# MATHEMATICAL ANALYSIS OF OIL INJECTED TWIN SCREW COMPRESSOR

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*Mathematical analysis of oil injected twin screw compressor is carried out on the basis of the laws of perfect gas, and standard thermodynamic relations. Performance of an oil injected twin-screw compressor depends on a large number of design parameters. A computer model for calculating compressor performance and to validate the results with experimental data is developed. The flow coefficients required to calculate leakage flow rates for simulation are obtained from efficiency versus clearance curves. Some numerical examples of P-V diagrams, influences of oil injection on volumetric efficiency etc for a given compressor are presented.*

## Nomenclature

| | |
|---|---|
| $L$ | length of rotor |
| $N$ | rotational speed of rotors |
| $n$ | number of lobes on rotors |
| $D$ | rotor diameter |
| $S$ | sealing line length |
| $A$ | heat transfer area, cross sectional area of rotor groove |
| $A_c$ | clearance area of leakage |
| $A_b$ | area of blowhole |
| $A_a$ | actual power input to the compressor |
| $A_e$ | experimental adiabatic power |
| $A_t$ | theoretical adiabatic power |
| $M_l$ | mass of oil in the working space |
| $M_g$ | mass of gas in the working space |
| $M_{ts}$ | mass of gas inducted during suction process at suction condition |
| $M_{t1}$ | mass of gas in suction chamber at the end of suction process |
| $M_t$ | theoretical mass flow rate at suction temperature |
| $M_1$ | actual gas mass sucked in to the suction chamber |
| $M_{il}$ | interlobe leakage mass leaked into the suction chamber during previous compression process |
| $m$ | leakage mass flow rate through flow path |
| $m_{gl}$ | total gas mass leakage during compression process |
| $m_b$ | leakage mass flow rate through blowhole |
| $m_d$ | leakage mass flow rate through discharge end clearance |
| $m_{il}$ | leakage gas mass flow rate through interlobe clearance |
| $m_t$ | leakage mass flow rate of oil through rotor tip-housing clearance |
| $P$ | pressure in the working space |
| $P_s$ | suction pressure |
| $P_b$ | pressure beyond working space |
| $T$ | temperature |
| $T_b$ | temperature of gas beyond working space |
| $T_g$ | temperature of gas in the working space |
| $T_l$ | temperature of oil in a working space |
| $T_{oil}$ | mean temperature of leaked oil in the suction chamber during suction process |
| $T_s$ | suction gas temperature |
| $T_1$ | temperature of the gas at the end of suction process |
| $H$ | enthalpy |
| $W$ | gas work |
| $Q$ | transferred heat between gas and oil |
| $R$ | gas constant |
| $R_m$ | effective gas constant of oil gas mixture |
| $C$ | flow coefficient |
| $V_t$ | rotor tip velocity |
| $t$ | time |
| $t_s$ | time required for suction process |
| $a$ | clearance between lobe tip and housing |
| $\varepsilon_r$ | gap between interlobe |
| $\varepsilon_d$ | discharge end clearance |
| $w_t$ | lobe tip width |
| $q$ | leakage volume flow rate through flow path |
| $h$ | heat transfer coefficient between gas and oil |
| $r$ | pressure ratio |
| $k$ | ratio of specific heats |
| $c_l$ | specific heat of oil |
| $c_p$ | specific heat of gas at constant pressure |
| $c_v$ | specific heat of gas at constant volume |
| $V$ | volume of the working space |
| $V_g$ | volume of gas in a working space |
| $V_{t1}$ | geometrical volume of one pair of male and female rotor grooves |
| $V_{tg}$ | theoretical volume displacement rate of the compressor |

### Greek Symbols

| | |
|---|---|
| $\rho$ | density |
| $\varphi$ | mass ratio of oil to gas |

$\phi$ male wrap angle
$\beta$ modified adiabatic index
$\mu$ dynamic viscosity
$\eta_{tv}$ theoretical Volumetric efficiency
$\eta_{ev}$ experimental Volumetric efficiency
$\eta_{ta}$ theoretical Adiabatic efficiency
$\eta_{ea}$ experimental Adiabatic efficiency
$\theta$ rotor rotational angle
$\Gamma$ time for compression process
T cycle time of compressor

**Subscripts**

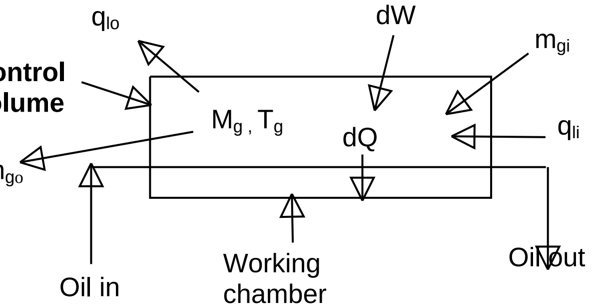
*g* gas
*l* oil
*m* male rotor
*f* female rotor
*i* gas or oil going in to the working space
*o* gas or oil going out of the working space

## Introduction

Screw compressor is a positive displacement machine that uses a pair of intermeshing rotors housed in a suitable casing to produce compression. These are capable of high-speed operation over a wide range of operating pressures. In screw machines, oil is deliberately injected into the compression chamber to get better sealing, lubrication and cooling effect. Rotary dual screw compressors are widely used in industry for gas compression and refrigeration. They are particularly suitable for small and intermediate size cryogenic refrigerators and liquefiers

## Thermal Analysis

Figure 1 reproduced from Ref [6] shows the schematic of the twin screw compressor, where the compression space has been identified. Figure 2 identifies the inlet and exit of fluids into this compression space.



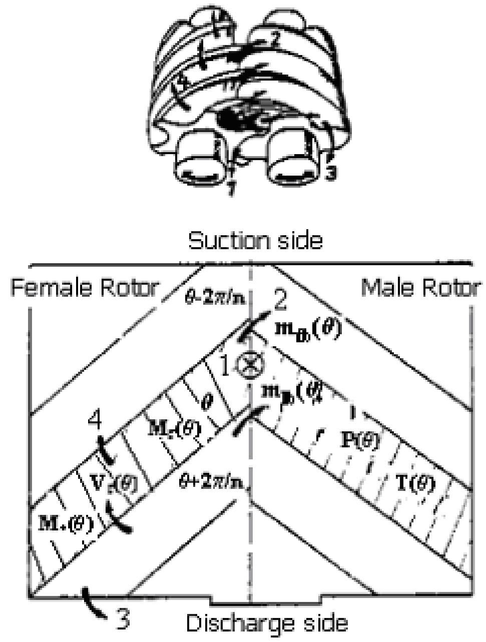




**Figure 1** Over view of different types of leakages through gaps of a twin-screw compressor during compression process [ 6 ]

1- interlobe gap leakage flow, 2 - leakage through trailing blowhole, 3 - leakage through discharge end clearance, 4 - Leakage through rotor tip housing clearance, $\theta$ - male rotor rotation angle

Thermodynamic properties of gas, and oil in the working chamber vary during compression process. The gas in the working chamber is compressed by the rotational movement of the rotors. The rate of change of mass of oil and gas is due to leakage. Leakage into the working chamber is from the leading chamber, which is at a higher pressure and leakage out of the chamber goes to the succeeding working chamber. It is assumed that the oil and gas are separate fluids, and only heat is exchanged between them.

**Figure 2** Schematic representation of the screw compressor working chamber at the beginning of compression process

The first law of thermodynamics for unsteady flow through a control volume may be expressed as

$$\frac{dE_v}{dt} = m_{gi} i_{gi} + \frac{dQ}{dt} - m_{go} i_{go} - \frac{dW}{dt}$$

where $E_v$ is the internal energy of the gas within the working chamber at any instant and $m_{gi}i_{gi}$, $m_{go}i_{go}$ are the enthalpy flow rates of gas into and out of the working chamber due to leakage. It is assumed that the potential and kinetic energies during compression process are negligible. From the above equation, the change of internal energy of gas in time dt may be written as

$$dU_g = dQ - dW + dH_g \qquad \text{(i)}$$

Change in internal energy of gas in time dt can also be expressed as a function of mass and temperature as

$$dU_g = c_v M_g dT_g + c_v T_g dM_g \qquad \text{(ii)}$$

The change of enthalpy in time 'dt' due to leakage is computed as;

$$dH_g = c_p T_{gb} dM_{gi} - c_p T_g dM_{go} \qquad \text{(iii)}$$

The gas work may be expressed in terms of geometrical volume change, and oil volume change due to the leakage. Since the oil is an incompressible fluid, the gas work can be expressed as

$$dW = PdV + Pq_{lo}dt - P_{bi}q_{li}dt \qquad \text{(iv)}$$

Heat exchange between the gas and oil is assumed to follow the Newton's law of cooling. Hence, the transferred heat between gas and oil in time 'dt' is

$$dQ = -hA(T_g - T_l)dt \qquad \text{(v)}$$

Substituting the equations (ii), (iii), (iv) and (v) in the equation (i), and rearranging, the rate of change of working gas temperature during compression process can be obtained as

$$\frac{dT_g}{dt} = -\frac{(k-1)T_g}{V_g}\left(\frac{dV}{dt} - \frac{P_b}{P}q_{li} + q_{l0}\right) + \frac{m_{gi}}{M_g}(kT_{gb} - T_g) - \frac{m_{go}}{M_g}(k-1)T_g - \frac{hA}{c_v M_g}(T_g - T_l) \qquad (1)$$

The equation of state of the gas may also be written as

$$PV_g = M_g R T_g$$

The differential form of the above equation is

$$\frac{dP}{dt} = \frac{1}{V_g}\left[-P\frac{dV_g}{dt} + RT_g\frac{dM_g}{dt} + RM_g\frac{dT_g}{dt}\right] \qquad \text{(vi)}$$

The rate of change of net gas volume can be expressed in terms of geometrical volume change and leakage oil volume flow rates into and out of the working space

$$\frac{dV_g}{dt} = \frac{dV}{dt} - q_{li} + q_{lo} \qquad \text{(Vii)}$$

The gas mass in the working chamber will vary continuously during compression process due to leakage. Therefore, the rate of change of gas mass is

$$\frac{dM_g}{dt} = m_{gi} - m_{go} \qquad (2)$$

Substituting $\frac{dT_g}{dt}$ value from equation (1), in to the equation (vi) and using equations (vii) and (2), the rate of change of pressure is obtained as

$$\frac{dP}{dt} = \frac{1}{V_g}\begin{bmatrix} -kP(\frac{dV}{dT} + q_{lo}) + (kP_b - P_b + P)q_{li} + k\frac{T_{gb}PV_g}{T_g M_g}m_{gi} \\ -k\frac{PV_g}{M_g}m_{go} - \frac{PV_g hA}{c_v M_g T_g}(T_g - T_l) \end{bmatrix} \qquad (3)$$

The rate of change of oil temperature in the working chamber may also be computed as

$$\frac{dT_l}{dt} = (T_{lb} - T_l)\frac{m_{li}}{M_l} + \frac{hA}{M_l c_l}(T_g - T_l) \qquad (4)$$

The oil mass in the compressor cavity will also vary continuously due to leakage and can be expressed as

$$\frac{dM_l}{dt} = m_{li} - m_{lo} \qquad (5)$$

The above differential equations (1) to (5) are adequate to simulate the compression process.

**Leakage Analysis**

The leakage of oil and gas mixture through leakage paths (except at lobe tip-housing clearance) during compression process, assumed to follow the path of a convergent nozzle, and can be rewritten as

$$m = (m_g + m_l) = \frac{CA_c P_1}{\sqrt{T_1}}\sqrt{\frac{2\beta(r^{\frac{2}{\beta}} - r^{\frac{\beta+1}{\beta}})}{(\beta - 1)R_m}} \qquad (6)$$

For $r > (\frac{2}{â + 1})^{\frac{â}{â-1}}$

or

$$m = (m_g + m_l) = \frac{CA_c P_1}{\sqrt{T_1}}\sqrt{\frac{â(\frac{2}{k-1})^{\frac{â+1}{â-1}}}{R_m}}$$

For $r \le (\frac{2}{â + 1})^{\frac{â}{â-1}}$

where $r = \frac{P_2}{P_1}$, $P_1$ & $P_2$ being upstream and down stream pressures respectively.

Due to the presence of oil, the exact properties of oil-gas mixture leaking through the leakage paths are not known. However, the properties of the oil-gas mixture coming out from the leakage paths are calculated based on the assumptions and comparison with experimental data. By comparison with the laboratory tests, the fallowing assumptions for different types of leakage paths have been shown to be the most appropriate [2].

(1) The gas/oil mixture in all leakage paths is homogeneous.
(2) The gas/oil mixture ratio is same in all leakage paths except at the lobe tip clearance and equal to the mixture ratio in the discharge port.

From leakage equation, the ratio of specific heats 'β' of the oil gas mixture will have the influence on leakage rate. Therefore, the apparent ratio of specific heats of oil-gas mixture may be estimated by the formula [1]

$$â = \frac{c_p + \varphi c_l}{c_v + \varphi c_l} \qquad (7)$$

Similarly, the modified gas constant for the mixture is [1]

$$R_m = \frac{R}{1 + \varphi} \qquad (8)$$

The oil to gas mass ratio in working chamber and through leakage paths may be written as

$$\varphi = \frac{M_l}{M_g} = \frac{m_l}{m_g} \qquad (9)$$

The Average leakage area is determined by multiplying sealing line length with an average gap (clearance) for each type of leakage. The discharge coefficient or flow coefficients are empirically selected to account for the presence of oil for each clearance leakage flow.

At the lobe tip, the clearance fills with the oil due to action of centrifugal force, and the oil leakage flow is in the single phase. Therefore, the leakage flow rate of oil can be calculated using the equation of incompressible viscous flow through a narrow gap [5].

$$m_{lt} = Sñ_l\left[\frac{V_t a}{2} - \frac{(P_1 - P_2)a^3}{12ì_l w_t}\right] \qquad (10)$$

The leakage gas in to the working chamber during compression process is the leakage through leading blowhole, and the clearance between discharge end of rotor and faceplate, from both male and female rotor leading cavities. Hence, the total leakage gas flow rate in to the chamber is

$$m_{gi} = (m_{bi} + m_{dmi} + m_{dfi})/(1+\varphi) \quad (11)$$

The total gas leakage going out of the working chamber is

$$m_{go} = (m_{bo} + m_{ilo} + m_{dmo} + m_{dfo})/(1+\varphi) \quad (12)$$

The oil mass leaking into the working chamber can be estimated by

$$m_{li} = \varphi m_{gi} + m_{tmi} + m_{tfi} \quad (13)$$

The leakage of oil from the working chamber to the adjacent groove can be estimated by

$$m_{lo} = \varphi m_{go} + m_{tmo} + m_{tfo} \quad (14)$$

The equations (6) to (14) are adequate to calculate the rate of change of gas and oil mass during compression process.
The heat transfer coefficient between oil and gas may be written as

$$h = \frac{kP_s V_{t1}}{(k-1)At_s} \frac{d\varsigma_v}{dT_s} \quad (15)$$

Equation (15) relates ‘h’ to the tangent of the $\eta_v$-$T_s$ curve. Applying experimental data to this equation, h can be determined. However, no exact information exists concerning the heat transfer area ‘A’. Therefore, the representative heat transfer area is defined [1] as;

$$A = V_{t1}^{2/3} \quad (16)$$

and

$$V_{t1} = (A_m + A_f)L$$

where, $V_{t1}$ is the geometrical volume of one composite male and female rotor cavities.

Total theoretical displacement volume rate of the compressor can be written as [6]

$$V_{tg} = (A_m + A_f)L \times n_m \times N_m \quad (17)$$

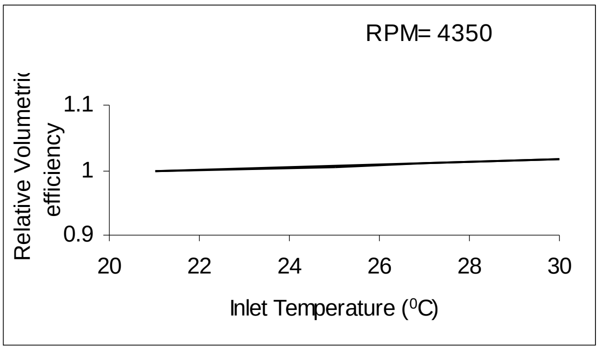



**Figure 3** Experimental volumetric efficiency against inlet temperature (Efficiencies are relative to inlet temperature of 21 $^0$C)

The amount of gas inducted in to the suction chamber depends on the temperature at the end of suction process. Solving the below quadratic equation (18), the temperature at the end of the suction can be found.

$$\begin{aligned} &- c_p M_{il} T_1^2 \\ &+ T_1\left[c_p M_{ts} T_s + c_p M_{il} T_s - hA(T_{oil} - T_s)t_s\right] \\ &- c_p M_{ts} T_s^2 = 0 \end{aligned} \quad (18)$$

The cycle time for the twin screw compressor is

$T = 360^0[1+1/n_m]+\varphi$

The time taken for suction process is

$$t_s = \frac{1}{N_m} \quad (19)$$

The time taken for compression process is

$$\Gamma = \frac{1}{N_m}\left(\frac{\phi}{360} + \frac{1}{n_m}\right) \quad (20)$$

**Efficiencies**

Numerically, it is easier to find the volumetric efficiency of the compressor in terms of mass discharge. Therefore, the volumetric efficiency based on the mass discharge rate is

$$\varsigma_{tv} = \frac{M_d}{M_{ts}} = \frac{M_{t1} - m_{gl}}{M_{ts}} \qquad (21)$$

where $M_d$ is the discharged gas mass and $m_{gl}$ is the net theoretical gas mass leakage during compression process.

Based on the experimental mass flow rate, the volumetric efficiency may be defined as

$$\varsigma_{ev} = \frac{m_a}{M_t} \qquad (22)$$

where $m_a$ is the experimentally measured discharged mass flow rate. The theoretical inducted mass flow rate into the compressor cavity is defined as

$\varepsilon_r$= 0.03mm
$\varepsilon_r$= 0.09mm
$\varepsilon_r$ =0.15mm
RPM=4350
$T_g$ = 293K
$T_l$ = 323K
$P_s$=1bar(a)
$P_d$ =10.3bar(a)
φ =300
φ=15
$\varepsilon_r$ = 0.03mm
$\varepsilon_d$ = 0.03mm
a = 0.03mm
$A_b$ =5.3mm$^2$

$$M_t = \frac{P_s V_t}{R T_s} n_m \times N_m$$

## Results and discussions

The effect of interlobe clearance on the P-V profile for the 5-6 rotor combination is shown in figure 2. It shows that the pressure during compression process decreases as the interlobe clearance increases. This is because of decreasing mass in the compression chamber due to more interlobe clearance leakage. The influence of oil injection quantity on volumetric efficiency is shown in figure 3. The influence of oil injection quantity will have marginal influence on volumetric efficiency beyond certain oil to gas mass ratio. The influence of RPM on volumetric efficiency is shown in figure 4. It shows that at higher lobe tip velocity i.e at higher RPM, the volumetric efficiency has little effect. This is because at lower RPM, the volumetric efficiency is less due more available leakage time. The variation of volumetric efficiency with discharge pressure is shown in figure 5. It gives an idea that the variation of volumetric efficiency at higher discharge pressures is appreciable. This is because the screw compressor doesn't have any clearance volume at the end of compression process.

RPM=4350
$T_g$ = 293K
$T_l$ = 323K
$P_s$=1.0 bar(a)
$P_d$ =10.3bar(a)
φ=300$^0$
$\varepsilon_r$= 0.03mm
$\varepsilon_d$ = 0.03mm
a = 0.03mm
$A_b$=5.3mm$^2$

$P_s$ =1.0 bar (a)
$P_d$ = 10.3bar(a)
$T_s$ =293K
$T_l$= 323K
φ =300$^0$
φ =15
$\varepsilon_d$ =0.03mm
$\varepsilon_r$ =0.03mm
$a_t$ = 0.03mm
$A_{bh}$ = 5.3mm$^2$

## Conclusions

The heat transfer coefficient between gas and oil for an oil injected twin screw compressor has been determined from experimental observations. A simulation model has been developed for evaluating the performance of oil injected dual screw compressor. The model allows wrap angle, blowhole sizes and process fluids to be varied, thus facilitating analysis of compressor.

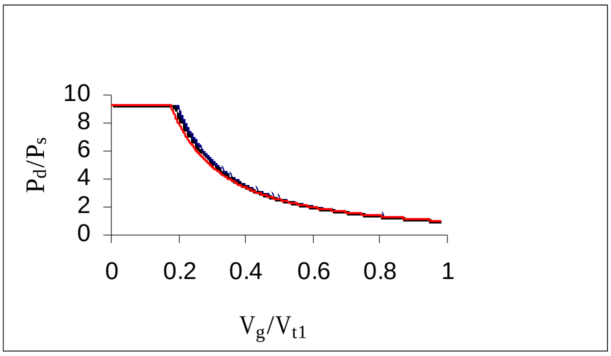



**Figure 4** Effect of interlobe clearance on P- V diagram profile

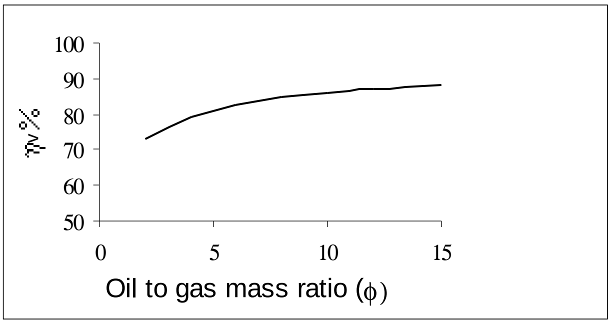



**Figure 5** Variation of volumetric efficiency with injected oil quantity

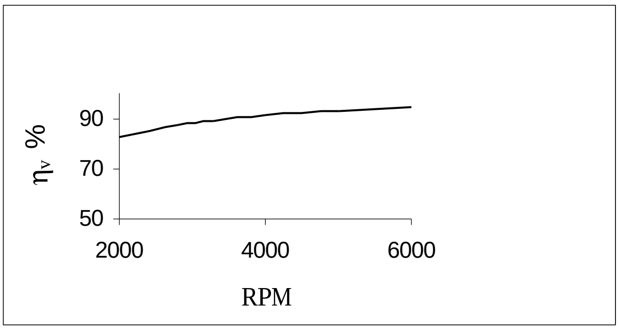


**Figure 6** Variation of volumetric efficiency with rotational speed of male rotor

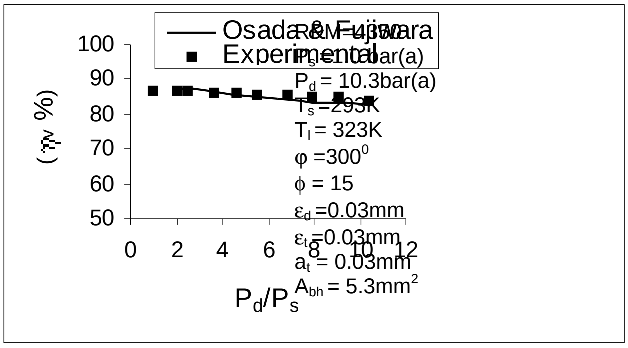



**Figure 7** Variation of volumetric efficiency with discharge pressure

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