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TRANSIENT BEHAVIOUR OF CROSSFLOW HEAT EXCHANGERS WITH LONGITUDINAL CONDUCTION AND AXIAL DISPERSION

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Abstract

Transient temperature response of the crossflow heat exchangers with finite wall capacitance and both fluids unmixed is investigated numerically for step, ramp and exponential perturbations provided in hot fluid inlet temperature. Effect of two-dimensional longitudinal conduction in separating sheet and axial dispersion in fluids on the transient response has been investigated. Conductive heat transport due to presence of axial dispersion in fluids have been analysed in detail and shown that presence of axial dispersion in both of the fluid streams neutralises the total conductive heat transport during the energy balance. It has also been shown that the presence of axial dispersion of high order reduces the effect of longitudinal conduction.

Keywords: axial dispersion, crossflow, heat exchanger, transient behaviour.

Introduction

Heat exchangers are generally designed to meet certain performance requirements under steady operating conditions. However, transient response of heat exchangers needs to be known for designing the control strategy of different HVAC systems, cryogenic and chemical process plants. Problems such as start-up, shutdown, failure and accidents have motivated investigations of transient thermal response in crossflow heat exchangers. It also helps the designer to find a solution of the time dependent temperature problems, essential for thermal stress analyses. Besides, transient testing techniques are often adopted for determining the thermal characteristics of heat exchangers. In these experiments one of the fluid streams is subjected to a specified perturbation in inlet temperature while the response at the outlet is recorded. Using this information, thermal characteristics of heat exchangers can be predicted if a suitably developed mathematical model is available.

In the past few decades numerous attempts have been made for modelling the dynamic behaviour of heat exchangers. The researchers not only considered heat exchangers of different types but also proposed various methodologies. A pioneering effort [1] for modelling the dynamic behaviour of an air-to-gas plate-fin type crossflow heat exchanger has been done. Based on a number of simplified assumptions like negligible local heat storage in the gas and air stream, the energy balance equations were formulated and solved using finite difference technique taking a specific example. The basic set of three differential equations for the simulation of transient heat exchange in a crossflow heat exchanger was proposed [2] and five independent non-dimensional parameters were identified. Next one of the fluids is assumed to be mixed, which renders the temperature variation in that stream one-dimensional and greatly simplifies the problem. Further simplifications were sought assuming a large wall capacitance and finally an approximate analytical solution was obtained using integral heat balance technique.

Based on the basic model [2] different researchers have analysed the transient response of crossflow heat exchangers relaxing one or more number of approximations. A step change of inlet temperature was considered and solution was obtained by finite difference technique using a transformation of variables [3, 4]. While infinite capacitance of one of the fluids was assumed in the former work, large wall capacitance was considered in the latter. On the other hand an analysis [5] has been forwarded for fluid streams of negligible capacitance subjected to step, ramp and exponential inputs. They studied the effect of longitudinal conduction in the separator sheet and observed a deterioration of thermal performance of the heat exchanger.

Parallel efforts were also made to find out the dynamic response of heat exchangers analytically, using Laplace transform technique. The response of gas-to-gas heat exchanger with large core capacitance in case of step input have been analysed [6, 7]. The work has been extended [8] and the two-dimensional temperature response as integrals of modified Bessel's function in space and time for a transient response with any arbitrary initial and inlet conditions were obtained. The results were presented for step, ramp and exponential inputs. Later the assumption of large wall capacitance is relaxed [9, 10] and the temperature response was derived for a deltalike perturbation in temperature as well as step input. The solution is again simplified [11] for the limiting case of small capacitance of the fluid stream compared to the core.

In all of the above works the transient response has been obtained by two or three-fold Laplace transform. Next the temperature fields have been obtained by inversion of the resulting Laplace equations. The task of inverting the transformed temperatures in the physical domain becomes progressively difficult with the increase in the number of simultaneous equations as well as independent variables. This has motivated the investigators to use single Laplace transform along with numerical inversion. A single Laplace transform is used in conjunction with the power series technique for determining the response of crossflow heat exchangers with large and finite core capacitance respectively [12, 13].

So far all the analysis made for the prediction of dynamic behaviour of crossflow heat exchangers are based on the assumption of ideal plug flow. However, in reality the flow field will have nonuniformities in the form of eddies, turbulent fluctuations and recirculating as well as stagnant fluid zones. This presents a significant deviation from the plug flow, but cannot also be considered as a situation where complete longitudinal mixing is present. The thermal dispersion phenomena in the fluid closely resemble conduction of heat with different time and space scales. The space and time scales of thermal dispersion are small compared to the dimension of the heat exchanger and the time scale of convective transport. An approximate analysis of the real situation can be made assuming a Fourier type apparent axial conduction. The axial dispersion model for the transport of mass in a fluid flowing through a conduit was originated [14] and analysed in general [15]. Exploiting the similarity between heat and mass transfer this model was extended for thermal transport and was implemented for the analysis of regenerators [16] and heat exchangers [17, 18]. All of these studies report a significant change in the heat exchanger performance, if the effect of axial dispersion is included. An analytical solution [19] was given for the temperature effectiveness and temperature distribution in crossflow heat exchangers with axial dispersion in one fluid. This analysis shows that the influence of axial dispersion on temperature effectiveness is significant for $Pe<20$, particularly when the heat exchanger is in balanced condition. Further the effect increases with the increase in NTU.

The present work investigates the transient performance of a direct transfer, single pass crossflow heat exchanger. The temperature response of the fluid streams as well as the separator plate has been obtained by solving the conservation equations by finite difference formulation for step, ramp as well as exponential variation in the hot fluid inlet temperature. The analysis has been done for the generalised case of unmixed fluid streams and finite capacitances of fluids and metal wall. Effect of various parameters particularly the two-dimensional conduction in wall, axial dispersion in fluids as well as their combined effect on the dynamic performance of the heat exchanger has been studied over a wide range.

Mathematical Formulation

A direct-transfer, two-fluid, crossflow, multilayer plate-fin heat exchanger is shown schematically in Fig. 1(a). The two fluids exchange thermal energy through a separating solid plate while flowing through this heat exchanger in directions normal to each other. The hot stream is subjected to a specified temperature variation at the inlet, while the cold stream enters the heat exchanger with a known constant temperature. Following assumptions are made for the mathematical analysis.

1. Both the fluids are single phase, unmixed and do not contain any volumetric source of heat generation.
2. The exchanger shell or shroud is adiabatic and the effects of the asymmetry in the top and bottom layers are neglected. Therefore the heat exchanger may be assumed to comprise of a number of symmetric sections as shown by dotted lines in Fig. 1(a) and in details in Fig. 1(b).
3. The thermo-physical properties of both the fluids and wall are constant and uniform.
4. The primary and secondary areas of the separating plate have been lumped together, so that the variation of its temperature is two-dimensional.
5. The thermal resistance on both sides comprising of film heat transfer coefficient of primary and secondary surfaces and fouling resistance are constant and uniform.
6. Dispersion coefficient of the fluid streams remains constant.
7. Heat transfer area per unit base area and surface configurations are constant.
8. Variation of temperature in the fluid streams in a direction normal to the separating plate (z-direction) is neglected.

Conservation of energy for wall and the two fluid streams considering longitudinal conduction in separating sheet and the axial dispersion in fluids can be expressed as given below,

$$MC\frac{\partial t_w}{\partial \tau} = (hA)_a(t_a - t_w) + (hA)_b(t_b - t_w) + k\delta L_a L_b\frac{\partial^2 t_w}{\partial x^2} + k\delta L_a L_b\frac{\partial^2 t_w}{\partial y^2} \qquad (1)$$

$$(hA)_a(t_w - t_a) = (LA_c\rho c)_a\frac{\partial t_a}{\partial \tau} + L_a(mc)_a\frac{\partial t_a}{\partial x} - A_{ca}D_aL_a\frac{\partial^2 t_a}{\partial x^2} \qquad (2)$$

$$(hA)_b(t_w - t_b) = (LA_c\rho c)_b\frac{\partial t_b}{\partial \tau} + L_b(mc)_b\frac{\partial t_b}{\partial y} - A_{cb}D_bL_b\frac{\partial^2 t_b}{\partial y^2} \qquad (3)$$

The Eq. (1)-(3) can be expressed in non-dimensional form as given below,

$$\frac{\partial T_w}{\partial \theta} = T_a + RT_b - (1+R)T_w + \lambda_a N_a\frac{\partial^2 T_w}{\partial X^2} + \lambda_b N_b R\frac{\partial^2 T_w}{\partial Y^2} \qquad (4)$$

$$V_a\frac{\partial T_a}{\partial \theta} = T_w - T_a - \frac{\partial T_a}{\partial X} + \frac{N_a}{Pe_a}\frac{\partial^2 T_a}{\partial X^2} \qquad (5)$$

$$\frac{V_b}{R}\frac{\partial T_b}{\partial \theta} = T_w - T_b - \frac{\partial T_b}{\partial Y} + \frac{N_b}{Pe_b}\frac{\partial^2 T_b}{\partial Y^2} \qquad (6)$$

Where non-dimensional terms are defined as,

$$X = \left[\frac{hA}{mc}\right]_a\frac{x}{L_a} = N_a\frac{x}{L_a}, \qquad Y = \left[\frac{hA}{mc}\right]_b\frac{y}{L_b} = N_b\frac{y}{L_b}, \qquad \text{where } N = \frac{hA}{mc}$$

$$\theta = \frac{(hA)_a\tau}{MC}, \qquad T = \frac{t - t_{b,in}}{t_{REF} - t_{b,in}}$$

Where t_{REF} is a reference temperature = $(t_{a,in})_{max}$.

Conductance Ratio, $R = \frac{(hA)_b}{(hA)_a}$, Capacitance Ratio, $V = \frac{LA_c\rho c}{MC}$

Longitudinal Heat Conduction Parameter, $\lambda_a = \frac{k\delta L_b}{L_a(mc)_a}$, $\lambda_b = \frac{k\delta L_a}{L_b(mc)_b}$

Axial Dispersive Peclet number, $Pe = \frac{(mc)L}{A_cD}$

NTU is defined as

$$\frac{1}{NTU} = C_{min}\left[\frac{1}{(hA)_a} + \frac{1}{(hA)_b}\right] \qquad (7)$$

Further N_a and N_b can be expressed as a function of non-dimensional heat exchanger parameters namely number of transfer units (NTU), conductance ratio (R) and capacity rate ratio [$E = \frac{(mc)_b}{(mc)_a}$].

For C_a=C_{min}

$$N_a = NTU\left[1+\frac{1}{R}\right], \qquad N_b = \frac{NTU}{E}(R+1) \tag{8}$$

and for C_b=C_{min}

$$N_a = E\ NTU\left[1+\frac{1}{R}\right], \qquad N_b = NTU(1+R) \tag{9}$$

The initial temperature of the hot fluid, cold fluid and the separating sheet are all equal to the ambient temperature, and so also the inlet temperature of the cold fluid. Then Eq. (1-3) are subjected to

$$T_a(X,Y,0) = T_b(X,Y,0) = T_w(X,Y,0) = 0 \tag{10}$$

Assuming the adiabatic boundary conditions at the fluid exit and at the edges of the separating sheets

$$\left.\frac{\partial T_a(X,Y,\theta)}{\partial X}\right|_{X=N_a} = 0, \tag{11}$$

$$\left.\frac{\partial T_b(X,Y,\theta)}{\partial Y}\right|_{Y=N_b} = 0, \tag{12}$$

$$\left.\frac{\partial T_w(X,Y,\theta)}{\partial X}\right|_{X=0} = \left.\frac{\partial T_w(X,Y,\theta)}{\partial X}\right|_{X=N_a} = \left.\frac{\partial T_w(X,Y,\theta)}{\partial Y}\right|_{Y=0} = \left.\frac{\partial T_w(X,Y,\theta)}{\partial Y}\right|_{Y=N_b} = 0 \tag{13}$$

Inlet temperature of hot fluid is subjected to an external perturbation as

$$T_a(0,Y,\theta) = \phi(\theta), \tag{14}$$

while inlet temperature of cold fluid remains unchanged as

$$T_b(X,0,\theta) = 0. \tag{15}$$

Solution may be obtained for any arbitrarily specified temperature function $\phi(\theta)$. However, dynamic response of heat exchanger is generally looked for step, ramp and exponential variation of temperature. Such variation may occur during operations or they may be especially created for the purpose of transient testing of heat exchangers. Though a ramp or an exponential function gives a continuous increase in temperature, such an increase for a prolonged duration is not feasible in reality. For instance the initial temperature rise may have the ramp or exponential nature in both designed and unforeseen transients, but the maximum value of temperature rise will generally not be unlimited. This aspect has not been considered by earlier researchers [5, 9], who have considered the continuous increase of temperature during the entire period of operation for both ramp and exponential functions. In the present case instead of continuous increase a limit of maximum temperature has been provided. The functional form of $\phi(\theta)$ is expressed as follows

$$\phi(\theta) = \begin{cases} 1; \quad \text{for step input} \\ \begin{bmatrix} \alpha\theta, \ \theta \le 1 \\ 1, \ \theta > 1 \end{bmatrix}; \text{for ramp input} \\ 1 - e^{-\alpha\theta}; \ \text{for exponential input} \end{cases} \tag{16}$$

where α is assumed to be unity in the present analysis.

The pictorial representation of the inlet temperature functions both as considered by the earlier researchers and as taken in the present work are depicted in Fig. 2.

Method of Solution

The conservation equations are discretised using the implicit finite difference technique [20]. Forward difference scheme is used for time derivatives, while upwind scheme and central difference scheme are used for the first and second order space derivatives respectively. The difference equations along with the initial and boundary conditions are solved using Gauss-Seidal iterative technique. The convergence of the solution has been checked by varying the number of space grids and size of the time steps. It has been observed that the space grids 50x50 along with time steps 50 give the grid independence. The solution gives the two-dimensional temperature distribution for both of the fluids as well as the separator plate. Additionally one may calculate the mean exit temperatures as follows.

$$\overline{T}_{a,ex} = \frac{\int_0^{Na} T_{a,ex}\, u \, dy}{\int_0^{Na} u \, dy} \quad \text{and} \quad \overline{T}_{b,ex} = \frac{\int_0^{Nb} T_{b,ex}\, v \, dx}{\int_0^{Nb} v \, dx} \tag{17}$$

To check the validity of the numerical scheme, the results of the present investigation have been compared with available analytical results. For balanced gas-to-gas crossflow heat exchangers, the variation of exit temperature is available [8] in the absence of core longitudinal conduction and fluid axial dispersion for a conductance ratio of 1 using Laplace transform. Fig. 3 depicts excellent agreements between the results of present investigation and those obtained analytically [8] for step, ramp and exponential inputs. It needs to be mentioned that for the comparison of the results the definition of ramp and exponential inputs prescribed in the present work has not been followed. They have been taken as suggested in [8].

Results and Discussions

Performance of the heat exchanger was studied over a wide range of parameters as well as for a sufficient time duration so that steady state conditions are obtained for each individual excitation. The results have been obtained for the range of NTU from 1 to 8, λ from 0 to 0.025, Pe from 1 to ∞, V from 0 to 10, E from 0.5 to 2, and R 0.1 to 10. Some of the salient results are discussed below.

Two-Dimensional Temperature Distribution

The temperature of the fluid streams as well as separator plate will continuously change in x-y plane till the heat exchanger attains steady state. The temperature profiles at any intermediate time steps can be obtained with the help of present scheme. Figure 4 shows some typical results of temperature variation of the wall and the two fluids at θ =1 for different input functions.

As the instantaneous value of $\phi(\theta)$ decides the hot fluid inlet temperature, it is 1 for both step and ramp functions, while it is much lower for exponential function at θ=1. Thereafter for all the three cases the hot fluid temperature decreases gradually till it reaches the exit plane. Interestingly the exit temperature of the hot fluid is highest in case of step input, lowest in ramp input, while it has some intermediate value for exponential input. This indicates the temperature variation for exponential input is more uniform compared to the previous two cases. The wall temperature at the inlet plane of hot fluid is highest for step input, while, ramp and exponential cases are in diminishing order. The cold fluid temperature builds up gradually, as it receives the stored thermal energy from the separating sheet. In the present example cold fluid temperature is more or less uniform over the entire area and is marginally different from its inlet value. This is due to the fact that time step θ=1 considered here is too small to produce any appreciable change in the temperature of the cold stream. However, the effect of input disturbance can still be appreciated as one may note relatively higher temperature in case of step input, the highest point of temperature being at corner D [Fig. 1(b)]. The similar profiles can be drawn for different time levels, which can give the complete insight of the temperature variation at different portion of the heat exchanger at transient condition. The temperature profiles may be useful in thermal expansion and stress analysis needed for the mechanical design of the heat exchanger. The variation of plate temperature is of particular interest for mechanical design.

Effect of Aspect Ratio

The effect of NTU on the transient performance of crossflow heat exchanger has been studied by most of the researchers [5, 9]. However, the ratio of the length to breadth or the aspect ratio may influence the transient performance of such heat exchangers even if NTU is kept constant. Such investigation for crossflow heat exchangers has so far not been made. From Eq. (8-9) and the definition of X and Y, the aspect ratio of the heat exchanger Y/X can be equated to N_b/N_a and hence to R/E. Thus for E=1, the effect of change in R will also give the effect of change in aspect ratio of the heat exchanger as shown in Fig. 5 for step, ramp and exponential inputs at different time levels. Figure 5 shows that with the increase in aspect ratio i.e. the length of travel of cold fluid, the steady state condition is reached early by both the fluid streams for a step input. This can be understood as the reduction in flow length/area of hot fluid, gives smaller area of heat transfer available for the fluid where disturbance is provided. Similar results are also observed for ramp and exponential inputs.

Effect of Heat Capacity Rate Ratio

As temperature transients are imparted to hot fluid, which has a higher heat capacity, the effect of disturbances will die out easily for E<1, i.e. when $(mc)_a > (mc)_b$,. Or in other words the cold fluid with a lower thermal capacity will behave like a gas following the transients of the other fluids closely and will reach the steady state within a small duration. This behaviour is depicted for step input in Fig. 6. Temperature responses for ramp and exponential inputs are not much different from those shown in this figure. A few more important observations may also be made from this figure. The difference between the exit temperatures of the two streams increases both during transient and at the steady state for a low value of heat capacity rate ratio. However, this effect diminishes very quickly with the increase in E. At a low value of E, the crossflow heat exchanger may exhibit an interesting behaviour. As the temperature of the cold stream increases rapidly, at the beginning of the transients cold stream exit temperature may be higher than that of hot stream though at the steady state there is a reversal of this. This crossover of temperature curves for both E=1 and 2 can be noticed from Fig. 6 for step change in input. Similar behaviour is obtained for ramp and exponential responses due to specific nature of the input functions.

Effect of Heat Capacity Ratio

When the heat capacity ratio ($V = \frac{LA_c \rho c}{MC}$) is small the effect of wall thermal capacity is more and the time taken to transfer the heat from one fluid to the other is less, so the mean exit temperature of the fluids quickly reaches the steady state value. As the capacitance ratio is increased, more time is taken by the fluid to reach the steady state [Fig. 7(a)]. In the presence of longitudinal conduction in wall and axial dispersion in fluids the effect of change of heat capacity ratio is not that much predominant [Fig. 7(b)]. The difference in mean exit temperatures due to increase in V is less and also it takes much smaller time to reach to its steady state value compared to the case when the effect of core longitudinal conduction and the fluid axial dispersion is absent. The effect is similar for step, ramp and exponential inputs.

Effect of Axial Dispersion

Nonuniformity in flow at the molecular level comes into picture due to eddy diffusion in the axial direction. This effect may be taken care of by modifying the ideal plug-flow model. An apparent axial heat conduction or axial dispersion term is introduced into the energy balance. Due to presence of molecular conductivity and conductivity due to eddy diffusion, an apparent heat conductivity or diffusion coefficient is considered, which leads to a dimensionless parameter termed as axial dispersive Peclet number ($Pe=mcL/A_cD$). A smaller value of Pe corresponds to a highly dispersive flow and $Pe \rightarrow \infty$ leads to the ideal plug flow model, while Pe other than ∞ represents axial dispersion in the fluid stream. One of the aims of the present work is to study the effect of axial dispersion of the fluids on the dynamic performance of heat exchangers in details. In the presence of axial dispersion there is an additional mode of heat transport in the direction of fluid flow. This not only changes the local and spatial temperature of the fluid streams, but may also have a significant effect on the energy balance. It is of particular interest to study the effect when the Peclet numbers are different for the two fluid streams. In the limiting condition this may lead to a situation when one of the fluids may have a high axial dispersion, while the effect is absent ($Pe=\infty$) in the other fluid.

In general the designers try to select the dimensions of the heat exchangers such that maximum exchange of energy takes place between the streams. This will result in a small temperature gradient in the individual fluid streams at the exit of the heat exchanger. It is the usual practice to impose a zero flux condition at the exit of the heat exchanger. Thermal energy will be transported away from the heat exchanger only through advection, whereas at the entry to the heat exchanger 'conductive' heat transport will also be present. An elaborate study of these effects on the performance of crossflow heat exchangers has been made and the salient results are presented below.

The effects of variation of Pe on dynamic behaviour of hot and cold fluids can be studied from Fig. 8. In the first investigation both the fluid streams are assumed to have identical Pe [Fig. 8(a)]. On decreasing Pe, the mean exit temperature of hot fluid increases showing reduction in the amount of heat transfer. Pe=1 leads to highly dispersive condition. The phenomena in the cold fluid is same in the initial phase, afterwards the mean exit temperature decreases with the reduction in Pe, which again indicates the less amount of heat transfer and in turn, deterioration in performance. The amount of deterioration in cold fluid is comparatively less compared to that in hot fluid in all the cases. Due to the phenomena involved with axial dispersion as described earlier, the fluid exit temperatures reach the steady state faster at low Pe i.e. with high axial dispersion. The behaviour is similar for step, ramp and exponential inputs. For a quick assessment of the energy balance one may refer to Fig. 8(a) in which the hot and the cold fluid temperatures are separately depicted on left and right hand co-ordinates respectively. For all the values of Peclet number, to start with, the exit temperatures of the fluid streams exhibit a large difference indicating sufficient storage of thermal energy in the separating plate. Gradually, as one approaches towards steady state this difference decreases. For Pe=1 it may be noted that the exit temperature curves coincide with each other indicating attainment of steady state where $T_{a,ex}=0.638$ and $T_{b,ex}=0.355$ (at $\theta=10$). Summation of these two temperatures tends to unity, which indicates that convective heat gain of one of the fluids is perfectly balanced by convective heat loss of the other. Though longer duration is needed to reach the steady state (as depicted in Fig. 8(a)) convective heat balance is satisfied in all the cases (not shown in the figure). From Fig. 8(b) it can be seen that convective heat balance is satisfied when Peclet number of both the fluid streams are of the same order. This includes all the cases from high axial dispersion to zero axial dispersion. However, when the axial dispersions are of different magnitudes in the fluid streams energy balance cannot be obtained considering only the convective heat transfer.

In practice the axial dispersive Peclet number of the fluid streams may have different values. To study the effect two different situations are considered. In the first case axial dispersion is present only in hot fluid while in the second case axial dispersion is present only in cold fluid. A part of the energy will be transported by conductive transport from or to the fluid having an axial dispersion. The value of conductive heat transfer will decrease with Peclet number as shown in Fig. 9. The hot fluid entering the heat exchanger gain energy due to this phenomenon, while the entering cold fluid will lose energy. It may be noted that at a particular Pe, the gain is equal to the loss. This gives a perfect convective heat balance as shown in Fig. 8 and 10. In Fig. 10 the cold fluid exit temperature has been plotted as a function of hot fluid exit temperature. A diagonal joining the corners (1,0) and (0,1) has been drawn. When the steady state is located in this diagonal, a convective heat balance is indicated between the two streams. This is the case when both the fluids have identical Peclet numbers (points A and H in the figure). When axial dispersion is absent in the fluids, the steady state located at point A indicates the minimum cold fluid exit temperature and the maximum exit temperature for the hot fluid for cases when both the streams are having identical Peclet numbers. With the decrease of Pe, exit temperature of cold fluid increases, while there will be an equal decrease in the exit temperature of hot fluid. A theoretical bound of the exit temperatures for such cases may be obtained from point A to R.

When the Peclet number of the fluids are not identical, the steady state will not be located on AR. When axial dispersion is present only in the hot fluid some additional amount of thermal energy will enter the stream due to temperature gradient at the inlet of the heat exchanger. Finally, a large portion of it will be transferred to the cold stream. The steady states (D, F, I) will be located on a line AQ indicating an increase in the cold fluid exit temperature. On the other hand if axial dispersion is present only in the cold fluid, it will lose certain amount of thermal energy while entering the exchanger. This will reduce the mean exit temperature of the cold fluid and will locate the steady state positions on a line AP below AR. Therefore, Fig. 10 gives a convenient way of representing steady state operation of a crossflow heat exchanger in the presence of axial dispersion. The steady states will be located on a space bounded by the lines AP, PR, RQ and QA. Line AR denotes identical Peclet numbers for both the fluids, value of Pe decreasing from A to R. AQ and AP are lines of infinite Pe for the hot and cold fluids respectively. When axial dispersion of different orders are present in hot and cold fluids, the steady state will be located either in ΔAQR or in ΔAPR. For example, G is located inside ΔAQP; it indicates the steady state for $Pe_a < Pe_b$. On the other hand E indicates the reverse situation.

Longitudinal Conduction in Wall

Longitudinal conduction in case of conventional heat exchangers is often an insignificant effect. But for compact heat exchangers used in cryogenics and similar applications, it may result in serious performance deterioration. It has been shown [9] that for crossflow heat exchanger the deterioration is significant only at higher NTU's (8 or more) and it increases with the increase in NTU. As the NTU values are very high for cryogenic heat exchangers (sometimes as high as 500) because of high effectiveness, consideration of longitudinal conduction becomes essential in such equipments. Moreover, cryogenic and other compact heat exchangers have short conduction lengths, which further enhances the effect of longitudinal conduction. In spite of the above facts extensive investigation has not been made to study the effect of longitudinal conduction on dynamic performance of compact crossflow heat exchangers. The sole work on this aspect is due to [9], which gives the effect of longitudinal conduction assuming large core capacity and negligible axial dispersion. In the present study it is intended to investigate the effect of longitudinal conduction in the presence of axial dispersion as well as for finite value of the core capacitance. Fig. 11 depicts the response of crossflow heat exchanger in the presence of core longitudinal conduction and axial dispersion for different values of core capacity. Though the mean exit temperatures at steady state are identical in all the cases, temperature of both the fluid streams exhibit significant difference in the transient phase for a change of capacitance ratio from 10 to 1. This clearly shows the effect of core capacitance on the performance of heat exchanger. It is interesting to note that the effect of capacitance ratio reduces as its value is decreased. Moreover, at a low value of capacitance ratio steady state is reached much faster.

From the previous figure it has been seen that the axial conduction in the wall influences the performance of the heat exchanger during the transient period. However, the combined effect of longitudinal conduction and axial dispersion is not clear from this. To study the combined effect of both these phenomena transient response of the heat exchanger was computed for the four cases namely (i) without longitudinal conduction and axial dispersion, (ii) with longitudinal conduction only, (iii) with both longitudinal conduction and axial dispersion, and (iv) with axial dispersion only. The changes in mean exit temperatures for the case (ii) and (iv) with respect to the case (i) and (iii) respectively have been computed. As the results obtained for step, ramp and exponential excitation depict similar trend, only step response has been presented in Fig. 12. Effect of longitudinal conduction is different for hot and cold fluids. In both the cases the effect of longitudinal conduction becomes prominent with the increase in NTU. In general, Initially the mean exit temperature of hot fluid increases at a rapid rate in the presence of longitudinal conduction. Subsequently it decreases and ultimately reaches to steady state value. This gives a peak at all the values of NTU. It signifies that the effect of longitudinal conduction cannot be neglected during the transient conditions. Presence of axial dispersion along with longitudinal conduction gives the same behaviour but the magnitude of the change is much smaller than that with pure longitudinal conduction.

The mean exit temperature of cold fluid exhibits a more complex behaviour in the presence of axial dispersion. In general there is decrease in the cold fluid exit temperature in the presence of longitudinal conduction. The relative change in temperature during the initial period of transients is more. Numerical value of the change in cold fluid exit temperature initially increases and then decreases finally leading to a steady state value. In the presence of axial dispersion the change in temperature first increases positively and finally assumes a small negative value. Axial dispersion in the cold stream diminishes the effect of longitudinal conduction.

Conclusion

A numerical scheme has been developed for determining the transient behaviour of crossflow heat exchangers using finite difference method. Dynamic performance of the heat exchanger has been studied in response to step, ramp and exponential excitation given to the inlet temperature of the hot fluid. Contrary to the conventional practice ramp and exponential input functions have been modified so that they reach a predefined maximum within a stipulated time period and then remain constant. The modified functions represent the transients generally observed in practice. Transient behaviour has been obtained for a combined effect of finite core capacity, two-dimensional longitudinal conduction in wall and axial dispersion in fluids. Two-dimensional temperature distributions of the two fluids and the separating sheet may be obtained from the simulation to form the basis of detailed mechanical design. Conductive heat transport due to the presence of axial dispersion in fluids have been analysed in detail and shown that presence of axial dispersion in both the fluid streams neutralises the total conductive heat transfer during the energy balance. It has also been shown that the presence of axial dispersion of high order reduces the effect of longitudinal conduction up to some extent. It has been observed that the longitudinal conduction plays an important role with the increase in NTU, similarly fluid axial dispersion is important when Pe is smaller. Both the conditions are very common in the case of compact crossflow heat exchangers having small sizes and working at high NTU and low values of Reynolds number. The detailed study of transient performance of crossflow heat exchangers will be useful for the detailed thermal and mechanical design and control of systems having such heat exchangers at different practical situations.

Nomenclature

A – area of heat transfer, m^2

A_c – area of cross-section, m^2

c, cp – specific heat of fluid, J/kg-K

C – specific heat of the wall material, J/kg-K

D – diffusion coefficient, W/m-K

E - capacity rate ratio $= \dfrac{(mc)_b}{(mc)_a}$

G – mass flux velocity, kg/m^2-s

h – heat transfer coefficient, W/m^2-K

k – thermal conductivity of the separating sheet, W/m-K

L – heat exchanger length, m

m – mass flow rate of fluid, kg/s

M – mass of the separating sheet, kg

N - $\dfrac{hA}{mc}$, dimensionless

NTU – number of transfer units, Eq. (7)

Pe - axial dispersive Peclet number $= \dfrac{(mc)L}{A_c D}$

R - conductance ratio $= \dfrac{(hA)_b}{(hA)_a}$

t – temperature, °C

$T = \dfrac{t - t_{b,in}}{t_{REF} - t_{b,in}}$, dimensionless temperature

$\overline{T}$ - mean dimensionless temperature

u, v – velocity in x and y direction

V - Capacitance Ratio $= \dfrac{LA_c\rho c}{MC}$

x, y – space direction

$X = (\dfrac{hA}{mc})_a \dfrac{x}{L_a}$, dimensionless length

$Y = (\dfrac{hA}{mc})_b \dfrac{y}{L_b}$, dimensionless length

Greek letters

$\theta = \frac{(hA)_a \tau}{MC}$, dimensionless time

ρ - density, kg/m^3

λ - longitudinal heat conduction parameter, $\lambda_a = \frac{k\delta L_b}{L_a (mc)_a}$, $\lambda_b = \frac{k\delta L_a}{L_b (mc)_b}$

δ - thickness of the separating sheet, m

τ - time, s

$\phi(.)$ – perturbation in hot fluid inlet temperature

Subscripts

a – hot fluid

b – cold fluid

w - wall

in - inlet

ex - exit

max - maximum

min - minimum

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List of Figure Captions

Figure 1 Crossflow heat exchanger (a) schematic representation, and (b) symmetric module considered for analysis.

Figure 2 Schematic representation of perturbation [$\phi(\theta)$] in inlet temperature of hot fluid.

Figure 3 Comparison of the solutions with the analytical results [8]. (a) Step, (b) ramp and (c) exponential inputs for E=R=1, V=λ=0, and Pe=∞.

Figure 4 2-dimensional temperature distribution of (a) hot fluid, (b) wall, and (c) cold fluid at time θ=1, for (A) step, (B) ramp and (C) exponential inputs with E=R=V=Pe=1, NTU=2, λ=0.025 with no flow transients.

Figure 5 Effect of aspect ratio (Y/X) on mean exit temperature of both the fluids with (a) step, (b) ramp, and (c) exponential inputs without longitudinal conduction axial dispersion.

Figure 6 Effect of heat capacity rate ratio on mean exit temperature of both the fluids for step input.

Figure 7 Effect of fluid capacitance ratio on step response of mean exit temperature of hot and cold fluids (a) without longitudinal conduction and axial dispersion, and (b) with longitudinal conduction and axial dispersion.

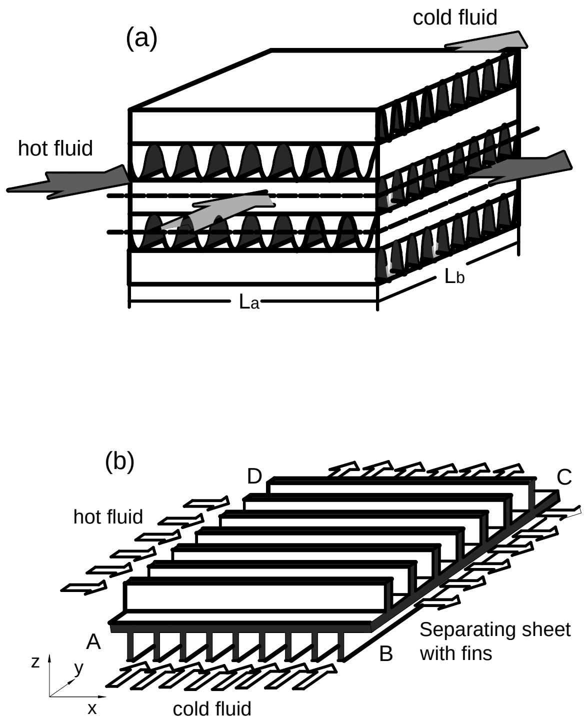
Figure 8 Effect of axial dispersive Peclet number, Pe on mean exit temperature of both the fluids.

Figure 9 Conductive heat transfer due to axial dispersion in any of the fluid streams.

Figure 10 Mean exit temperature of hot and cold fluids due to axial dispersion for step change in hot fluid inlet temperature.

Figure 11 Effect of capacitance ratio on step response of hot and cold fluid mean exit temperatures.

Figure 12 Change in exit temperature of (a) hot and (b) cold fluid due to longitudinal conduction with and without fluid axial dispersion for step input.



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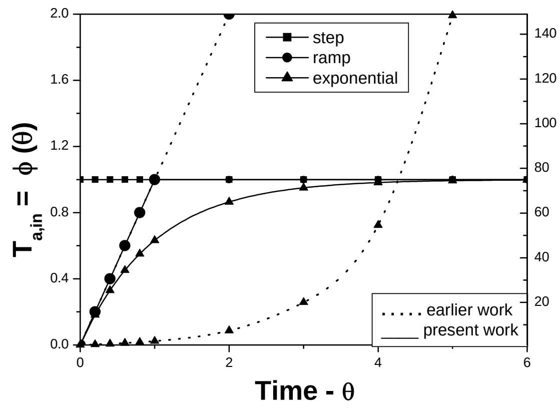

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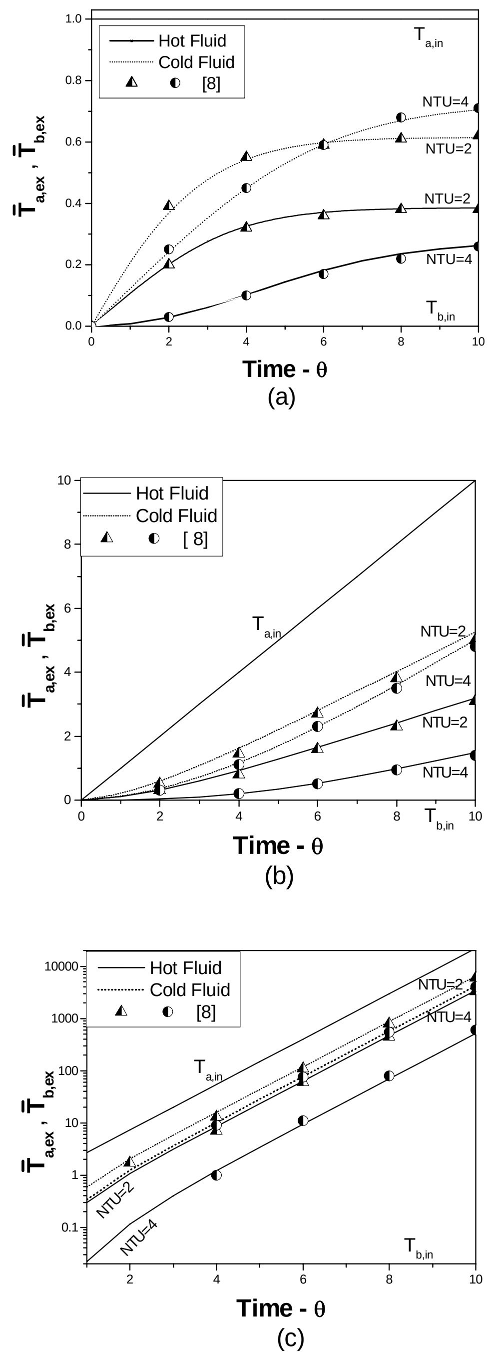
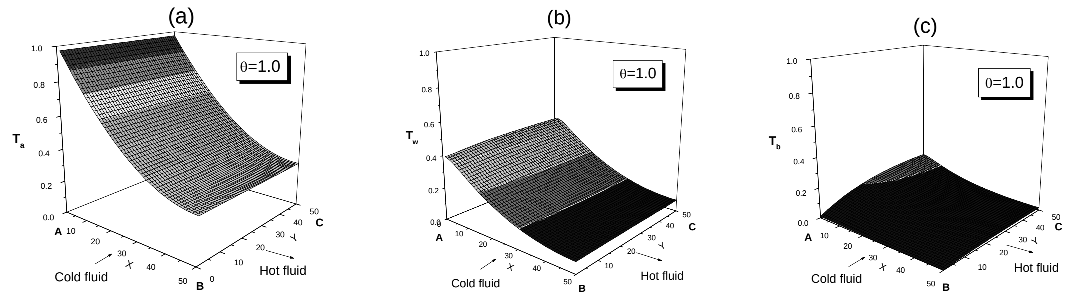
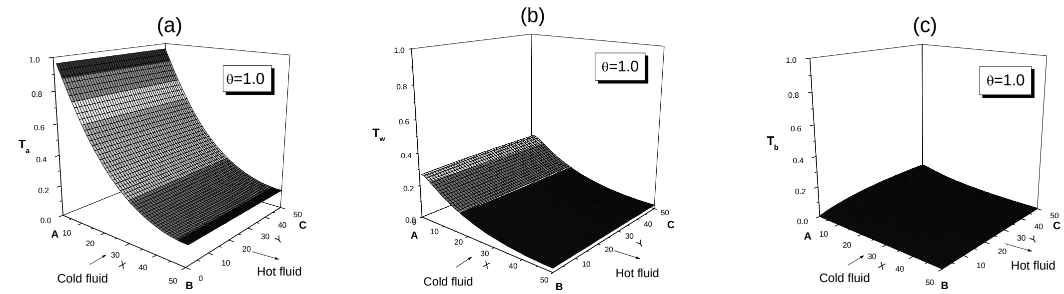


Fig.3, Mishra, JHT

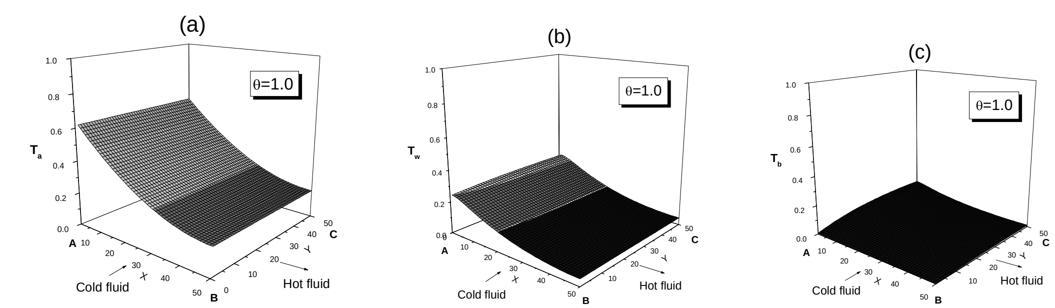
(A) Step



(B) Ramp



(C) Exponential



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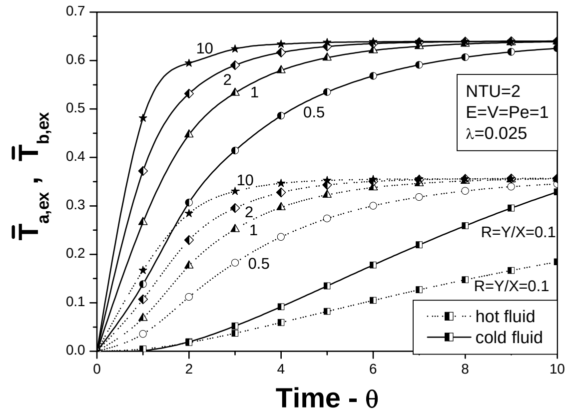
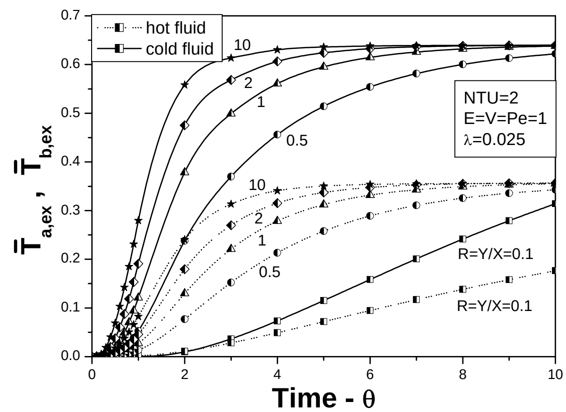


(a)



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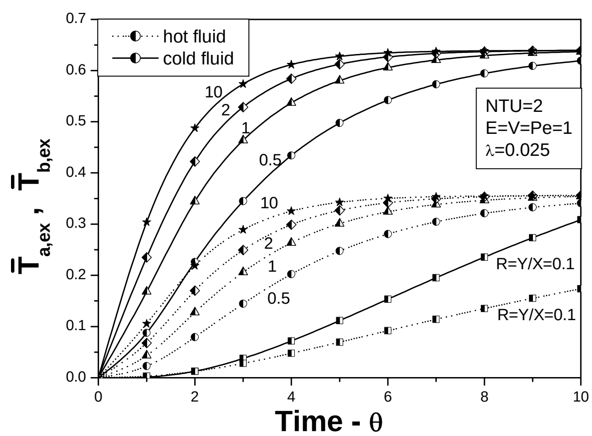
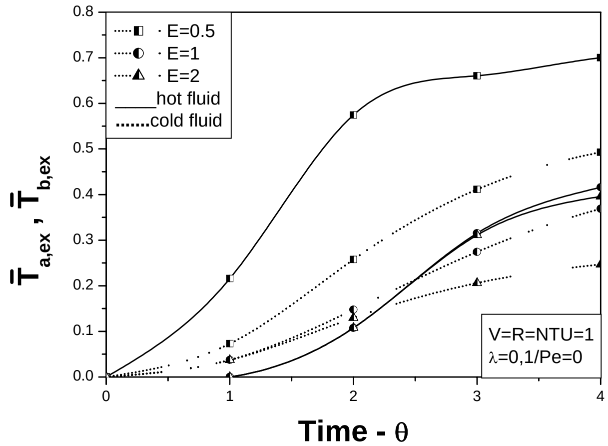


(c)

Fig.5, Mishra, JHT



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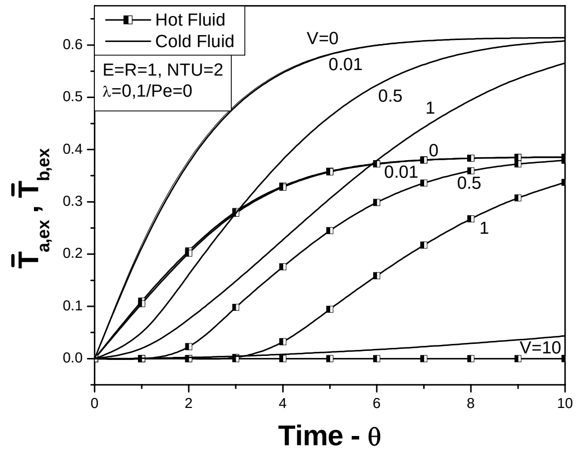
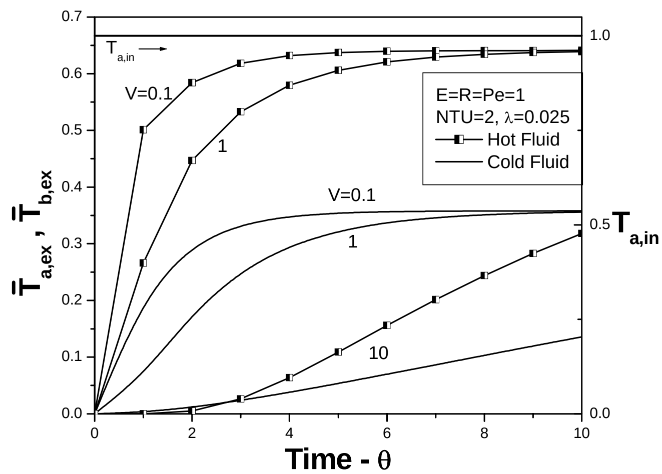


(a)



(b)

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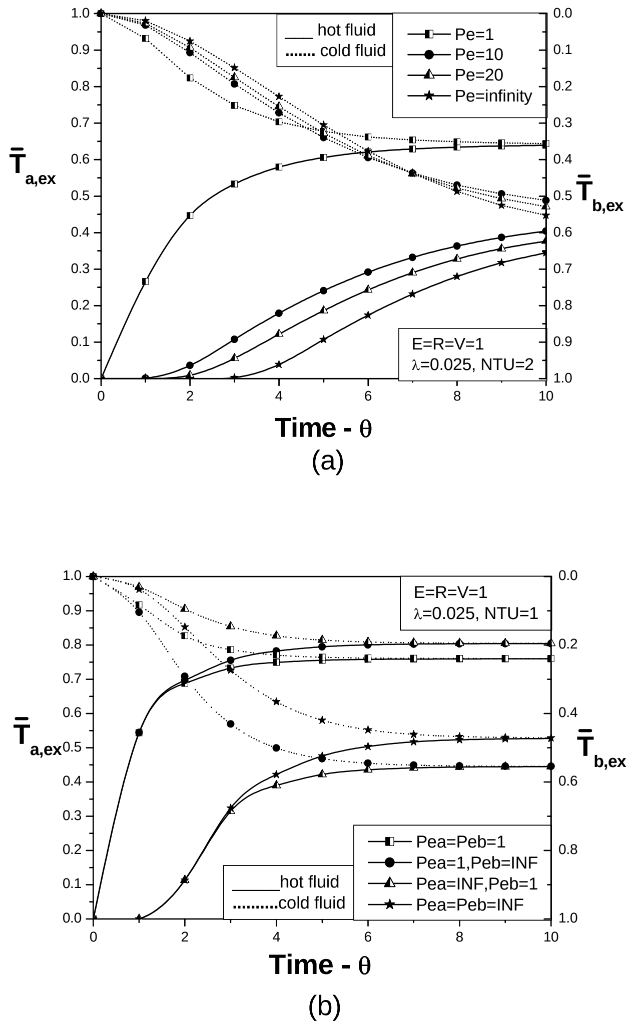

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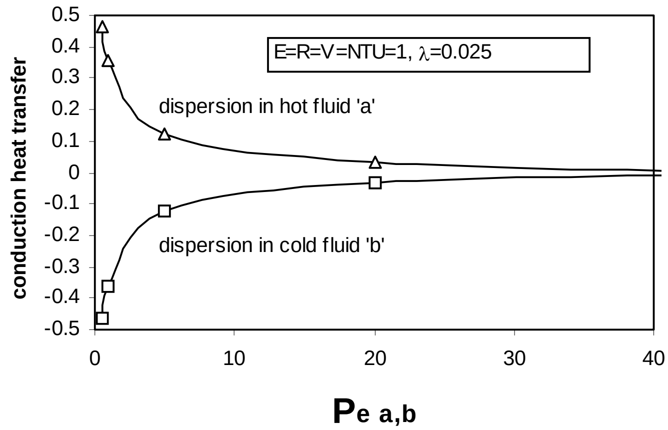

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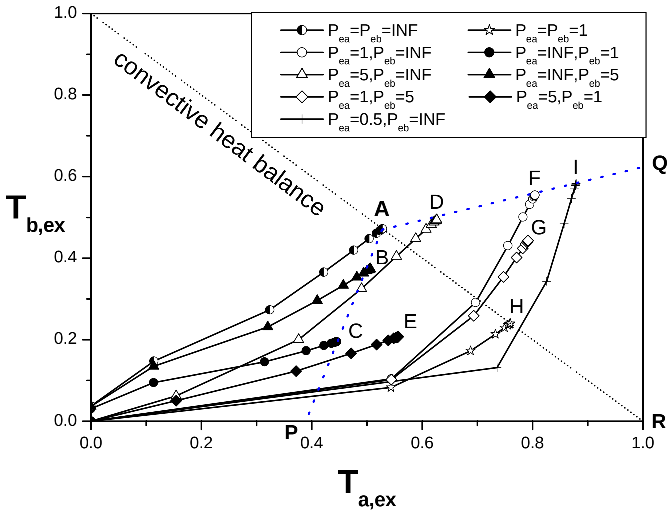

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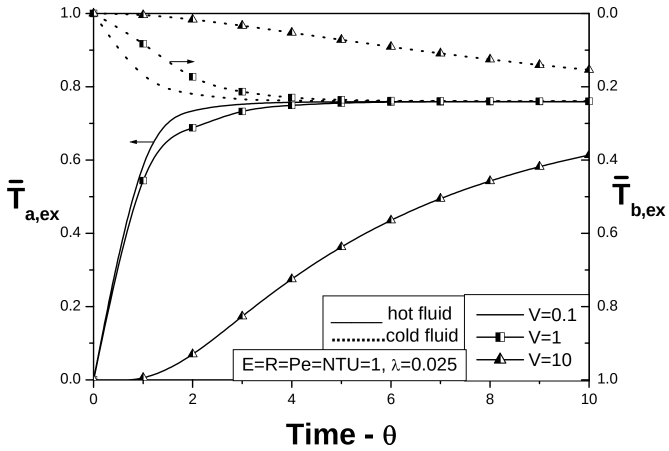

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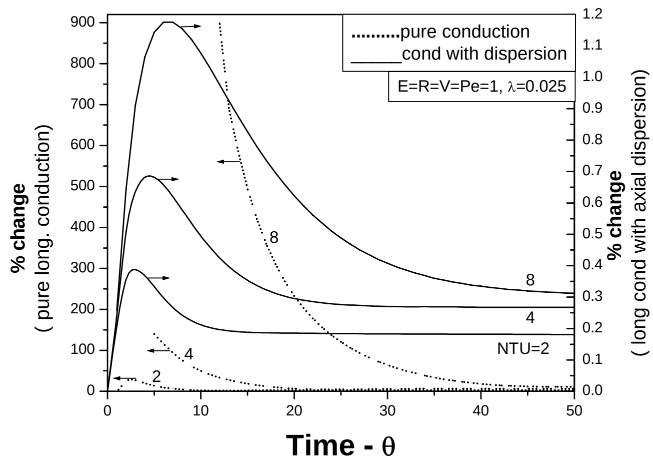
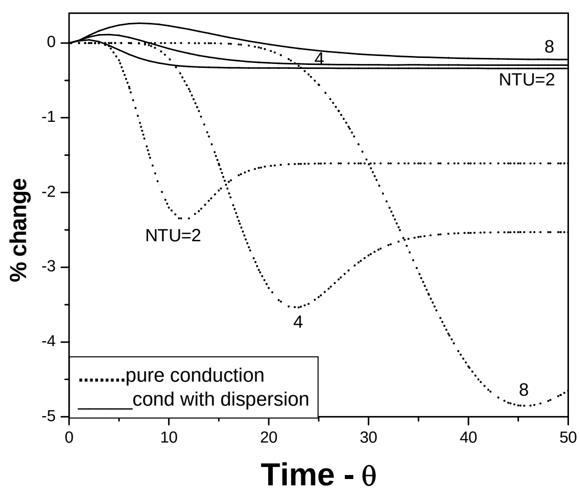


(a)



(b)

Fig.12, Mishra, JHT