FREE VIBRATION ANALYSIS OF CANTILEVER BEAMS WITH TRANSVERSE OPEN CRACKS

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Abstract—For many engineering applications, beams are essential models for the structural elements and have been studied extensively. The structures planned to support high speed engines and turbines are subjected to vibration. Mild steel specimens of square area of cross section are considered for the experiment and the experimental results are contrasted with numerical analysis using Finite Element Method (FEM) in MATLAB environment. The crack considered is transverse crack which open in nature. Due to the presence of crack, the total flexibility matrix is established by adding local additional flexibility matrix to the flexibility matrix of the corresponding intact beam element .The local additional flexibility matrix is obtained from Linear Elastic Fracture Mechanics theory. Numerical study to validate the accuracy of the present (FEM) analysis is done. Experimental work is carried out to study the affect of presence of crack in uniform beams of square area of cross section for cantilever boundary condition.

Index Terms—Beam, Natural frequency, Vibration

I. Introduction

Plenty of literature is available on vibration of beams without cracks. Limited analytical and numerical studies exists on vibration of beam with cracks. Shen and Pierre (1986) presented a finite element approach to predict the changes in the first few Eigen frequencies, Eigen modes due to presence of crack. Eight nodes Isoperimetric element is used to model across the thickness of the beam. Shifrin and Rutolo (1999) proposed a new technique for enumerating natural frequencies of a beam with a random number of transverse open cracks. Cracks are characterized as massless rotational springs. Compared to the substitute methods which make use of continuous model of beam, the computation time required here was reduced due to the decreased dimension of the matrix. The experimental investigations of the effects of cracks on the first two modes of vibrating beams for both hinged-hinged and fixed–free boundary conditions is elaborated by Owolabi (2003). The Frequency Response Function (FRF) amplitudes and changes in natural frequencies obtained from the measurements of dynamic responses of cracked beams as a function of crack depth and location of crack are used for the detection of crack. Zheng *et al*. (2004) obtained the natural frequencies and mode shapes of cracked beam using Finite Element Method(FEM). The total flexibility matrix is established by adding overall additional flexibility matrix to the flexibility matrix of the corresponding intact beam element .The results when compared with analytical results show more accuracy than when the local additional flexibility matrix was used in the place of overall additional stiffness matrix. Khiem *et al.* (2004) obtained numerical results for a cantilever beam with crack. Main focus is laid on the detection of multi-crack for structures by natural frequencies. Accuracy in detecting the crack depth is more if more natural frequencies are measured. Chen *et al.*(2005) performed experimental investigation for spotting the location and size of crack. The intersection of curves of stiffness versus location of crack for the first three natural frequencies obtained from the vibration of the cantilever beam with crack predicts the crack location and crack depth .Patil *et al.* (2005) verified a method to envisage the location and depth of crack experimentally for cantilever beams with edge crack. The energy approach method is used for analysis and the crack is represented as a rotational spring. For a particular mode, varying crack location, a plot of stiffness versus crack location is obtained. The intersection of these curves consequent to the three modes gives the crack location and the associated rotational spring stiffness. Lee (2009) presented a simple method to recognize multiple cracks in a beam using the finite element method. The method for identifying double and triple cracks is illustrated by numerical examples.

The present study deals with both numerical and experimental investigation on vibration of practically important stepped beams. Mild steel specimens of square area of cross section are considered for the experiment and the experimental results are contrasted with numerical analysis using Finite Element Method (FEM) in MATLAB environment. The crack considered is open transverse crack in nature. Due to the presence of crack, the total flexibility matrix is established by adding local additional flexibility matrix to the flexibility matrix of the corresponding intact beam element in line with Zheng *et al.* (2004) .The local additional flexibility matrix is obtained from Linear Elastic Fracture Mechanics theory.

The convergence study is done for the cantilever uniform beam of square cross-section with single crack with the case considered in Lee et al (2000). A 300mm cracked cantilever beam of cross section (20 x 20) mm with young's modulus, E= 206GPa and mass density, ϱ =7750 kg/m^3. It is observed that convergence starts when the number of elements is 14 and convergence up to 30 numbers of elements. As per the convergence study, 20 elements are considered for the discretization of whole structure.

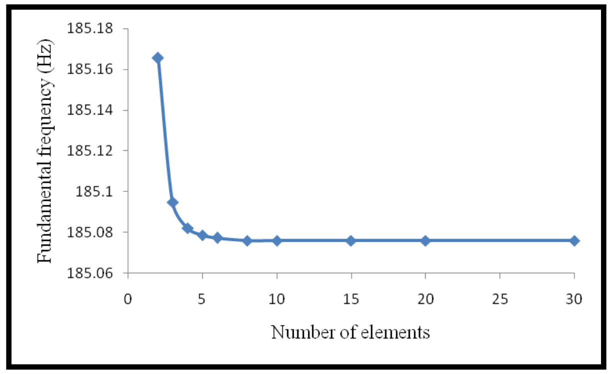


Figure 1: Convergence of fundamental frequency of uniform cantilever beam with single crack.

The present FEM formulation is validated with literature. The variation of natural frequency with respect to the uniform cantilever beam with single crack is studied and compared with **Shiffrin (1999)** as shown in the **Table 1.**

The material properties of the beam are, Elastic modulus of the beam, E = 210MPa**,**

Poisson's Ratio, υ = 0.3**,** Density, ϱ = 7800 kg/m3**,** Beam Width, b = 0.02m**,** Beam depth, h = 0.02 m**,** Beam length, L = 0.8m, Position of the crack from clamped end x_1= 0.12m, Crack depth a_1=0.002 m.

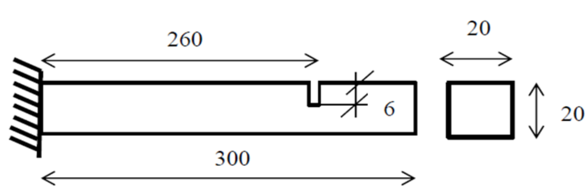
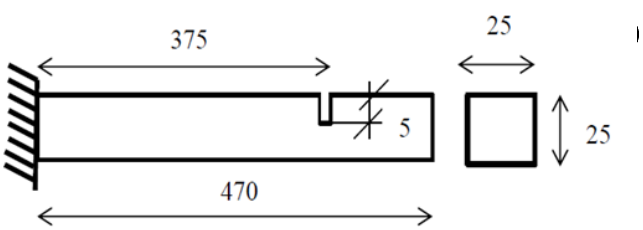


Figure 2: Cracked cantilever beam (mm)

Table 1: Comparison of natural frequency drawn between **Shiffrin (1999)** and present **FEM** analysis.

MODE	Natural Frequency (Hz) Shiffrin (1999)	Present analysis FEM (Hz)
MODE1	26.123	26.168
MODE2	164.092	164.109
MODE3	459.607	459.558

Free vibration analysis of the uniform beam shown in Fig 5.5 is carried out for fixed-free boundary condition. The variation of non-dimensional first natural frequency with relative location of the crack (*x/L*) for different crack depths of the Fixed-Free beam is plotted in Fig



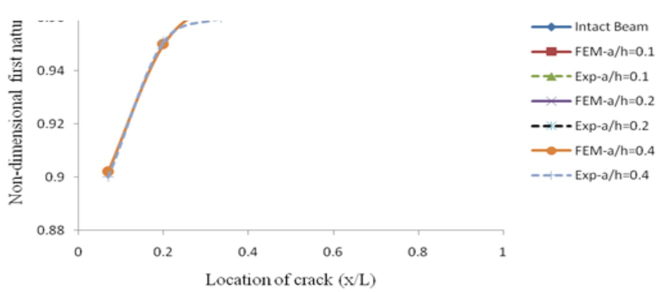


Figure 4: Comparison of FEM and experimental results for non-dimensional first natural frequencies of single cracked cantilever beam with location of crack (x/L) for varying crack depth ratios.

For all the different locations of crack considered, the fundamental frequency is more affected when crack is located at x=0.0325L , the first mode of non-dimensional frequency decreases by 0.15%, 0.92%, 9.70%compared to intact beam for the crack depth ratios 0.1,0.2,0.4 respectively. It is observed that as the crack position moves away from the fixed end, the non-dimensional first natural frequency increases and at the free end it is almost similar to intact beam. The effect of crack is effective when it is near to fixed end which could be made cleared by the fact that bending moment is maximum at the fixed end, thus, resulting in considerable loss of stiffness

It is observed that the presence of crack has significant effect on the second mode non-dimensional frequency for all the cases of crack positions except for the crack location at (x/L= 0.20). When the crack is located at x/L= 0.20, the non-dimensional frequency of second mode is barely affected, the cause for this zero influence was that the nodal point for the second mode was located here. The crack located at x/L=0.0325 brings about 0.164%, 1.01%, 12.22% decrease in second mode of non-dimensional frequency compared to intact beam for the crack depth ratios 0.1,0.2,0.4 respectively. It is also noticed that for the crack locations x/L= 0.34 to 0.85, the second mode non-dimensional frequency is diminished from maximum.

Numerical study to validate the accuracy of the present (FEM) analysis is done. Experimental work is carried out to study the effects of crack in uniform and stepped beams of square area of cross section for cantilever and free-free boundary condition.

The results obtained from experimental are checked for accuracy with the present analysis by plotting non-dimensional frequencies for first three modes as function of crack depth ratios for different locations of cracks.

Conclusion: Experimental results showing the effects of various parameters on the natural frequencies of vibration of cracked beams are presented. The experimental results are compared with Finite element based numerical results.

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