Crack Effects on Rotors Using Mode-I Failure Model with Transfer Matrix Approach

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Abstract.
This paper presents, a crack identification in a rotor dynamic system based on changes in natural frequencies predicted from the transfer matrix approach using a crack point matrix defined in terms of stress intensity factors. A local flexibility of the system due to crack is employed and changes in natural frequencies of the system are illustrated. Effect of cracked and it is location along the length are studied by using some interpolation formula. A generalized transfer matrix approach is applied after validating with the finite element model. Analytical results are compared with those obtained from numerical model through ANSYS. Fatigue analysis of cracked rotor shaft system containing the bearing at ends is performed using solid modeling tool. The results are compared and conclusions are drawn.
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Keywords: Identification, Transfer matrix, Breathing, Behavior, Single plane model.

1 Introduction

Dynamic behavior of mechanical elements is important and should be monitor to access reliability of machinery. Most of the components used in high speeds often possess flaws due to cracks formed by cyclic loading, vibration and other types of loads. Due to loss of integrity, the dynamic properties such as natural frequencies of such systems are drastically effected and requires a special attention. Compare to various non destructive techniques such as ultrasonic testing, visual examination etc, vibration based online monitoring for identification of crack location and severity has become more popular today. Vibration based techniques follow two rules for detection of cracks. First one monitors synchronous vibration amplitude and phase. If such 1X amplitude and phase exists, it implies the presence of crack. The second rule states that there also exist 2X vibration in cracked rotor at half of the resonance speed. There are several papers published on this subject and still various new methods are still under investigation. The constant rotation of the shaft with a crack has a periodic time varying stiffness characteristics. However, transfer cracks open and closed alternatively leading to breathing behavior. Here also the shaft stiffness variation is time periodic. Many investigation studied such parametric instability conditions in cracked rotor dynamics.

Meng and Gasch [1] studied stability of cracked Jeffcott rotor supported on journal bearing. Gasch [2] given an overview of cracked Laval rotor stability diagram. Nonlinear-dynamic stability analysis of shaft disk system with-crack was presented in some works [3-5]. Most of these works focused on simple systems having few degrees of freedom. Sinou [6] conducted the stability analysis by applying perturbation to non linear periodic solution and identified effect of crack on the instability region. When crack depth increases, the model must include additional flexibility term so as to generate an integral model. The stress intensity factors in the theory of fracture mechanics may be used for modeling open or breathing edge cracks. Local flexibility coefficients depend on the size of crack and crack length geometry. Often cracked section is replaced by single rotation spring representing local flexibility of crack. Investigators divided the edged crack problem into two categories. These are (1) direct problem where the effect of crack on the natural frequencies is studied (2) inverse problem where the vibration data is used to predict location and size of the cracks. Cavelini et al. [7] calculated the additional flexibility introduced by the crack using linear fracture mechanics model. Tsai and Wang [8] investigated the position and size of crack on a stationary shaft by modeling crack as joint of local spring. Darpe [9] detected fatigue transverse crack in rotating shaft by using detection methodology exploiting both the typical non linear breathing phenomenon of the crack and the coupling of bending-torsional vibration due to presence of crack for its diagnosis. According to Gomez et al. [10], who detected crack in a rotating shaft by applying Wavelet Packet transverse energy combined with ANN (Artificial Neural Network) with RBF (Radial Basis Function) architecture. Attar [11] investigated mode and frequencies of stepped beam consisting an arbitrary number of transverse cracks by using analytical approach and also calculated position and depth of crack opting transverse matrix method. Broda et al. [12] investigated longitudinal vibrations of beams with breathing cracks numerically and experimentally both. In all the above mentioned works, it is assumed that the crack depth is relatively small in comparison with rotor diameter. Also most of the literature dealt with open crack modeling. In practice, the crack depth may elongate towards the centre line of the shaft and it requires additional flexibility consideration using fracture mechanics theory. In this line, present paper formulates an approach based on mode-1 failure to model the crack in terms of its depth function and the frequency analysis of the rotor bearing system is carried out using transfer matrix approach. The breathing crack behavior is analysed in detail with variations in natural frequencies and mode shapes. The reminder of the paper is organized in the following format. Section 2 deals with mathematical modeling of the crack and rotor system. Section 3 presents results and discussion and brief conclusions are given in section 4.

2 Mathematical modeling

This section deals with description of the breathing crack model along with a transfer matrix method employed in the present work. When the crack width is very small, it is known as gaping fatigue crack which is different from notch. Gaping cracks are difficult to manufacture. The shaft should be subjected to prolonged cyclic bending fatigue to initiate the crack. Compliance of a cracked shaft is a function of uncracked shaft compliance and additional compliance induced by the crack. Dimarogomas et al. [13] employed the fracture mechanics techniques for estimating crack compliance, where the strain energy release rate was used along with linear elastic fracture mechanics.

2.1 Additional flexibility due to crack

A shaft element of length L having a transverse crack of depth h is located at the middle position as shown in Fig.1.

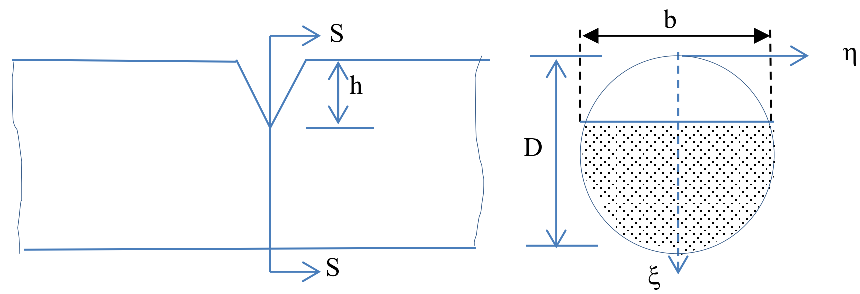

Fig.1 Cracked shaft element

By considering that there are 6 degrees of freedom at each station of the shaft element(i=1-6) the method is explained. If D= 2R is the diameter of the cross section, E is elastic modulus and υ is Poisson ratio of the material then the additional strain energy U_c is given by the integration of the strain energy density function over the cracked area A_c as follows

$$U_c = \frac{(1-\upsilon^2)}{E} \int_{A_c} \left[\left(\sum_{i=1}^{6} K_{Ii} \right)^2 + \left(\sum_{i=1}^{6} K_{IIi} \right)^2 + (1+\upsilon) \left(\sum_{i=1}^{6} K_{IIIi} \right)^2 \right] dA_c \quad (1)$$

Here K_I, K_{II}, and K_{III} are the stress intensity factors corresponding to opening, sliding and shearing modes of the crack, 'i' denotes the applied load. Based on the stress distribution on the cross section of the crack, these stress intensity factors are determined. As mode-I (opening) is predominant on the crack flexibility it is only considered in this work. This is expressed as

$$K_I = \sigma_i \sqrt{\pi a}\, f\left(\frac{h}{h_x}\right) \quad (2)$$

Where σ_i is stress at the crack due to applied load P_i and f is a shape function called crack configuration factor. The additional flexibility induced within the shaft due to crack is obtained as $c_{ij}=\frac{\partial^2 U_c}{\partial P_i \partial P_j}$. The elements of local flexibility matrix depends only on the degrees of freedom being considered for moments and forces applied on the crack cross section. The full compliance matrix is 6×6, however in present study the flexural vibration of beam in one plane (X-Y) is only considered so that the load P_5 (moment) only comes in equations. That is the component of flexibility matrix $C=c_{55}$ only exists in this case. The detailed derivation of this component for a circular sections is given in appendix.

2.2 Transfer matrix method

Transfer matrix method is useful techniques for solving the natural frequencies of the present rotor. Any section or station of the rotor is described by a state vector whose coordinates are physical quantities defining the vibration state of the section. In a 2-D bending, the state vector S is defined in terms of transverse displacement y, slope θ, bending moment M, and shear force Q. The method consists in relating the ends of entire shaft line by means of several transfer matrices. Then the boundary conditions can be applied and natural frequencies or critical speeds are calculated. Several papers have been recently focused on the applications of transfer matrix method in rotor dynamics. Albuquerque and Barbosa [14] used transfer matrix method to predict the bending critical speeds of hydro-generator shaft. Ghasemalizadeh et al.[15] analysed a shaft system using transfer matrix method by considering gyroscopic effect. Albuquerque and Barbosa[16] predicted bending critical speed of a hydrogenerator shaft using transfer matrix method. More recently, Lee and Lee [17-18] determined the solution to the free vibration characteristics of a tapered and twisted Bernoulli beam by finding the roots of differential equation. Fig.2 shows the rotor model in present work. Here, the station locations are also shown.

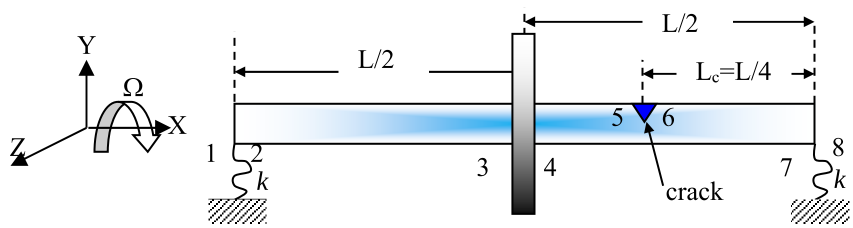

Fig.2 Cracked rotor under study with station numbers

There are 8 stations and the rotating shaft is assumed to be bending in one plane. In applying the method, there are two sets of matrices (1) field matrix which relates the distributed effects such as shaft elements (2) point matrix which relates the lumped phenomena such as springs, crack, or disk etc. Following, field and point matrices are used to compute overall transfer matrix[19].

Field matrix for the shaft elements -

$$[R_{23}(L/2)]=[R_{45}(L/4)]=[R_{67}(L/4)]=[R]=\begin{bmatrix} S & \frac{T}{a} & -\frac{U}{a^2EI} & -\frac{V}{a^3EI} \\ aV & S & \frac{T}{aEI} & -\frac{U}{a^2EI} \\ -a^2EIU & -aEIV & S & \frac{T}{a} \\ -a^3EIT & -a^2EIU & aV & S \end{bmatrix} \tag{3}$$

with $a=\sqrt{\omega}\sqrt[4]{\frac{\rho A}{EI}}$. Here, ρ and E are density and elastic modulus of shaft while A and I are cross section and moment of inertia of the section. Also, ω is natural frequency, while other terms are defined as:

$$S(al)=\frac{1}{2}\left[\cosh(al)+\cos(al)\right] \tag{4a}$$

$$T(al)=\frac{1}{2}\left[\sinh(al)+\sin(al)\right] \tag{4b}$$

$$U(al) = \frac{1}{2}\left[\cosh(al) - \cos(al)\right] \tag{4c}$$

$$V(al) = \frac{1}{2}\left[\sinh(al) - \sin(al)\right] \tag{4d}$$

Point matrix for bearing support with translation =[P_{12}]=[P_{78}]= $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ k & 0 & 0 & 1 \end{bmatrix}$ (5)

Point matrix for the disk with mass m, diametral moment of inertial I_d and polar moment of inertia I_p-

$$[P_{34}] = P_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \omega\left(-I_p\Omega + I_d\omega\right) & 1 & 0 \\ -m\omega^2 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

Here, Ω is speed of the shaft.

Point matrix for the cracked portion (discontinuity between the slopes revealed by bending moment and rotation)

$$[P_{56}] = P_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & C & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

The resultant overall transformation matrix is given by

$$[T]=[P_{78}][R_{67}][P_{56}][R_{45}][P_{34}][R_{23}][P_{12}]$$

$$= \begin{bmatrix} T_{11} & T_{12} & T_{13} & T_{14} \\ T_{21} & T_{22} & T_{23} & T_{24} \\ T_{31} & T_{32} & T_{33} & T_{34} \\ T_{41} & T_{42} & T_{43} & T_{44} \end{bmatrix} \tag{8}$$

As both ends of the shaft after the support springs have free end-conditions, the moment and shear become zero. Thus, the resultant characteristic equation is obtained as $T_{31}T_{42}$-$T_{32}T_{41}$=0. The natural frequencies of the system are obtained by solving this equation for the frequency parameter *a*.

2.3 Breathing crack

So far only the studies of rotor with transverse open crack is studied. As shaft start to revolve the crack smoothly transits from fully open to fully closed mode. So, we use a cosine function to model this behavior.
Here Mayes' model is employed as:

$$K_{maye\xi} = a_1 + b_1 \cos\theta$$
$$K_{maye\eta} = a_2 + b_2 \cos\theta \tag{9a}$$

Or $$K_{maye} = K_{maye\xi} + iK_{maye\eta} \tag{9b}$$

Where $K_{maye\xi}$, $K_{maye\eta}$ are shaft stiffness with crack along ξ and η , and $\theta = \Omega t$ is angular displacement.

$$a_1 = \frac{1}{2}\left(K_0 + K_\xi\right), b_1 = \frac{1}{2}\left(K_0 - K_\xi\right)$$
$$a_2 = \frac{1}{2}\left(K_0 + K_\eta\right), b_2 = \frac{1}{2}\left(K_0 - K_\eta\right) \tag{10}$$

Here if $\theta = \pi$, $K_{maye\xi} = K_{\xi}, K_{maye\eta} = K_{\eta}$ and if $\theta = 0$, $K_{maye\xi} = K_0 = K_{maye\eta}$. First case is called, fully open crack and second case is called fully closed crack.

3 Results and Discussion

Initially the problem is solved as direct analysis approach where the crack parameters are varied and the corresponding changes in natural frequencies are predicted. Dimensions of the rotor under consideration are given in Table 1.

Table 1 Geometric and material properties of system

Parameter	Value
Diameter of the shaft(mm)	15
Diameter of the disk(mm)	128
Thickness of the disk(mm)	10
Length of the shaft(mm)	328
Elastic modulus of shaft and disk(GPa)	200
Material density(Kg/m^3)	7850
Bearing stiffness k (N/m)	1×10^5

A computer program is developed to obtain the natural frequencies of the system with variable crack dimensions at different speeds of operation and the support stiffness values. Without crack, the intact model of the rotor is tested for its natural frequencies using transfer matrix method. A comparison of the first three frequencies using finite element analysis using Timoshenko beam theory, ANSYS solution is depicted in Table 2.

Table 2 First three natural frequencies (in Hz) of intact rotor

Method	Mode-1	Mode-2	Mode-3
Present transfer matrix	52.91	160.10	483.90
Timoshenko finite element	52.86	160.01	481.62
ANSYS	52.92	160.56	483.68

3.1 Effect of crack depth

Fig.3 shows variation of fundamental natural frequencies as function of local crack flexibility parameter.

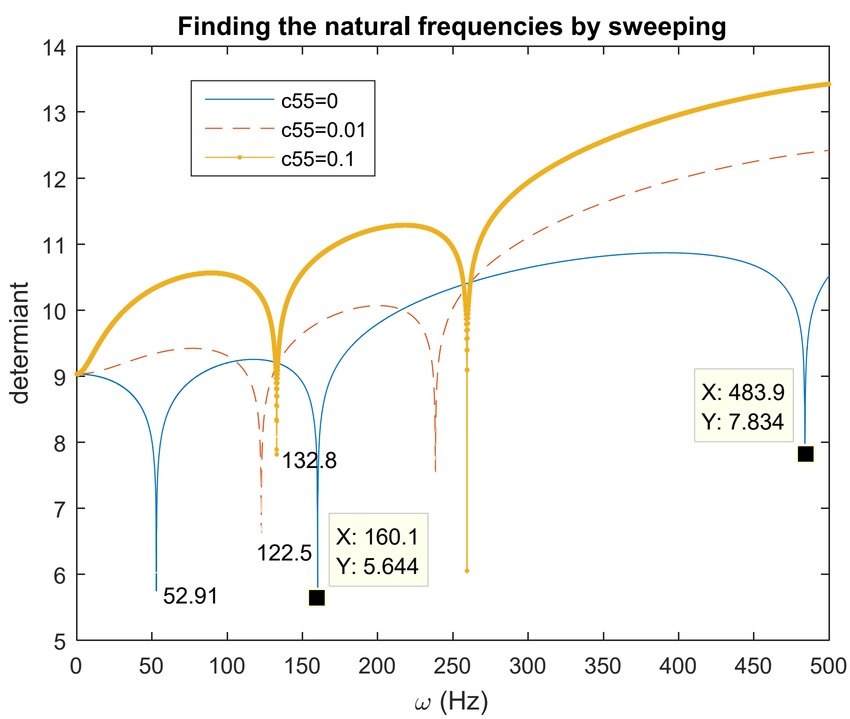

Fig.3 The natural frequencies as a function of local flexibility parameter

As is obvious, it can be seen that as flexibility reduces from 0 to 0.01, the natural frequencies first increases and then decrease. In the next step, the relative crack depth (h/D) is varied from 0.1 to 0.5 and the corresponding c55 values are computed and the three natural frequencies are obtained in a similar manner as shown in Table 3. It is seen that there is a marked effect of crack depth on the natural frequency. In this paper value of c_{55} is calculated by using the best fit formula [21] for solid circular shaft as a function of h/D. Table 3 shows the change in the value of natural frequencies at different mode and by changing the value of c_{55} obtained from above reference and crack location is fixed at 82mm from the right end of shaft.

Table 3. Variation of natural frequency with h/D

h/D	c_{55} (10^-6)	Natural frequencies(Hz) (Mode-1)	Natural frequencies(Hz) (Mode-2)	Natural frequencies(Hz) (Mode-3)
0.1	1.55	52.91	159.9	484.2
0.2	7.818	52.96	160	485
0.3	20.4	53.01	160	486.5
0.4	42.85	53.16	160.2	489.4
0.5	84.02	53.41	160.5	494.8

It is clear that the natural frequencies with c_{55} have some change with respect to uncracked rotor from mode 3 onwards.

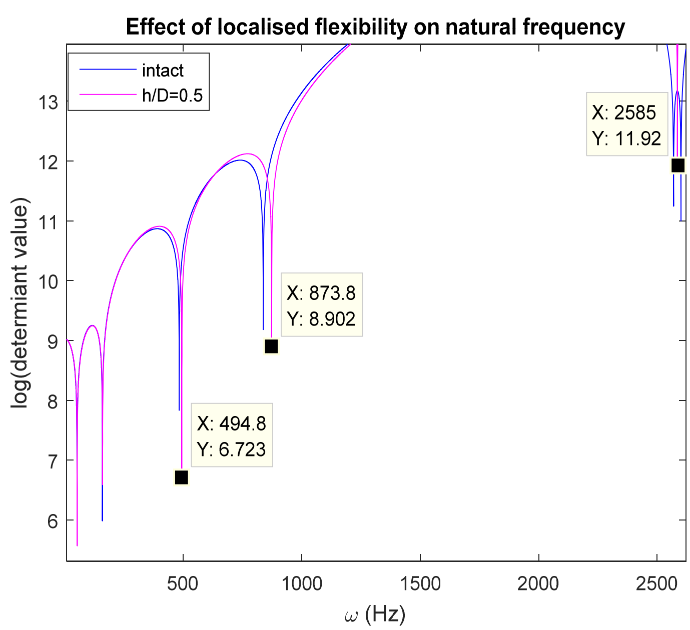

Fig.4 Variation of natural frequencies of cracked beam

3.2 Effect of crack location

Table 4 shows the effect of crack location on the natural frequencies of the system and the variation are shown in table at the fix value of h/D ratio of the crack. From table 4, it is clear that the natural frequencies vary as a function of crack location.

Table 4. Variation of natural frequencies (Hz) with the change of crack location

Non dimensional crack location L_c/L	(Mode-1)	(Mode-2)	(Mode-3)
0.167	99.26	257.3	565.7
0.25(initial location)	122.4	238.4	623.6
0.334	132.6	214.5	--

3.3 Modeling of crack in 2-D

Ansys software is used to model the crack in the present shaft and analyse the system with two ends supported on spring elements. Fig.5 shows the 2-D model of the rotor with crack. Plane 183 elements are used to mesh the model.

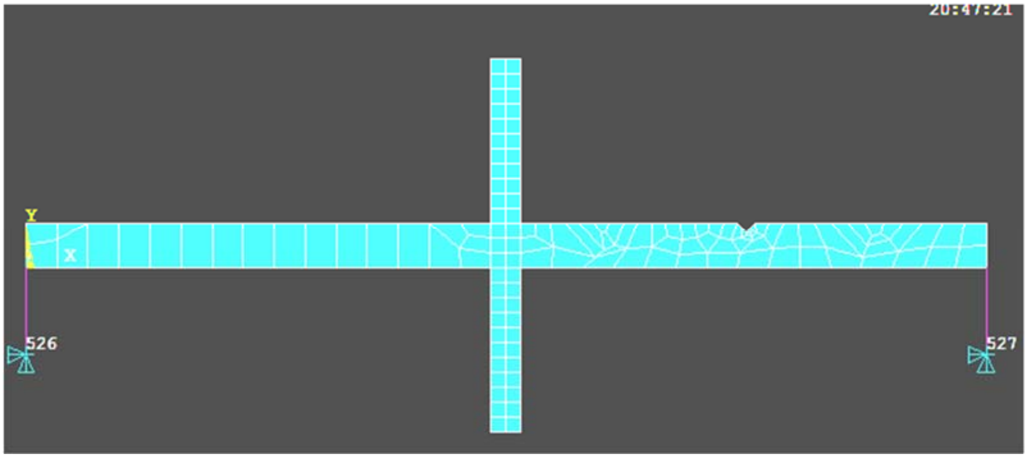

Fig.5 2-D model for rotor and shaft

Table 2 First five frequencies obtained from ANSYS

Mode	Intact	Fatigue crack(depth h=3mm, width b=6 mm)
1	10.315	10.323
2	19.420	19.433
3	646.97	642.87
4	1453.1	1441.1
5	2423.1	2419.5

3.4 Fatigue analysis of system on difference condition

Using 3D solid modeling fatigue analysis is done for with and without crack (depth 2mm and width 3mm) in ANSYS. Material used is stainless steel and rotation speed 500rps. Result about life of system is shown in fig (5a and 5b). And it is clear that life of system decreases after introduction of crack.

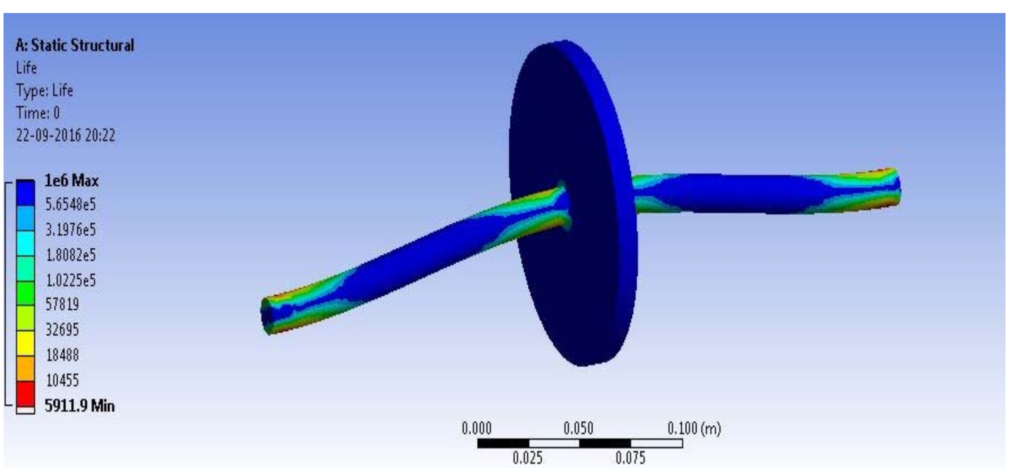

Fig 5a. Shown fatigue life of system without crack

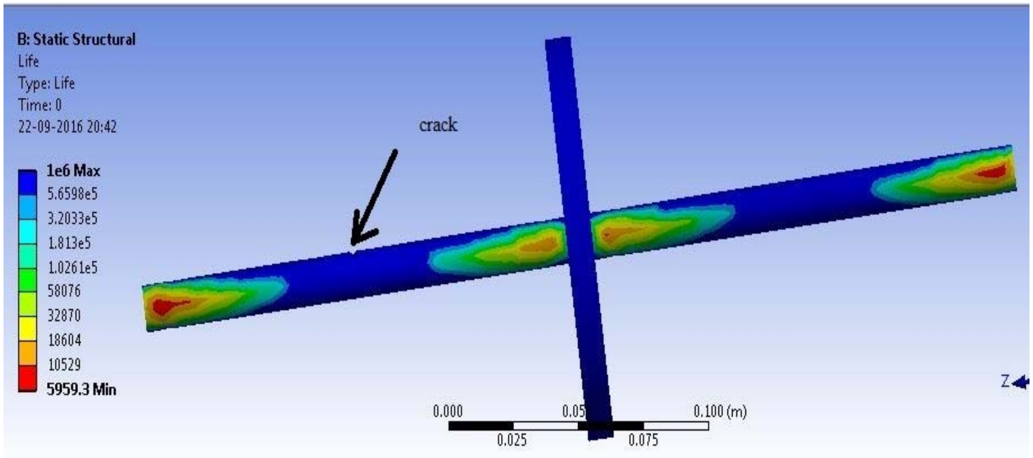

Fig 5b. Shows fatigue life of system with crack

4 Conclusions

In the context of this paper in which various analysis is done for finding natural frequency of shaft and rotor system at different position and dimension of shaft, it is clear that with variation of crack depth natural frequency increases. It can be also concluded from fatigue analysis that fatigue life of any shaft rotor system decreases with introduction of crack.

Appendix: The flexibility coefficient derivation

Stress Intensity Factor,

$$K_{I5} = \sigma_5 \times \sqrt{\pi\xi} F_{I5}$$

Here, $\sigma_5 = \frac{P_5 h}{2I}$ with P_5 as bending moment and I is moment of inertia of cross section

ξ and η are spatial variables to measure the crack length and width along crack plane

So, $c_{55} = \frac{\partial^2}{\partial P_5^2}\left[\int \frac{K_{I5}^2}{E} dA\right]$

$$= \frac{16D}{EI}\int_0^{a/D}\left\{\int_{-P_1}^{P_1}(1-4\varphi^2)\left(2\phi+\sqrt{1-4\varphi^2}-1\right)[H(\lambda)]^2 d\varphi\right\}d\phi$$

Here $I = \frac{\pi D^4}{64}$ and $P_1 = \sqrt{\phi-\phi^2}$ Where, $\phi = \frac{\xi}{D}$, $\varphi = \frac{\eta}{D}$

$$H(\lambda) = \frac{\sqrt{\left(\frac{2}{\pi\lambda}\right)\tan\left(\frac{\pi\lambda}{2}\right)}\left(0.923+0.199\left(1-\sin\left(\frac{\pi\lambda}{2}\right)\right)^4\right)}{\cos\left(\frac{\pi\lambda}{2}\right)}$$

Where $\lambda = \frac{2\phi+\sqrt{1-4\varphi^2}-1}{2\sqrt{1-4\varphi^2}}$ for circular section

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