# Integrating Principal Component Analysis, Fuzzy Linguistic Reasoning and Taguchi Philosophy for Quality-Productivity Optimization in Manufacturing Context: A Case Study

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***Abstract***: *Manufacturing process often requires optimization of machining parameters in order to improve cost and production time and also to improve the product quality as well as to increase productivity. In this context, present work demonstrates a multi-response optimization problem for selection of optimal cutting parameters (optimal process environment) for machining (turning) of nylon 6, as a case study; by using Principal Component Analysis (PCA) followed by fuzzed linguistic reasoning in combination with Taguchi's robust design technique. In this study, three controllable process parameters: cutting speed, feed, and depth of cut have been considered for obtaining desired Material Removal Rate (MRR) of the process and favorable multiple surface roughness features for the machined product; based on $L_9$ orthogonal array experimental design. The study has been aimed to search an appropriate process environment for simultaneous optimization of quality-productivity favorably. Various surface roughness parameters of statistical importance (of the machined product) have been considered as product quality characteristics whereas; MRR has been treated as productivity measure for the said machining process. To avoid assumptions, limitations, uncertainties and imprecision in application of existing multi-response optimization techniques; Principal Component Analysis (PCA) has been proposed to convert correlated responses into uncorrelated quality indices (called individual principal components); next, a fuzzy inference system (FIS) has been proposed for meaningful and feasible aggregation of individual principal components into an equivalent single quality index, thereby, converting such a multi-objective optimization problem into an equivalent single objective optimization situation. A Multi-Performance Characteristic Index (MPCI) has been defined based on the FIS output. MPCI has been optimized finally using Taguchi method. The study exhibits application feasibility of the proposed approach with satisfactory result of confirmatory test.*

***Keywords:*** *Principal Component Analysis (PCA), Taguchi's robust design, fuzzy inference system (FIS), Multi-Performance Characteristic Index (MPCI)*

## 1. Introduction

Nylon is widely used in a variety of application fields for their outstanding mechanical properties including high wear and abrasion resistance, superior strength and stiffness. Nylon's toughness, low coefficient of friction and wide size range availability make it an ideal replacement for a wide variety of materials from metal to rubber. Therefore, machining aspects of nylon is an emerging area of research.

Quality and productivity are two important but conflicting criteria in any machining operations. In order to ensure high productivity, extent of quality is to be compromised. It is, therefore, essential to optimize quality and productivity simultaneously. Productivity can be interpreted in terms of material removal rate (MRR) in the machining operation and quality represents satisfactory yield in terms of product characteristics as desired by the customers. Dimensional accuracy, form stability, surface smoothness, fulfillment of functional requirements in prescribed area of application etc. are important quality attributes of the machined product. Increase in productivity results reduction in machining time which may results quality loss. On the contrary, an improvement in quality results in increasing machining time thereby reducing productivity.

Optimization aspects of machining have been amply highlighted in literature [1-5], but to a limited extent. In most of the cases optimization has been done on a single objective function and it has been assumed that responses are uncorrelated. Literature highlights that Taguchi method is very popular in product/process optimization as it requires a well-balanced experimental design i.e. orthogonal array (limited number of experiments) which saves experimental time as well as cost. Not only this, Taguchi approach finds optimal at discrete levels of the process parameters; which can easily be adjusted in the machine/ setup. The traditional Taguchi method is widely used for optimizing the process parameters of a single response problem. Optimization of a single response may result unsatisfactory yield for remaining response features. But, overall performance of the manufactured product is often evaluated by several quality characteristics/responses. Under such circumstances, a unique optimal solution needs to be identified to optimize multiple responses simultaneously. In this context, Principal Component Analysis (PCA), a fuzzy expert system [6-8] coupled with Taguchi method has been proposed. PCA has been applied to eliminate response correlation thereby converting correlated multi-responses into equal or less number of uncorrelated indices called principal components (PCs). These PCs have been fed to a fuzzy inference system which works on a rule-base based on input-out mapping relationship and provides a single output. This unique FIS output has been termed as Multi-Performance Characteristic Index (MPCI). MPCI has been optimized finally by Taguchi method. This procedural concept avoids vagueness, uncertainty in assigning response weights. FIS can efficiently take care of this aspect into its internal hierarchy.

## 2. Experimentation

Experiments have been performed in following steps.

[1] Checking and preparing the Centre Lathe ready for performing the machining operation.
[2] Cutting nylon 6 bars by power saw and performing initial turning operation in Lathe to get desired dimension of the work pieces.
[3] Calculating weight of each specimen by the high precision digital balance meter before machining.
[4] Performing straight turning operation on specimens in various cutting environments involving various combinations of process control parameters like: spindle speed, feed and depth of cut.
[5] Calculating weight of each product (after machining) by the digital balance meter.
[6] Measuring the machining time to calculate MRR.
[7] Measuring surface roughness and surface profile with the help of a portable stylus-type profilometer, *Talysurf* (Taylor Hobson, Surtronic 3+, UK)

Samples of nylon 6 bars with dimensions of $\phi 50 \times 150$ (cutting length of 50 mm) have been used as working material. Single point HSS Tool of *INDOLOV SHRIRAM IK-20* has been used for the machining operation. Three cutting parameters (spindle speed, feed, depth of cut) varied in three different levels have been used to optimize the machining condition. Table I indicates selected process control parameters and their limits. In the present study, interactive effects of process parameters have been assumed negligible. The most suitable array based on Taguchi's method has been found as $L_9$ orthogonal array (Table II). The manually operated lathe *PINACHO (180×750) of Tussor Machine Tool India Pvt. Ltd. Coimbatore, India* has been used for the machining. The weight of the work piece has been measured in a high precision digital balance meter: Model: DHD 200 Macro single pan DIGITAL reading electrically operated analytical balance made by *Dhona Instruments*. The measured roughness parameters: $R_q$, $R_a$, $R_t$, $R_{ku}$, $R_z$, $R_{sm}$ along with material removal rate (MRR) have been shown in Table III.

## 3. Data Analysis

Experimental data (corresponding to Table III) have been converted into corresponding S/N ratios. For all surface roughness parameters, a Lower-the-Better (LB) criterion and for MRR, a Higher-the-Better (HB) criterion has been selected. These S/N ratios have then been normalized again based on Higher-the-Better (HB) criteria.

Pearson's correlation coefficient has been evaluated next. In all cases nonzero values of correlation coefficients (Table IV) depict that responses (S/N ratios of all output features) are inter-correlated. In order to avoid response correlation, PCA has been applied to convert correlated responses into uncorrelated quality indices called principal components (PCs). After finding Eigen values, Eigen vectors and correlation coefficients; factor analysis has been carried out to summarize the data structure in a few dimensions of the data and also to explain the dimensions associated with large data variability. It has been observed that first five principal components (PCs) can satisfactorily explain 99.9% data variation. Therefore, only these PCs have been considered for further analysis. Remaining PCs have been ignored.

Individual principal components (PC1 to PC5) thus computed (Table V) have been normalized next by using Higher-the-Better (HB) criteria. Normalized PCs have been fed as inputs in Fuzzy Inference System (FIS) (Fig. 1). The output of the fuzzy inference system has been defined as MPCI (Table VI). This Multi-Performance Characteristic Index (MPCI) has been finally optimized by using Taguchi methodology. Higher- the- Better (HB) criterion has been used for optimizing (maximizing) the MPCI (Eq. 1).

$$\frac{S}{N}(Higher - The - Better) = -10\log\left[\frac{1}{n}\sum_{i=1}^{n} = \frac{1}{y_i^2}\right] \tag{1}$$

$y_i$ represents measured response data in $ith$ experiment and $n$ is the total number of trial.

In calculating MPCI in FIS system, various membership functions (MFs) have been assigned to the five input variables: The selected membership functions for input variables are given below.

PC1: “Low”, “Medium” and “High”
PC2: “Low”, “Medium” and “High”
PC3: “Low”, “Medium” and “High”
PC4: “Low”, “Medium, and “High”.
PC5: “Low”, “Medium, and “High”.

Nine membership functions have been selected for MPCI: “Very very Low”, “Very Low”, “Moderately Low”, “Low”, “Relatively Medium”, “Medium”, “High”, “Very High” and “Very Very High” (Fig. 2). 27 fuzzy rules have been explored for fuzzy reasoning (Fig. 3; Table VII). Fuzzy logic converts linguistic inputs into linguistic output. Linguistic output is again converted to numeric values (MPCI) by defuzzification method. Numeric values of MPCIs have been tabulated in Table VI with corresponding S/N ratio as well as mean. S/N ratios of MPCIs have been calculated using Higher-the-Better (HB) criterion. Fig. 4 represents optimal parametric combination ($N_2$ $f_2$ $d_3$). Optimal result has been validated by satisfactory confirmatory test. Predicted value of S/N ratio of MPCI becomes -2.58814 and predicted mean is 0.645111 (highest among corresponding to all entries in Table VI). So, quality has improved using the said optimal setting validated by confirmatory experiment.

## 4. Conclusion

In this study, PCA and fuzzy rule based integrated optimization module has been developed using five input variables (PCs) with single output i.e. MPCI. By this way a multi-response optimization problem has been converted into an equivalent single objective optimization problem which has been further solved by Taguchi philosophy. The proposed procedure is simple, effective in developing a robust, versatile and flexible mass production process. Response correlation is eliminated by PCA analysis. PCs can be aggregated further to compute an overall performance index (MPCI). In the proposed model it is not required to assign individual response weights. Degree of influence of various process control factors can be investigated easily. Accuracy in prediction of the model analysis can be subsequently increased by assigning adequate fuzzy rules as well as by increasing number of membership functions in the fuzzy inference system. This approach can be recommended for continuous quality improvement and off-line quality control of a process/product in any manufacturing/ production environment.

The main highlights of this research are given below:

1. Development of an integrated methodological framework for correlated multi-response optimization. A case study has been chosen to optimize contradicting requirements of quality and productivity.

2. The methodology described above seeks to overcome limitations of existing common optimization approaches well documented in literature.
3. The aforesaid concept of multi-response optimization incorporates various aspects that are valid and influential in practical field but generally being assumed to impose negligible effect in existing optimization practices.

TABLE I: Domain of experiment

| Sl. No. | Factors | Notation | Unit | Level 1 | Level 2 | Level 3 |
|---|---|---|---|---|---|---|
| 1 | Cutting speed | N | [RPM] | 360 | 530 | 860 |
| 2 | Feed rate | f | [mm/rev] | 0.083 | 0.166 | 0.331 |
| 3 | Depth of cut | d | [mm] | 2 | 3 | 4 |

TABLE II: Design of experiment

| Sl. No. | Factorial combination (Coded form) | | |
|---|---|---|---|
| | N | f | d |
| 1 | 1 | 1 | 1 |
| 2 | 1 | 2 | 2 |
| 3 | 1 | 3 | 3 |
| 4 | 2 | 1 | 2 |
| 5 | 2 | 2 | 3 |
| 6 | 2 | 3 | 1 |
| 7 | 3 | 1 | 3 |
| 8 | 3 | 2 | 1 |
| 9 | 3 | 3 | 2 |

TABLE III: Experimental data

| Sl. No. | $R_q$ (μm) | $R_a$ (μm) | $R_t$ (μm) | MRR($mm^3$/min) | $R_{ku}$ | $R_z$ (μm) | $R_{sm}$ (mm) |
|---|---|---|---|---|---|---|---|
| 1 | 1.613333 | 1.35 | 8.433333 | 1436.839 | 2.406667 | 7.176667 | 0.044367 |
| 2 | 5.013333 | 4.19 | 39.8 | 3992.6746 | 9.826667 | 19.96667 | 0.098433 |
| 3 | 5.563333 | 4.76 | 23.53333 | 9909.7919 | 2.066667 | 21 | 0.161667 |
| 4 | 2.333333 | 1.786667 | 18.39667 | 4290.9832 | 17.9 | 10.96 | 0.054667 |
| 5 | 3.1 | 2.64 | 12.63333 | 7693.0652 | 2.166667 | 11.2 | 0.082433 |
| 6 | 5.336667 | 4.653333 | 20.06667 | 5298.241 | 1.803333 | 18.63333 | 0.16 |
| 7 | 1.066667 | 0.858 | 6.823333 | 6048.7008 | 3.143333 | 5.046667 | 0.052667 |
| 8 | 3.476667 | 2.976667 | 13.36667 | 4762.783 | 2.19 | 12.33333 | 0.081167 |
| 9 | 4.696667 | 4.243333 | 18.03333 | 18843.154 | 1.616667 | 15.9 | 0.160667 |

TABLE IV: Check for response correlation

| Correlation (between the responses) | $R_q$ | $R_a$ | $R_t$ | MRR | $R_{ku}$ | $R_z$ |
|---|---|---|---|---|---|---|
| $R_a$ | 0.998 | | | | | |
| $R_t$ | 0.848 | 0.821 | | | | |
| MRR | 0.461 | 0.475 | 0.254 | | | |
| $R_{ku}$ | -0.165 | -0.219 | 0.347 | -0.331 | | |
| $R_z$ | 0.986 | 0.976 | 0.912 | 0.402 | -0.030 | |
| $R_{sm}$ | 0.890 | 0.903 | 0.636 | 0.704 | -0.428 | 0.849 |

TABLE V: Individual principal components (PCs)

| Sl. No. | PC1 | PC2 | PC3 | PC4 | PC5 |
|---|---|---|---|---|---|
| 1 | 0.852283 | -0.1208 | 0.147181 | 0.004384 | 0.000828 |
| 2 | 4.236844 | -1.15717 | 0.641488 | 0.189084 | -0.08866 |
| 3 | 4.535799 | -1.79554 | -0.31347 | 0.268568 | -0.04346 |
| 4 | 2.135723 | -0.625 | 0.978295 | 0.060893 | -0.05386 |
| 5 | 2.666602 | -1.20492 | -0.1642 | 0.12605 | -0.01159 |
| 6 | 4.268112 | -1.53786 | -0.36042 | 0.259788 | -0.03239 |
| 7 | 0.231981 | -0.55 | 0.139251 | 0.028989 | -0.0035 |
| 8 | 2.844211 | -1.06057 | -0.13043 | 0.12451 | -0.01086 |
| 9 | 4.09653 | -1.9704 | -0.49477 | 0.257081 | -0.0292 |

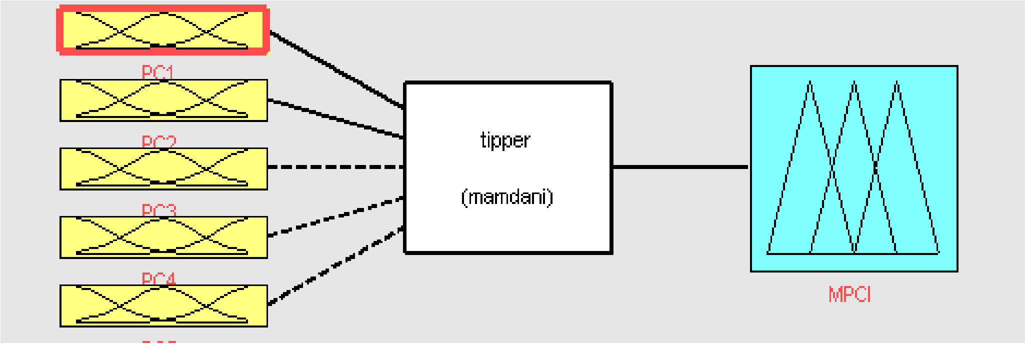


Fig.1: Proposed FIS architecture

TABLE VI: Individual principal components (normalized) and computed MPCI

| Sl. No. | Nor C1 | Nor PC2 | Nor PC3 | Nor PC4 | Nor PC5 | MPCI | S/N Ratio | MEAN |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.144128307 | 1 | 0.43579272 | 0 | 1 | 0.126 | -17.9926 | 0.126 |
| 2 | 0.930537258 | 0.439678849 | 0.77135632 | 0.69913394 | 0 | 0.5 | -6.0206 | 0.500 |
| 3 | 1 | 0.09453936 | 0.12307671 | 1 | 0.505095655 | 0.5 | -6.0206 | 0.500 |
| 4 | 0.442337943 | 0.727400519 | 1 | 0.21390016 | 0.388878956 | 0.334 | -9.5251 | 0.334 |
| 5 | 0.565688651 | 0.413862457 | 0.22440965 | 0.46053508 | 0.861232791 | 0.593 | -4.5389 | 0.593 |
| 6 | 0.937802435 | 0.233855969 | 0.09120439 | 0.96676559 | 0.628799392 | 0.484 | -6.3031 | 0.484 |
| 7 | 0 | 0.767949827 | 0.43040938 | 0.09313584 | 0.951635974 | 0.281 | -11.0259 | 0.281 |
| 8 | 0.606956428 | 0.491906358 | 0.24733464 | 0.45470581 | 0.869390309 | 0.636 | -3.9309 | 0.636 |
| 9 | 0.897935043 | 0 | 0 | 0.95651894 | 0.66444663 | 0.414 | -7.6600 | 0.414 |

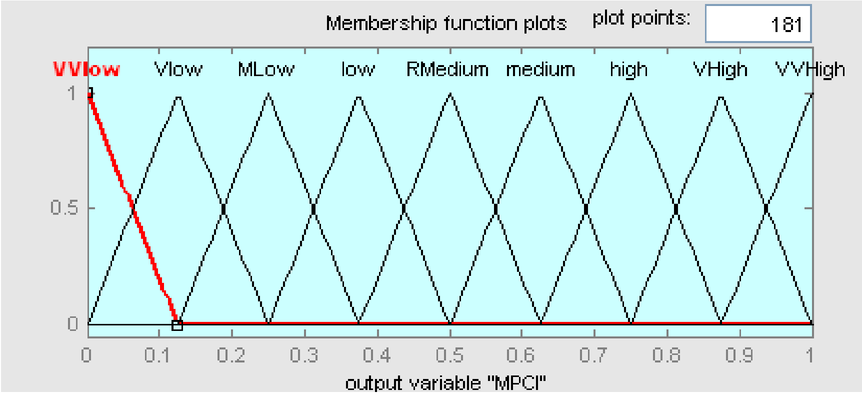


Fig. 2: MFs for MPCI

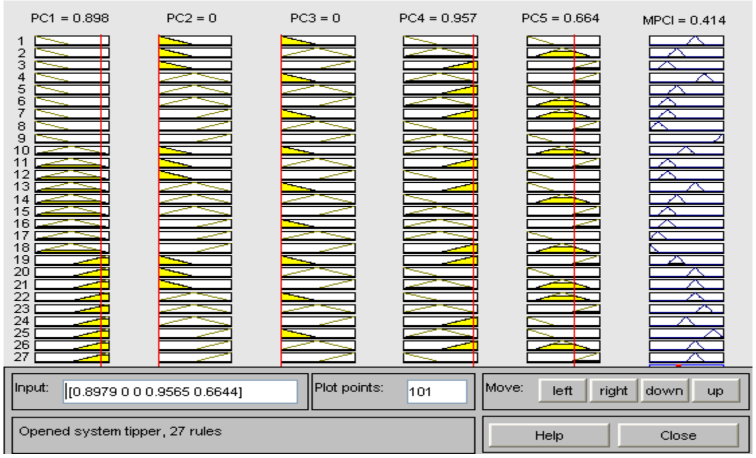


Fig. 3: Fuzzy rule base

TABLE VII: Fuzzy rule matrix

| Sl. No. | IF Nor PC1 | AND Nor PC2 | AND Nor PC3 | AND Nor PC4 | AND Nor PC5 | THEN MPCI |
|---|---|---|---|---|---|---|
| 1 | low | low | low | low | Low | Medium |
| 2 | low | low | Medium | Medium | Medium | Low |
| 3 | low | low | High | High | High | Moderately low |
| 4 | low | Medium | low | Medium | High | High |
| 5 | low | Medium | Medium | High | Low | Low |
| 6 | low | Medium | High | Medium | Medium | Moderately low |
| 7 | low | High | low | High | Medium | Moderately low |
| 8 | low | High | Medium | Low | High | Very low |
| 9 | low | High | High | Medium | Low | Very very high |
| 10 | Medium | low | low | Medium | Medium | Relatively medium |
| 11 | Medium | low | Medium | High | High | Moderately low |
| 12 | Medium | low | High | Low | Low | Moderately low |
| 13 | Medium | Medium | low | High | Low | Medium |
| 14 | Medium | Medium | Medium | Low | Medium | Low |
| 15 | Medium | Medium | High | Medium | High | Moderately low |
| 16 | Medium | High | low | Medium | High | Moderately low |
| 17 | Medium | High | Medium | Medium | Low | Very low |
| 18 | Medium | High | High | High | Medium | Very very low |
| 19 | High | low | low | High | High | Low |
| 20 | High | low | Medium | Low | Low | Medium |
| 21 | High | low | High | Medium | Medium | Medium |
| 22 | High | Medium | low | Low | Medium | Medium |
| 23 | High | Medium | Medium | Medium | High | High |
| 24 | High | Medium | High | High | Low | Relatively medium |
| 25 | High | High | low | Medium | Low | Very high |
| 26 | High | High | Medium | High | Medium | Medium |
| 27 | High | High | High | Low | High | Medium |

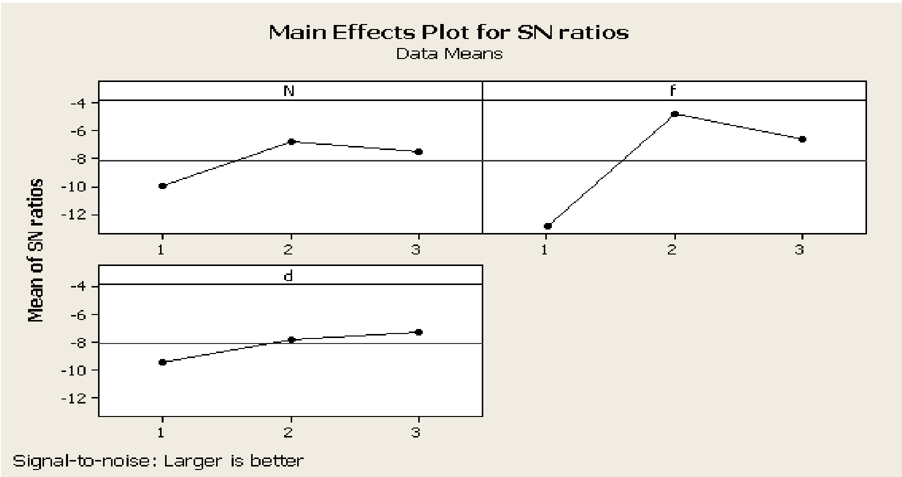

Fig. 4: S/N ratio plot of MPCI (Evaluation of optimal setting)

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