Method of Moments Analysis of Circular Loop Linear Array and Circular Loop Circular Array

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***Abstract*— In this paper, Method of Moments technique is used to analyze the Circular loop array antenna in the VHF (Very High Frequency) band with a loop thickness of 0.04m and radius, 0.1λ. RWG (Rao-Wilton-Glisson) edge elements are used as basis function and Galerkin's method is used to cast the problem as a matrix equation. The corresponding radiation pattern in the far zones for both types of antennas are plotted in MATLAB. The variation of gain with the number of antenna elements (N), distance between the adjacent elements (d) and radius of circular array (R) is tabulated and the variation of Antenna gain vs Frequency is plotted.**

Keywords—Method of Moments ; RWG elements; Circular loop linear array; Circular loop circular array

I. INTRODUCTION

Antenna arrays are becoming extremely important in wireless communications. An increasing demand for better quality, increased capacity and new value added services on the wireless mobile communication network has led to many innovations in the antenna design. Arrays of antenna have a highly directive nature.

VHF band is widely used in Air Traffic Control communication, TV broadcasting and FM radio broadcasting. Doppler Very High Frequency Omni Range (DVOR) which enables an aircraft to get its position coordinates is a phased circular array antenna that operates on a frequency band of 108MHz-117.95MHz.

Method of Moments is a novel approach to reduce the hefty integral equations to a set of matrix equations. Method of Moments technique is used to obtain the non-uniform current distributions and resulting radiation patterns [1]-[2].A full wave Method of Moment analysis of infinite arrays of stripline fed tapered slot antenna has been designed [3]. Ito, in 2004 patented a VHF wideband loop antenna [4]. This paper combines the individual works on Method of Moments and circular loop design in VHF and presents a study of the radiation characteristics of Circular loop linear array antenna and Circular loop circular array antenna in the VHF range using Method of Moments.

II. PROBLEM FORMULATION AND SOLUTION

A. Method of Moments

Method of Moments (MoM) converts the integro-differential equations into matrix system of linear equations.

$$A(x) = B \qquad (1)$$

Where, A: Integro differential operator
x: unknown function to be solved
B: known function

The unknown function x that is to be solved is then approximated by the basis function

$$x = \sum_{n=1}^{N} I(n) b(n) \qquad (2)$$

Where, b: basis function
x: unknown function
I : unknown complex coefficients
N: total number of basis functions

Now, imposing boundary conditions and making the inner product of weighting function (w) and residual zero.

$$B - A\left(\sum_{n=1}^{N} I n b n\right) : \text{Residual/ Error} \qquad (3)$$

$$< w\, ,\, B - A\left(\sum_{n=1}^{N} I n b n\right) > = 0 \qquad (4)$$

To simplify the calculation Galerkin's method or Collocation technique is used. In this paper Galerkin's method is applied wherein the basis function and the weighting functions are taken to be equal.

$$b_n = w_n \qquad (5)$$

After simplifying equation (4) we get the equations in the form of,

$$[Z][I]=[V] \qquad (6)$$

$$[I] = [I_1\ I_2\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ I_N]^T \qquad (7)$$

$$[V] = [<B,w_1>\ <B,w_2>\ \_\ \_\ \_\ \_\ \_\ \_\ \_ <B,w_n>]^T \qquad (8)$$

$$[Z]=\begin{bmatrix} <w_1,A(b_1)> & <w_1,A(b_2)> & \ldots\ldots\ldots & <w_1,A(b_N)> \\ <w_2,A(b_1)> & <w_2,A(b_2)> & \ldots\ldots\ldots & <w_2,A(b_N)> \\ & & \vdots & \\ <w_N,A(b_1)> & <w_N,A(b_2)> & \ldots\ldots\ldots & <w_N,A(b_N)> \end{bmatrix} \quad (9)$$

B. RWG Elements

The method of analysis of the above rely on RWG (Rao-Wilton-Glisson) edge elements [5]. First the given surface is divided into separate triangles. Every pair of triangles which shares a common edge, constitutes a RWG edge element. One of the triangles has + sign while the other has – sign. Basis function for the RWG element is defined as-

$$f_m \begin{cases} L_m/2A_m^{+}\ \rho^{+}(r) & r \text{ in } T_m^{+} \\ L_m/2A_m^{-}\ \rho^{-}(r) & r \text{ in } T_m^{-} \end{cases} \quad (10)$$

Here, L is the edge length and $A^{-/+}$ is the area of the minus/plus triangle. Vector rho connects the free vertex of the +/- triangle to the observation point r. The surface electric current on the surface of the antenna (a vector) is the sum of the contributions made by every edge element with unknown coefficients. These coefficients are found from the moment equations. The moment equations are a linear system of equations with the impedance matrix Z. The basis function of the edge element approximately corresponds to a small but finite electric dipole of length d=|r(c-)-r(c+)|, here index c denotes the middle of the triangle. The division of the antenna into RWG elements thus corresponds to the division of the antenna into small dipoles [5]. Impedance matrix Z implies the relation between different electric dipoles [5].

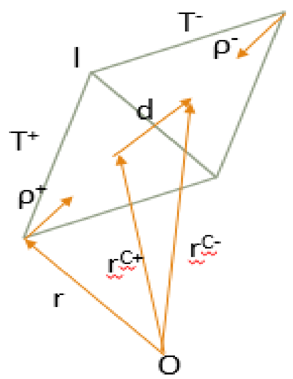


Fig1. RWG edge element

Using the basis function b_n the surface current density J [5], can be computed as,

$$J=\sum_{n=1}^{N} I(n)b(n) \quad (11)$$

Where N is the number of inner edges. The impedance matrix can be calculated by performing integration over the triangle. Barycentric subdivision of triangle is applied (Fig 2). A triangle is divided into 9 small triangle using 1/3rd rule. Then the integral of a function g over the triangle T_m is given by

$$\int_{T=1}^{N} g(r)ds = \frac{Am}{9}\sum_{1}^{9} g(r') \quad (12)$$

Where r' are the mid points of the sub triangle and A_m is the area of the primary triangle.

The average power of the radiated field in the far zones is given by the vector product of E and H* which is called Poynting vector.

$$W(r) = \frac{1}{2}\mathrm{Re}[E(r) \times H^*(r)] \quad (13)$$

Radiation intensity U is defined as radiation density multiplied by r^2,

$$U=Wr^2 \quad (14)$$

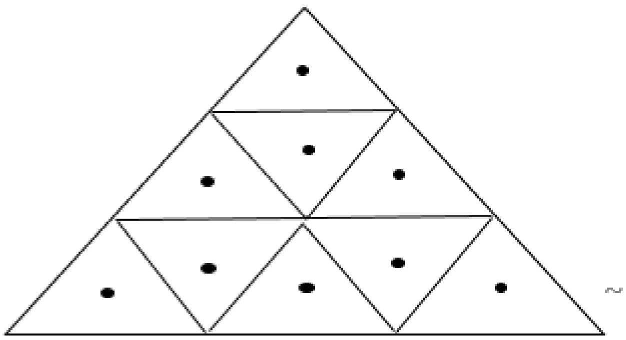


Fig2. Barycentric subdivision of triangles

The radiation density and radiation intensity is calculated over a large sphere of radius 100m. Directivity is defined as

$$D=10*\log_{10}\left(\frac{U}{Uo}\right) \quad (15)$$

And gain is defined as

$$G=\frac{\max(U)}{Uo} \quad (16)$$

C. Design of Circular Loop

A circular loop is nothing but the variation of the angle subtended by the rectangle at the center.

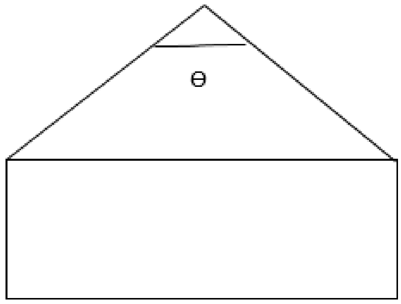


Fig3. Rudimentary step to create a loop

Where, Θ= 2πk/N where N are total number of rectangles and k=1,2,3…. N. Along with the creation of circular loop a coordinate matrix is maintained. The feed point is taken at [-1 0 0].

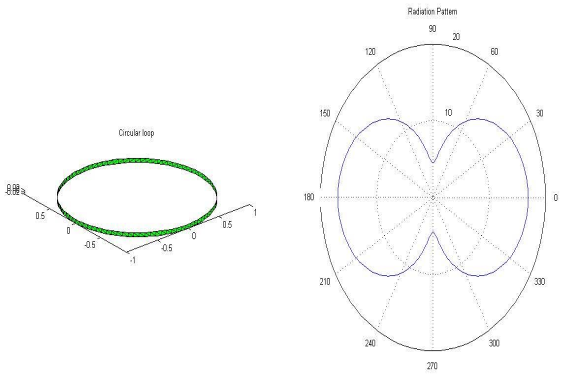
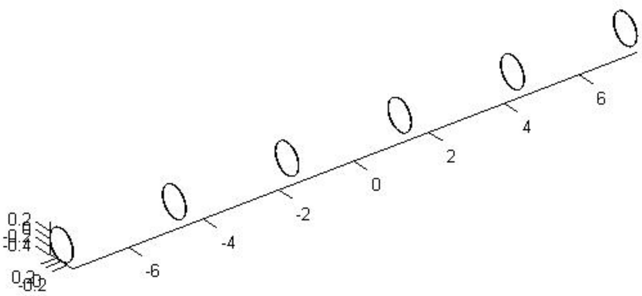


Fig4. Circular loop Antenna

D. *Circular Loop Linear Array*

Once the circular loop is created we need to replicate the design at a separation distance, d between every element over the entire length L with individual feed points,

*L= (N-1)*d, where N is the number of elements*



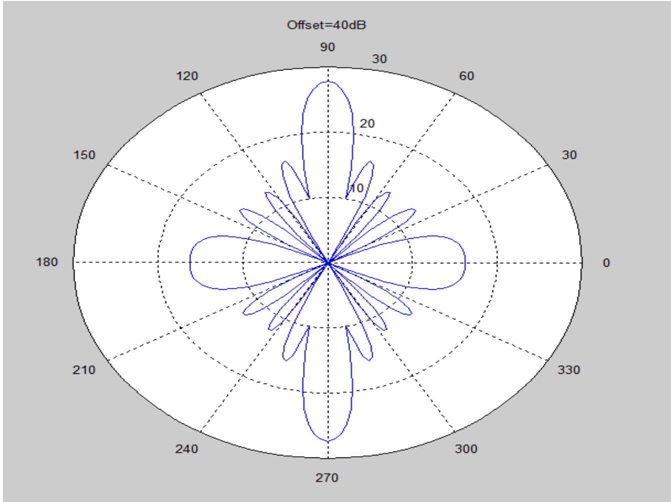
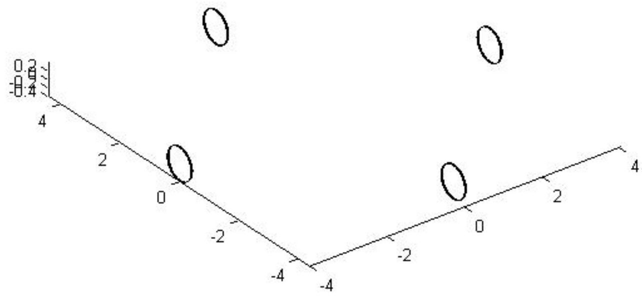


Fig 5. Arrangement and radiation plot of circular loop linear array for N=6 and d=3m

E. *Circular Loop Circular Array*

After creating a circular loop, replicating the loop along a circle will form a circular loop array with individual feed points.

$\theta = 2\pi/N$ *N: Number of elements*



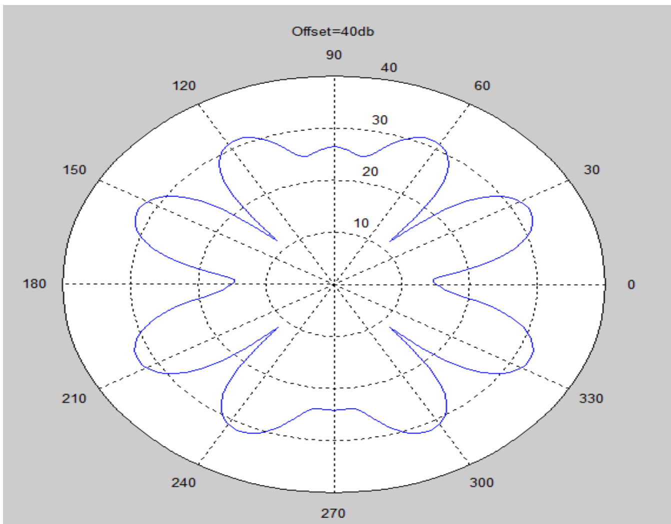


Fig6. Arrangement and radiation plot of circular loop circular array for N=4 and R=4

III. Results

S.No	N	d (m)	Gain (dB)
1.	4	3	8.97
2.	4	8	9.76
3.	6	1	12.49
4.	6	3	12.73

TABLE 1. Variation of Gain with respect to N and d for Circular loop Linear Array (75MHz)

S.No	N	R (m)	Gain (dB)
1.	4	4	8.62
2.	4	7	9.77
3.	6	8	12.76
4.	6	14	14.12

TABLE 2. Variation of Gain with respect to N and R for Circular Loop Circular Array (75MHz)

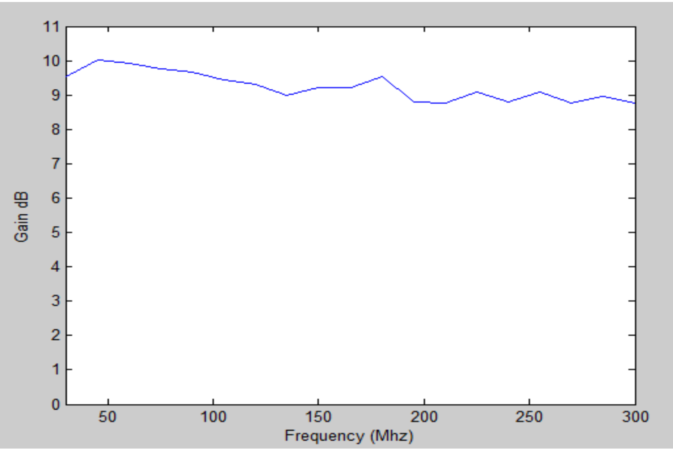


Fig 7. Variation of Gain of Circular loop Linear array antenna over the entire VHF range (N=4 and d=3)

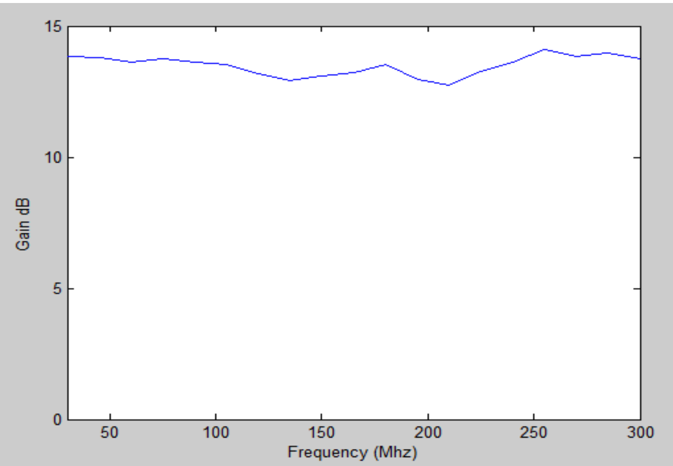


Fig 8. Variation of Gain of Circular loop circular array antenna over the entire VHF range (N=6 and R=14)

IV. Conclusion

The Method of Moments technique was applied to analyze Circular Loop Linear Array and Circular Loop Circular Array antennas. The results were obtained for VHF range using RWG elements. The variation of antenna gain with N (Number of array elements), d (distance of separation between elements) and R (Radius of array) was tabulated along with the variation of antenna again for frequencies in VHF band. It was observed that circular loop linear array antenna has greater directivity in comparison to circular loop circular array antenna. Moreover, the desired directivity can be achieved by choosing the appropriate values for N, d and R. The gain vs frequency plots show the variation of gain of antenna over the entire VHF range. For N=4 and d=3 the gain varies between 8dB to 10dB for linear array and for N=6 and R=14 it varies between 13dB to 15dB for circular array.

While plotting radiation patterns an OFFSET of 40dB was added as MATLAB's polar function does not take into account the negative values of gain, hence the plots are plotted after adding an OFFSET of 40dB.

Another approach to model a loop in MATLAB is by using the PDE toolbox of MATLAB. The mesh created then can be used to design different antenna designs.

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