Finite Element Buckling Analysis of Thin Plates with Complicated Geometry

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ABSTRACT: The plates used as structural elements in the field of aerospace, offshore, ocean, mechanical, nuclear, and civil engineering take different shapes due to their functional and structural requirements as well as for aesthetic considerations. Their application with complex configuration is of practical importance and requires accurate and efficient analysis of their stability. In this paper, finite element buckling analysis of different complicated shaped thin plates is presented. In the formulation, the arbitrary planform of the whole plate is mapped into a square domain where a cubic serendipity shape function is used to represent the complicated geometry and an ACM plate bending element is considered for the displacement function. Many researchers have used different elements to analyze plates but these elements are limited to solve a particular type of geometry only. This element is capable to model different geometries just like isoparametric element without the shear locking problem and generation of spurious mechanisms which is inherent in the isoparametric formulation. The versatility of the element is proved by undertaking different plate geometries. New results are presented as no such geometries are analyzed in any previous published literatures.

1 INTRODUCTION

Different arbitrary shaped plates are widely used in the field of aerospace, offshore, ocean, mechanical, nuclear, and civil engineering due to their functional and structural requirements as well as for aesthetic considerations. Therefore, it is required to calculate the buckling load at which the structure becomes unstable. This paper presents buckling analysis of elliptic, right angled triangle and diamond-shaped plates with different boundary conditions.

Some of the recent works done on the above topic is discussed. Rao et al. (1992) proposed an empirical formula following Rayleigh Ritz technique for calculating the critical loads of elliptical plates. Buckling factors of triangular plates with different translational and rotational elastic restraint using p-Ritz method is presented by Xiang (2002). Different irregular straight-sided quadrilateral thin plates are analyzed by Karami and Malekzadeh (2002) applying differential quadrature (DQ) methodology. Rahai et al. (2008) substituted modified buckling mode shapes in the elastic energy formulation and presented a new procedure to find the buckling load of stepped and perforated rectangular plates. Liu and Pavlović (2008) analytically solved simply supported rectangular plates under arbitrary loads using Ritz energy technique. Sectorial plates are analyzed by Coman (2009) employing interactive boundary layer analysis. Bui et al. (2011) investigated buckling load factor of rectangular plates with and without circular cut outs using an improved Moving Kriging interpolation meshfree method. Deformation theory and incremental theories are used to study elastic/plastic buckling analysis of rectangular plates by Kadkhodayan and Maarefdoust (2014). Papkov and Banerjee (2015) studied free vibration and buckling analysis of orthogonal clamped rectangular plates by enhancing the method of superposition which reduces the boundary value problem to an infinite system of linear algebraic equations.

Barik and Mukhopadhyay (1998) have analyzed free flexural vibration of arbitrary plates. They have used an ACM plate element with three degrees of freedom ($w, \frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}$) to represent the displacement function and cubic serendipity function to define the geometry. In the present paper, the above element is considered for buckling analysis of complex geometries. Elliptical and right angled triangular plate examples are validated with published results. New results for semicircular semielliptical and diamond shaped plates under different boundary conditions and loadings are included.

2 MAPPING OF THE PLATE

The formulation developed in Barik and Mukhopadhyay (1998) is followed here. The arbitrary shape of the whole plate is mapped approximately into a $[-1, +1]$ region in the $s-t$ plane with the help of the cubic serendipity shape function. Then the mapped square plate in $s-t$ plane is discretized into a number of elements and each element is again mapped with the same cubic serendipity shape function to a natural coordinate element of domain $[-1, +1]$ in $\xi - \eta$ plane.

From the mapping we have,

$$\begin{Bmatrix} \frac{\partial w}{\partial x} \\ \frac{\partial w}{\partial y} \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} \frac{\partial w}{\partial \xi} \\ \frac{\partial w}{\partial \eta} \end{Bmatrix} \tag{1}$$

$$\text{where, } [J] = \begin{Bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{Bmatrix} \tag{2}$$

3 DISPLACEMENT INTERPOLATION FUNCTION

The ACM plate bending element with three degrees of freedom ($w, \frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}$) at each node is considered for the present investigation. The interpolation functions for the bending are mentioned in detail in Barik and Mukhopadhyay (1998).

4 ELEMENT MATRICES AND EQUATIONS OF EQUILIBRIUM

The equilibrium equation for buckling analysis of an elastic system undergoing small displacement is given by

$$[K_e] \{\delta\} - \lambda [K_G] \{\delta\} = \{0\} \tag{3}$$

where $[K_e]$ is the global elastic stiffness matrix, $[K_G]$ is the geometric stiffness matrix and $\{\delta\}$ is the displacement vector in the global coordinate system.

The stiffness matrix of the plate element is given by

$$\{K_e\} = \int\int [B]^T \, [D] \, [B] \, |J| \, d\xi \, d\eta \tag{4}$$

The geometric stiffness matrix of the plate element is given by

$$\{K_G\} = \int\int [B_G]^T \, [\sigma] \, [B_G] \, |J| \, d\xi \, d\eta \tag{5}$$

where

$$[B_G] = J^{-1} \left[\left[\frac{\partial N_w}{\partial \xi} \right] \left[\frac{\partial N_w}{\partial \eta} \right] \right]^T \tag{6}$$

$$[\sigma] = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} \tag{7}$$

5 BOUNDARY CONDITIONS

As a general case, the stiffness matrix for a curved boundary supported on elastic springs continuously spread along the boundary line can be obtained as

$$[K_b] = \int [N_b]^T [N_k] |J_b| \, d\lambda_1 \tag{8}$$

where λ_1 is the direction of the boundary line in the $\xi - \eta$ plane and J_b =Jacobian $= ds_1/d\lambda_1$. The jacobian is the ratio of actual length to the length of mapped domain at any segment of boundary length. For details refer Barik and Mukhopadhyay (1998).

6 RESULTS AND DISCUSSION

6.1 *Elliptical plate*

Buckling analysis of an elliptical plate with a = semi-major axis and b = semi minor axis is validated with Rao et al. (1992) shown in figure 1 and 2 respectively.

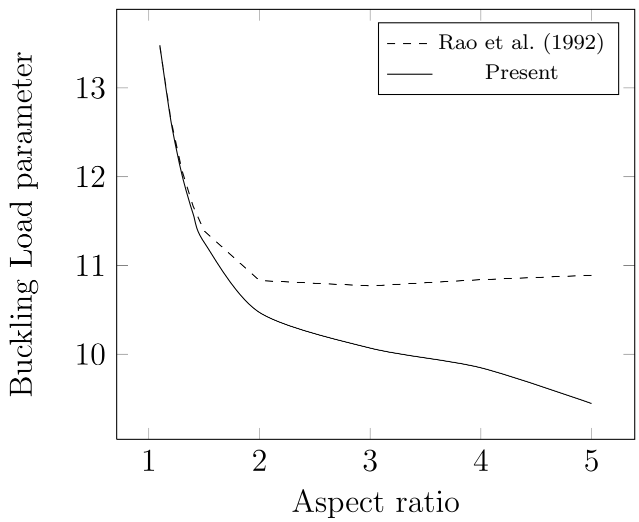


Figure 1: Comparison of buckling load parameter ($\lambda b^2/D$) with aspect ratio (a/b) for uniformly compressed all edges clamped elliptical plate

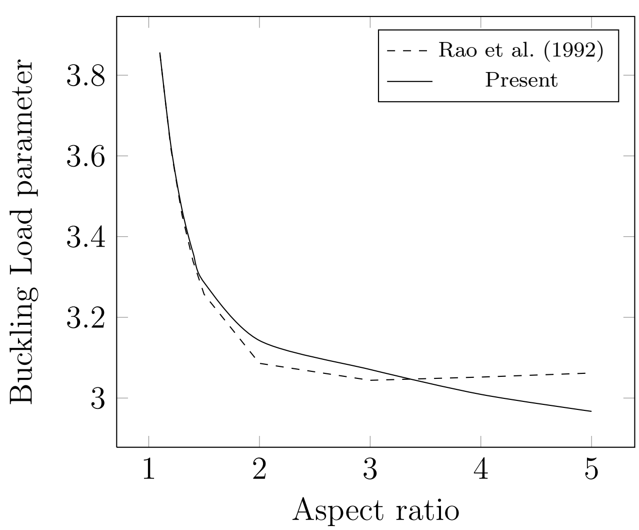


Figure 2: Comparison of buckling load parameter ($\lambda b^2/D$) with aspect ratio (a/b) for uniformly compressed all edges simply supported elliptical plate

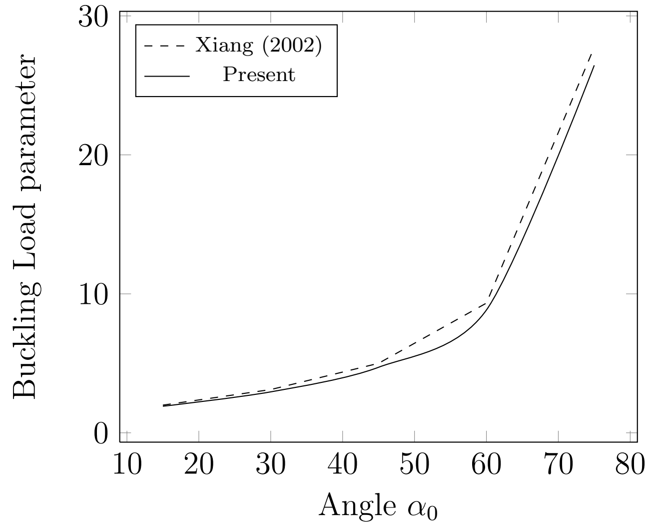


Figure 3: Comparison of buckling load parameter ($\lambda b^2/\pi^2 D$) with Angle α_0 for simply supported right angled triangular plate

Table 2: Different parameters used

Thickness	h	$6.5\,cm$
Poisson's ratio	ν	0.33
Young's modulus of elasticity	E	$7 \times 10^8\ N/cm^2$
Rigidity Modulus	D	$Eh^3/12(1-\nu^2)$
Semi-major axis length	a	
Radius of semicircle	r	$200\,cm$

6.2 *Right angeled triangle*

Right angeled triangle plate is analyzed for different boundary conditions by Xiang (2002). The present formulation is applied for the and the results are compared. The figure 3 presents for the simply supported case and the table 1 shows case-5 with angle $\alpha_0 = 15^\circ, 30^\circ$ for different translational (S_w) and rotational (S_r) elastic restraints.

6.3 *Semicircular Semielliptical Plate*

The buckling load for a simply supported (SSSS) and clamped (CCCC) plate shown in figure 4 which is a combination of semicircular (left part) and semielliptical (right part) shape is presented for aspect ratio 1.125 and 1.25 for all edges compressive load λ in table 3. The different parameters used for the problem is given in table 2.

6.4 *Diamond-shaped Plate*

The buckling load for a diamond-shaped plate shown in figure 5 is presented for aspect ratio 1.0 and 1.5 and different boundary conditions (BC)

Table 1: Comparison of buckling load parameter ($\lambda b^2/\pi^2 D$) with Angle α_0° for different elastic restrained right angled triangular plate

Angle		(S_w, S_r)						
α_0		$(0,0)$	$(10^3,0)$	$(0,10^3)$	$(10^3,10^3)$	$(10^6,0)$	$(0,10^6)$	$(10^6,10^6)$
15°	Xiang (2002)	0.8257	3.7706	2.0324	5.9114	3.7886	1.8593	6.0123
	Present	0.8330	3.6226	2.4398	5.2031	3.6286	2.4404	5.5781
30°	Xiang (2002)	1.8088	6.0491	3.7018	8.6851	6.1109	3.7079	8.9190
	Present	1.9718	5.9662	4.6734	8.6836	5.9589	4.6749	8.7253

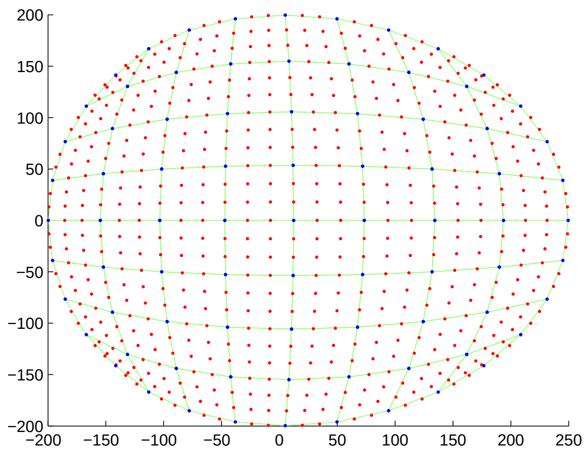
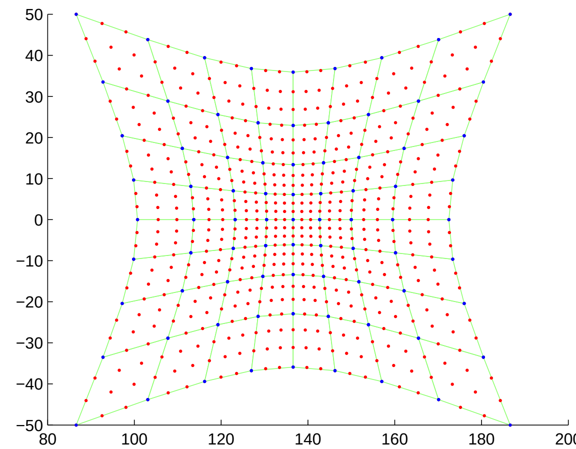


Figure 4: A typical 8×8 mesh discretization with boundary nodes of Semi-circular Semi-elliptical Plate

Table 3: Comparison of buckling load parameter ($\lambda r^2/D$) with aspect ratio (a/r) of Semi-circular Semi-elliptical Plate

(a/r)	BC	Buckling load parameter
1.125	SSSS	4.0546
	CCCC	13.8679
1.25	SSSS	3.9072
	CCCC	13.2325



$r_1 = r_2 = 100\,cm$

Figure 5: A typical 8×8 mesh discretization with boundary nodes of Diamond Shaped Plate for $r2/r1 = 1.0$

for all edges compressive load λ in table 4. The different parameters used for the problem is given in table 2. The mixed boundary condition SCSC represents top and bottom edge simple supports and other two as clamped edges.

6.5 *Spinning-top shaped Plate*

The buckling load for a spinning-top shaped plate shown in figure 6 is presented for aspect ratio 1.0

Table 4: Comparison of buckling load parameter ($\lambda r^2/D$) with aspect ratio (a/r) of Diamond-shaped Plate

(a/r)	BC	Buckling load parameter
1.0	SSSS	41.0438
	CCCC	98.5689
	SCSC	72.4925
1.5	SSSS	36.8487
	CCCC	109.7127
	SCSC	94.5842

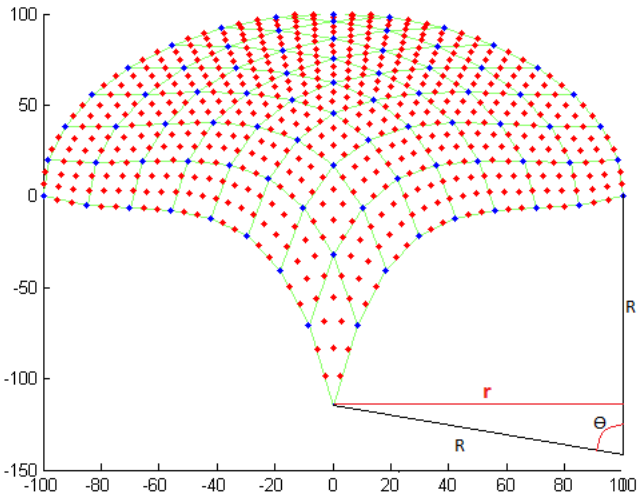


Figure 6: A typical 8×8 mesh discretization with boundary nodes of Spinning-top shaped plate for $\theta = 60°$

Table 5: Comparison of buckling load parameter $(\lambda r^2/D)$ with angle θ of Spinning-top shaped plate

θ	BC	Buckling load parameter
45°	SSSS	22.1538
	CCCC	33.9273
	SCSC	22.3933
60°	SSSS	17.1595
	CCCC	33.4403
	SCSC	19.2625

and 1.5 and different boundary conditions for all edges compressive load λ in table 5. The different parameters used for the problem is given in table 2.

7 CONCLUSIONS

Different authors have investigated on the buckling behavior of plates. It is observed that most of them developed or used methods that can solve only a particular type of geometry. Kirchoff theory can not handle arbitrary thin plates. Thus isoparametric element which is based on Mindlin's theory is often used by authors while using finite element method. But, when this element is used to analyze this plates, inconsistency in results are seen due to shear locking problems, even after applying reduced/selective integration. Therefore, the element used in this present paper may be preferred for the analyses of thin plates as no such above problems are faced and is capable to accommodate different general and complex geometries.

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