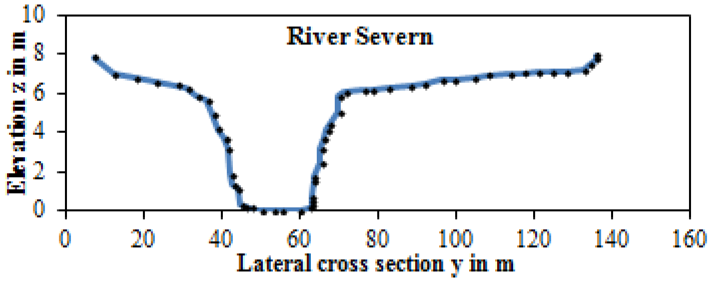


Figure 2. Cross sectional details of river Severn (Knight 1989)

4. RESULTS AND ANALYSIS

4.1 Application of analytical method for flow distribution

After estimation of percentage shear force on flood plains $\left(\%S_{fp}\right)$, individual shear on left flood plain $\left(\%S_{fpl}\right)$ and on right flood plain $\left(\%S_{fpr}\right)$ using equation (2f), (2g) and (2h) respectively, the three unknowns i.e., X_{mc}, X_{fpl} and X_{fpr} can be assessed. Equation 4a and 4b can then be utilized to estimate zonal apparent shear force in every sub compartments. It has been observed that, this method is predicting the zonal apparent shear force for each channel with more precision of average error not more than 2%. For representing these results graphically, percentage of apparent shear force on main channel ($\%ASF_{mc}$) has been computed for each relative flow depth and compared with the measured apparent shear force.

The results of predicted $\%ASF_{mc}$ from current approach with their measured values for four homogenous compound channels data sets have been presented in Figure 3(a). Similarly the predicted $\%ASF_{mc}$ against their observed values for two non-homogenous compound channels have been graphically demonstrated in Figure 3(b).

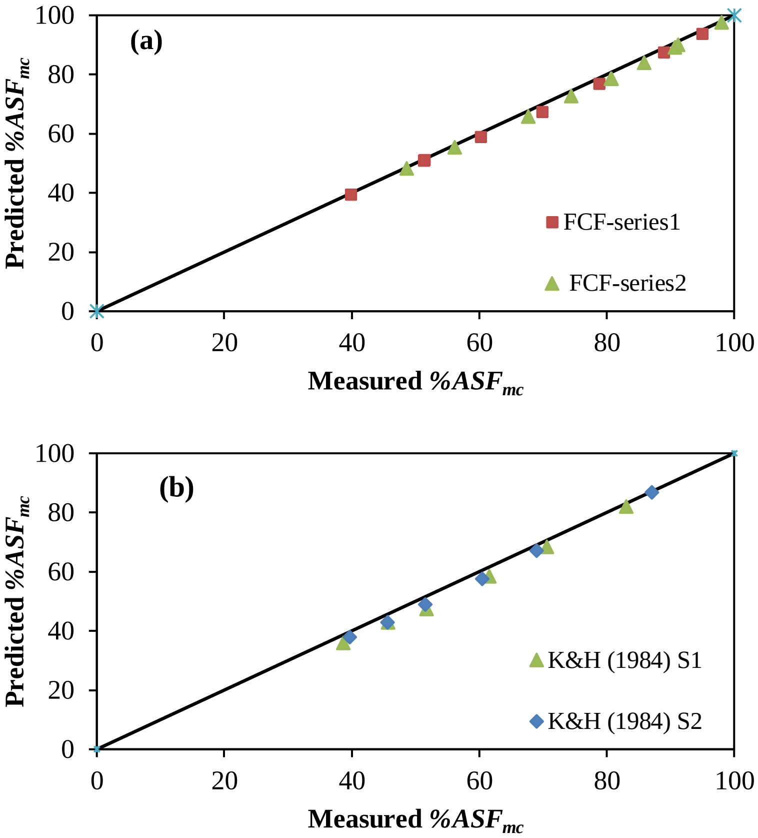


Figure 3. Predicted $\%ASF_{mc}$ from present approach with their observed values for (a) homogenous channels and (b) non-homogenous channels

This Figure 3 indicates the efficacy of the present approach for predicting the apparent shear force in main channel in terms of $\%ASF_{mc}$ with sufficient accuracy as the predicted outputs are on the line of good agreement. The overall apparent shear force for all considered channels were estimated and outlined in Figure 4 and Figure 5.

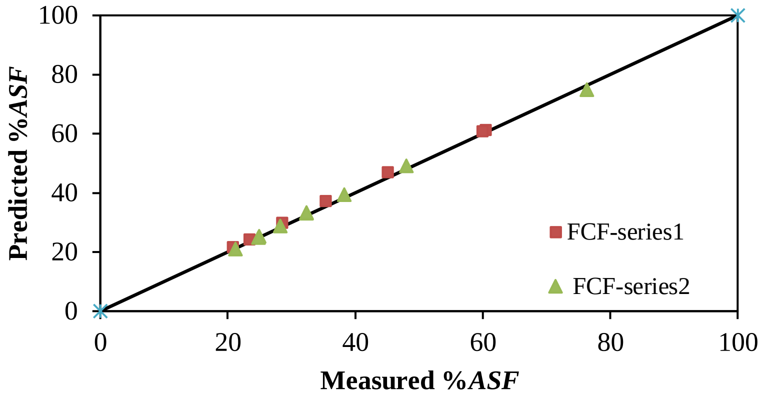


Figure 4. Predicted apparent shear force (%*ASF*) from current approach with their observed values for homogenous data sets FCF channels

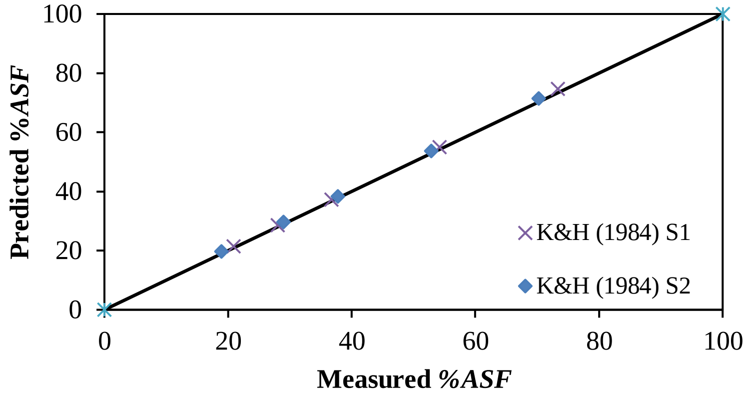
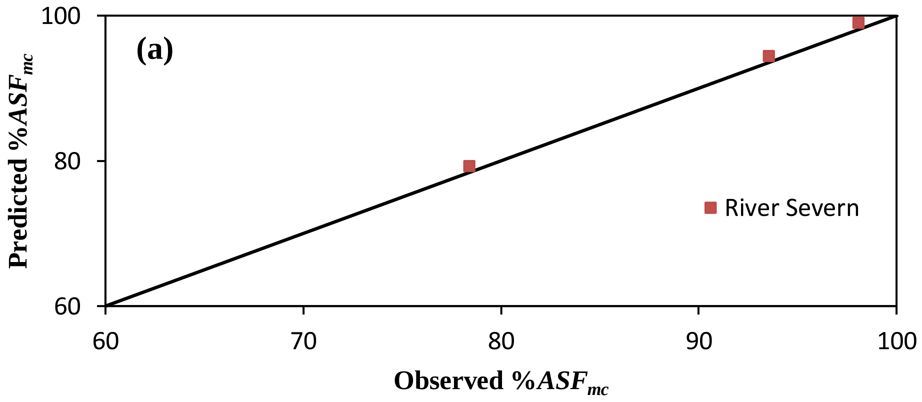


Figure 5. Predicted apparent shear force (%*ASF*) from current approach with their observed values for non-homogenous data sets Knight and Hamed (1984) channels

Figure 4 and Figure 5 provide an evidence of usefulness of current prediction approach for total apparent shear force for channels. It has been clearly seen that there are negligible deviation of predicted results from the line of good agreement. Therefore it seems that, the developed approach for apparent shear force distribution will work very well. In addition to the above symmetrical compound channel data sets, the zonal apparent shear force in each subsection has been obtained for unsymmetrical river section i.e., Severn. Previously due to the identical flood plains exist in symmetric cross sections; the apparent shear force in each flood plain is equal. To evaluate the predictive strength of the new approach for unsymmetrical case, the total apparent shear force and its distribution in every sub-compartment have been calculated for three flow depths and the results have been shown in Figure 6.



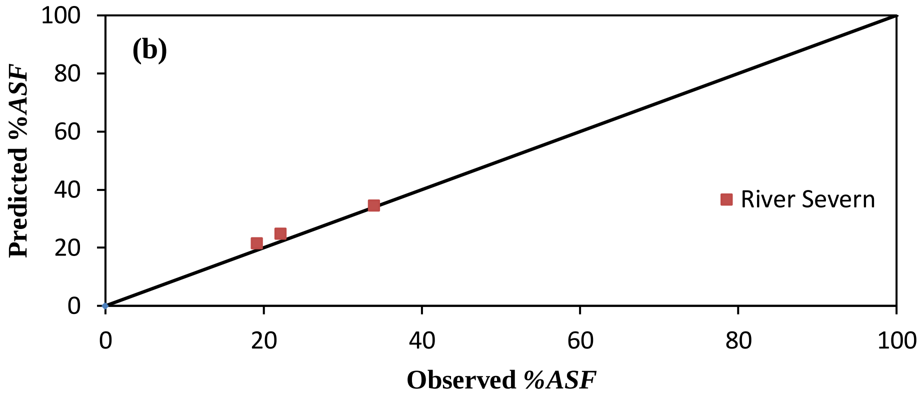


Figure 6 (a). Predicted $\%ASF_{mc}$ and **(b)** total apparent shear force $(\%ASF)$ from current approach with their observed values for river Severn

Figure 6 indicates the significance of the present model to compute the percentage apparent shear force in main channel $(\%ASF_{mc})$ and total apparent shear force $(\%ASF)$ for river Severn with acceptable errors of 1% and 6% respectively.

5. CONCLUSIONS

Three expressions for interaction length of vertical interface have been developed for suitably account the momentum exchange occurs between the main channel and floodplain. These expressions are utilised to adjust the wetted perimeters of left flood plain, main channel and right flood plain for balancing the shear force generated at interfaces.

The new approach is primarily dependent upon the percentage shear on flood plains which deploy corresponding area of flood plains as their independent parameters.

The present approach is significant for predicting total apparent shear force and zonal apparent shear force for both symmetrical and unsymmetrical compound channels associated with homogenous or non-homogenous roughness on the subsection perimeters.

The applicability of the model has been verified with small scale and large scale data sets and has been found to give satisfactory results with minimum error when compared with models of previous investigators.

The errors found for river Severn by the new approach illustrates that this method is capable of providing zonal apparent shear force and total discharge with more accuracy than the other existing model chosen for comparison and hence can be applicable for other real word cases.

REFERENCES

Ghosh S, Jena SB (1971) Boundary shear distribution in open channel compound. Proceedings of the Institution of Civil Engineers 49(4):417-30.

Khatua KK, Patra KC, Mohanty PK (2011) Stage-discharge prediction for straight and smooth compound channels with wide floodplains. Journal of Hydraulic Engineering 138(1):93-9.

Khatua KK, Patra KC, Nayak P, SAHOO N (2012) Stage–discharge prediction for meandering channels. International Journal of Computational Methods and Experimental Measurements 1(1):80-92.

Khatua KK, Patra KC (2007) Boundary shear stress distribution in compound open channel flow. ISH Journal of Hydraulic Engineering 13(3):39-54.

Khatua KK, Patra KC (2009) Flow distribution in meandering compound channel. ISH Journal of Hydraulic Engineering 15(3):11-26.

Knight DW, Demetriou JD (1983) Flood plain and main channel flow interaction. Journal of Hydraulic Engineering 109(8):1073-92.

Knight DW, Hamed ME (1984) Boundary shear in symmetrical compound channels. Journal of Hydraulic Engineering 110(10):1412-30.

Knight DW (1989) River channels and floodplains [R]. Final Report for severn trent water authority, severn trent water authority. 1989:1-00.

Mohanty PK, Khatua KK, Dash SS (2014) Flow prediction in two stage wide compound channels. ISH Journal of Hydraulic Engineering 20(2):151-60.

Mohanty PK, Khatua KK (2014) Estimation of discharge and its distribution in compound channels. Journal of Hydrodynamics, Ser. B. 26(1):144-54.

McGahey C, Samuels PG, Knight DW (2006) A practical approach to estimating the flow capacity of rivers–application and analysis. Lisbon, London: Taylor & Francis; 2006 Sep 14.