Kinematic Control of a Mobile Manipulator

B.B.V.L .Deepak[1,2] , Dayal R. Parhi[1] and Ravi Praksh[2]

[1]Department of Mechanical Engineering, National Institute of Technology-Rourkela
[2] Department of Industrial Design, National Institute of Technology-Rourkela, India
{bbv, DRKPARHI, 113ID1274}@nitrkl.ac.in

Abstract: Proper motion planning algorithms are necessary for intelligent robotic systems in order to execute their specific tasks. To solve this problem, current research work introduces the inverse kinematic models for mobile manipulators. In general a systematic closed form solution is not available in the case of inverse kinematic models. To obtain elucidation for inverse kinematic problem is more complex as compared to direct kinematics problem. The current research work aims to combine the functionality of a robot arm with an autonomous platform. It means development of an autonomous wheeled mobile robot on which the robot arm is mounted. The purpose of this work is to integrate both the segments (i.e. mobile manipulator & mobile platform), such that the system can perform the constrained moves of the arm in the mean while as the platform is moving.

Key words: Robotic manipulator, wheeled mobile platform, inverse kinematic models, end effector constraints & geometric wheel constraints.

1. Introduction

Proper motion planning algorithms are necessary for robotic systems may be of manipulator or mobile platforms, in order to execute their specific tasks [1]. Motion planning of industrial robots is a critical issue because of its end effectors path constraints [3]. Whereas, the motion control of mobile robots or the mechanical behavior of the robot depends upon the wheel geometric constraints while the robot is in motion [4-5]. To sustenance the progress and to enlarge the solicitation potential of robotic manipulators (industrial robotics), it is coherent to integrate locomotion features with manipulation capabilities, hereby developing wheeled mobile manipulators [6-7].

Matched to conventional industrial robotic arms, mobile manipulators adapt to environmental changes for performing wide range of manufacturing tasks. Another benefit of this category of robotic systems is that the existing industrial environments do not have to be altered or modified as in the case of Automated Guided Vehicles (AGV’s), where permanent cable layouts and/or markers are required for navigation [6]. In past [5], [8], authors dealt with kinematic models of wheeled mobile robots to generate trajectory within its environments. The developed kinematic models according to the wheel geometric constraints: Wheel sliding constraint and Wheel rolling constraints. But the kinematic analysis of manipulators is quite different from as compared to wheeled mobile robots. Kinematics deals with joint space coordinates, link reference frames and end-effector reference frames [9-10]. To obtain solutions of inverse kinematics, there are several algorithms have been developed subjected to end-effector constraints [11]. Motion planning of mobile robot deals with generation of safest and shortest pats while reaching its target position [12-14]. There are several motion planning techniques have been developed based on to artificial intelligence algorithms [16-17]. But these techniques are not suitable for mobile manipulator control [18-19].

In this paper we propose inverse kinematic solutions for both the mobility platform and robotic manipulator. The purpose of this work is to integrate both the segments (i.e. mobile manipulator & mobile platform), such that the system can perform the constrained moves of the arm in the mean while as the platform is moving.

2. Mechanical Design Architecture of Mobile Manipulator

The main characteristic of an automated mobile manipulator is its flexible operational workspace. Coordinated motion of the manipulator and mobile platform leads to a wide range of redundancy. Owing the velocity restrictions enforced on the mobile base, the WMM is a non-holonomic system. So it is required to develop a kinematic controller to make the robot system follows a desired end-effector and platform trajectories in its workspace coordinates simultaneously.

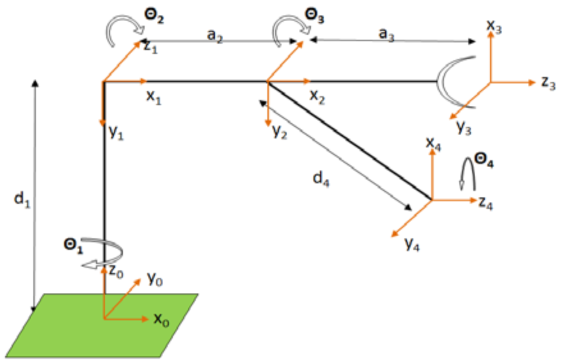


Fig.1 Link coordinate frame of the manipulator

Table 1 Kinematic parameters of the manipulator

Axis	$\boldsymbol{\theta}$	***d (mm)***	**a *(mm)***	$\boldsymbol{\alpha}$
1	θ_1	$d_1 = 70$	0	$-\pi/2$
2	θ_2	0	$a_2 = 100$	0
3	θ_3	0	$a_3 = 70$	$-\pi/2$
4	θ_4	$d_2 = 45$	$a_4 = 0$	0

In the current research work, a wheeled mobile manipulator is considered with a 4-axix manipulator equipped on a non-holonomic differential wheeled mobile platform. For the considered 4-axis manipulator coordinate frames for the manipulator are assigned as shown in the Fig.1 and corresponding kinematic parameters are represented in Table.1.

The arm equation is obtained using D-H notation [2] as represented in Eq.(4), which is function of θ_1, θ_2, θ_3 and θ_4. The arm equation consists of six elements; three corresponds to the end-effector's position and the remaining three represents yaw, pitch and roll orientations of the tool.

Using a homogeneous coordinate transformation matrix, the relation between adjacent links is given in Eq.(1).

$$T_i = Rot(z,\theta_i) * Trans(0,0,d_i) * Trans(a_i,0,0) * Rot(x,\alpha_i) \quad (1)$$

$$= \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

Here $C_i=\cos(\theta_i)$, $S_i=\sin(\theta_i)$

On replacing the kinematic parameters illustrated in Table 1 into Eq. (2), individual transformation matrices T_0^1 to T_4^5 can be found and the global transformation matrix T_0^5 of the robot arm is fund according to the Eq. (3).

$$T_{base}^{tool} = T_{base}^{wrist} * T_{wrist}^{tool} = \begin{bmatrix} m_x & n_x & o_x & p_x \\ m_y & n_y & o_y & p_y \\ m_z & n_z & o_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} R(\theta)_{3x3} & P_{3x1} \\ 0 & 1 \end{bmatrix} \quad (3)$$

Where $T_{base}^{wrist} = T_0^1 * T_1^2$ and $T_{wrist}^{tool} = T_2^3 * T_3^4$

Where (p_x, p_y, p_z) represents the position and $R(\theta)_{3x3}$represents the rotation matrix of the end effector. Tool configuration is six-dimensional because arbitrary specified by three position co-ordinates (x, y, z) and orientation co-ordinates (yaw, pitch, roll).

$$X = \begin{Bmatrix} p_x \\ p_y \\ p_z \\ y \\ p \\ r \end{Bmatrix} = \begin{Bmatrix} c_1(a_2c_2 + a_3c_{23} - d_4s_{23}) \\ s_1(a_2c_2 + a_3c_{23} - d_4s_{23}) \\ d_1 - a_2s_2 - a_3s_{23} - d_4c_{23} \\ -[exp(\theta_4)/\pi]c_1s_{23} \\ -[exp(\theta_4)/\pi]s_1s_{23} \\ -[exp(\theta_4)/\pi]c_{23} \end{Bmatrix} \quad (4)$$

The time derivative of the end-effector's position gives the linear velocity of the end-effector. The position of the end-effector $[p_x \quad p_y \quad p_z]^T$is a function of ($\theta_1$, θ_2, θ_3) because θ_4 indicates the orientation of the tool.

$$\begin{Bmatrix} v_x \\ v_y \\ v_z \end{Bmatrix} = \frac{d}{dt}\begin{Bmatrix} p_x \\ p_y \\ p_z \end{Bmatrix} = = [J_M]_{3x3} \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{Bmatrix} \quad (5)$$

Where $[J_M]_{3x3}$ is manipulator velocity Jacobean matrix and is equal to:

$$\begin{bmatrix} -s_1(a_2c_2 + a_3c_{23} - d_4s_{23}) & -a_2^2s_2c_1 - a_3^2s_{23}c_1 - d_4c_{23}c_1 & -a_3s_{23}c_1 - d_4c_{23}c_1 \\ c_1(a_2c_2 + a_3c_{23} - d_4s_{23}) & -a_2s_1s_2 - a_3s_1s_{23} - d_4s_1s_{23} & s_1(-a_3s_{23} - d_4s_{23}) \\ 0 & -a_3c_{23} + d_4s_{23} & -a_3c_{23} + d_4s_{23} \end{bmatrix} \quad (6)$$

2.1. Inverse Kinematic Model

This section describes the development of inverse kinematic models of an arm based on its link coordinate systems. From Eq.(9), it is observed that there is a possibility of getting two wrist angles ($\pm\theta_3$) for the same tool position. Here the elbow angle (θ_2) depends on θ_3, two elbow angles will obtain corresponds to each θ_3. The base angle can be found easily by Eq.(7).

Base angle $\theta_1 = arctan\left({p_y}/{p_x}\right)$ (7)

Where $p_x\ and\ p_y$ are determined from Eq. (8) and from the arm equation, the global pitch angle θ_{23} can be found as follows:

$$\theta_{23} = arctan\left({(c_1 y + s_1 p)}/{r}\right) \quad (8)$$

The wrist angle can be found as follows:

$$\theta_3 = \pm arccos\left({(\|b\|^2 - {a_2}^2 - {a_3}^2)}/{p_x}\right) \quad (9)$$

Where $\|b\|^2 = {b_1}^2 + {b_2}^2;\quad and\quad b_1 = c_1 p_x + c_2 p_y + d_4 s_{23}$ & $b_2 = d_1 - d_4 c_{23} - p_z$

Once θ_3 is known then elbow angle θ_2 can be found from the global pitch angle θ_{23}.

$$\because \theta_{23} = \theta_3 + \theta_2 \Rightarrow \theta_2 = \theta_{23} - \theta_3 \quad (10)$$

The final joint parameter θ_4 can be found from the arm Eq. (14) as follows

Tool roll angle $\theta_4 = \pi * ln\sqrt{(y^2 + p^2 + r^2)}$ (11)

2.2. Velocity Jacobean of mobile platform

There are three constraints for a differential platform: first one corresponds to move the platform in the direction of axis of symmetry and the remaining two are rolling constraints not allow the wheels to slip. The motion equation of a differential mobile platform is a function of left wheel and right wheel velocities (v_{Lt}, v_{Rt}) as represented in the Eq.(15).

$$\dot{\xi}_I = \begin{Bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{Bmatrix} = \frac{1}{2s}\begin{bmatrix} s*cos\psi & s*cos\psi \\ s*sin\psi & s*sin\psi \\ -1 & 1 \end{bmatrix} * \begin{Bmatrix} v_{Rt} \\ v_{Lt} \end{Bmatrix} \quad (12)$$

While moving the mobile platform with an heading angle ψ, the following kinematic Eq.(16) is used which relates the linear velocity of the mobile platform reference frame to the wheel velocities.

$$\begin{Bmatrix} V_x \\ V_y \end{Bmatrix} = [J_{MP}]_{2x2} \begin{Bmatrix} \dot{\theta}_{rt} \\ \dot{\theta}_{lt} \end{Bmatrix} \quad (13)$$

Where $[J_{MP}]_{3x3}$ is mobile platform velocity Jacobean matrix and $\dot{\theta}_{rt}$ & $\dot{\theta}_{lt}$ are angular velocities of right and left wheels respectively

Velocity Jacobean matrix $[J_{MP}] = \frac{1}{2r}\begin{bmatrix} cos\psi & cos\psi \\ sin\psi & sin\psi \end{bmatrix}$

2.3. Velocity Jacobean of Mobile Manipulator

The differential kinematics of the mobile manipulator is obtained by combining the kinematic Eqs. (5) & (13) of a 4-axis manipulator and the differential mobile platform as shown in Eq. (14). The first three parameters in the above equation relate to the manipulator and the remaining two corresponds to the differential platform.

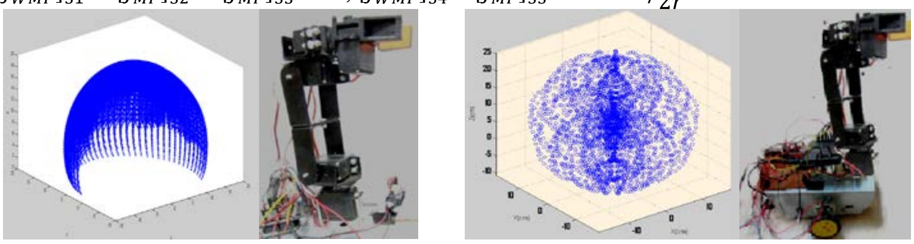
$$\{\dot{q}\} = \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_{rt} \\ \dot{\theta}_{lt} \end{Bmatrix} = [J_{WMP}]_{5x5} \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_{rt} \\ \dot{\theta}_{lt} \end{Bmatrix} \qquad (14)$$

Where $[J_{WMP}]$ is the Velocity Jacobean of Mobile Manipulator and is represented as follows:

$$[J_{WMP}] = \begin{bmatrix} [J_{WMP}]_{11} & [J_{WMP}]_{12} & [J_{WMP}]_{13} & [J_{WMP}]_{14} & [J_{WMP}]_{15} \\ [J_{WMP}]_{21} & [J_{WMP}]_{22} & [J_{WMP}]_{23} & [J_{WMP}]_{24} & [J_{WMP}]_{25} \\ [J_{WMP}]_{31} & [J_{WMP}]_{32} & [J_{WMP}]_{33} & [J_{WMP}]_{34} & [J_{WMP}]_{35} \\ [J_{WMP}]_{13} & [J_{WMP}]_{13} & [J_{WMP}]_{43} & [J_{WMP}]_{44} & [J_{WMP}]_{45} \\ [J_{WMP}]_{13} & [J_{WMP}]_{13} & [J_{WMP}]_{53} & [J_{WMP}]_{54} & [J_{WMP}]_{55} \end{bmatrix}$$

Where

$[J_{WMP}]_{11} = -s_1(a_2c_2 + a_3c_{23} - d_4s_{23})$; $[J_{WMP}]_{12} = -a_2^2s_2c_1 - a_3^2s_{23}c_1 - d_4c_{23}c_1$;
$[J_{MP}]_{13} = -a_3s_{23}c_1 - d_4c_{23}c_1$; $[J_{WMP}]_{14} = [J_{MP}]_{15} = 0$;
$[J_{WMP}]_{21} = c_1(a_2c_2 + a_3c_{23} - d_4s_{23})$; $[J_{WMP}]_{22} = -a_2s_1s_2 - a_3s_1s_{23} - d_4s_1s_{23}$
$[J_{WMP}]_{23} = s_1(-a_3s_{23} - d_4s_{23})$; $[J_{WMP}]_{24} = [J_{MP}]_{25} = 0$; $[J_{WMP}]_{31} = 0$
$[J_{WMP}]_{32} = -a_3c_{23} + d_4s_{23}$; $[J_{WMP}]_{33} = -a_3c_{23} + d_4s_{23}$; $[J_{WMP}]_{34} = [J_{MP}]_{35} = 0$
$[J_{WMP}]_{41} = [J_{MP}]_{42} = [J_{MP}]_{43} = 0$; $[J_{WMP}]_{44} = [J_{MP}]_{45} = {(cos\psi)}/{2r}$
$[J_{WMP}]_{51} = [J_{MP}]_{52} = [J_{MP}]_{53} = 0$; $[J_{WMP}]_{54} = [J_{MP}]_{55} = {(sin\psi)}/{2r}$



(a) (b)

Figure4: Workspace generated by (a) Fixed Manipulator and (b) WMM

Fig.4 represents the workspace generated by the mobile manipulator, when it is at a specific position. The developed hybridised system extends the workspace of a fixed manipulator by two times as shown in Figure5.

3. Conclusion

This study integrates the kinematic models of a 4-axis manipulator and a differential mobile platform. The motion of the developed WMM is controlled by five parameters in which three parameters gives the velocity information of the manipulator and the remaining two corresponds the differential mobile platform. Finally, comparison has been performed in between the theoretical result obtained from the current analysis with the experimental results of a fabricated real mobile manipulator (4 DOF).

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