

An Improved Approach for Flow Prediction in Compound Open Channel Flow of Uniform Roughness

Pradhan Siprarani ¹Khatua Kishanjit K. ²

¹Ph.D. Research Scholar Department of Civil Engineering, N.I.T. Rourkela, India

²Associate Professor, Department of Civil Engineering, N.I.T. Rourkela, India.

Email: pradhan.siprarani@gmail.com, 512ce1006@nitrkl.ac.in.

ABSTRACT

Most natural rivers consist of a main channel and the adjacent floodplains. The various subdivision methods fail to determine the discharge capacity in rivers with overbank flow because of the ignorance of the strong interaction between the main channel and the shallow floodplains. An experimental investigation is performed to study the effect of momentum transfer in terms of an apparent shear stress in rough and smooth compound channels for different hydraulic, geometric conditions and roughness ratios. A new method to calculate flow in compound channels is proposed. The approach is based on a new model on the momentum transfer coefficient at the vertical interface and the horizontal interface between adjacent flow compartments, typically between the main channel and floodplain of a two-stage channel. The new approach is found to give better results as compared to the other approaches when applied to the present experimental data sets, large channel of FCF data sets and some natural river data sets.

Keywords: Compound Channels, Momentum Transfer coefficient, Stage-Discharge relationship, Apparent shear stress

1. INTRODUCTION

A major area of uncertainty in river channel analysis is that of accurately predicting the capability of river channels with floodplains, which are termed compound channels. Cross-sections of these compound channels are generally characterized by a deep main channel, bounded on one or both sides by a relatively shallow floodplain, which is rougher and has slower velocities than as compared to that of main channel. Due to interaction between the main channel and floodplains, there are bank of vertical vortices found in many experiments by Myers (1990), Knight and Shiono (1991), along the vertical interface between the main channel and the flood plain, which lead to extra resistance in terms of consumption a lot of energy. Due to this extra resistance, the prediction of stage-discharge curve become difficult and more difficult when there is large different roughness between the main channel and floodplain. There, flow structure and flow resistance become very complex and flow shows very strongly three-dimensional characteristics, see Knight (1999), Rameshwaran and Naden (2003). In predicting the flood-water level, conventional methods for estimating the discharge capacity of a compound channel have been based on standard uniform flow formulas, such as Chezy, Manning, and Darcy-Weisbach equations, by either treating the cross section as a single channel or by dividing it horizontally, vertically, or diagonally into non interacting subareas. For divided channel methods, the total discharge is then obtained by adding the individual discharges in each subarea (Chow 1959; Shiono et al. 1999). However, if the compound section is treated as a single channel, the discharge capacity is underestimated; if the section is vertically divided into subsections of simple shape, the value obtained is a serious overestimation of capacity because the method fails to take account of the extra flow resistance resulting from momentum transfer (Myers and Brennan 1990). Knight and Hamed (1984) compared the accuracy of several subdivision methods for compound straight channels by including and excluding the vertical division line in determining the wetted perimeters of the main channel and the floodplains. However, their results show that the conventional calculation methods result in larger error. Apart from the case of the vertical

division, Wormleaton et al. (1982) and Wormleaton and Hadjipanos (1985) also discussed the horizontal division through the junction point between the main channel and the floodplains. However, their studies also show that these subdivision methods cannot well assess the discharge in compound channels. For this type of channels, how to predict its stage-discharge curve is still fraught with difficulty.

This paper mainly focuses on the modeling of momentum transfer coefficient for three-stage trapezoidal compound cross-section. The cross section is divided into vertically the main channel and floodplain, and floodplain. By separately calculating the apparent shear stress caused by lateral momentum exchange, the average velocity of each sub-section is derived via the balance of force. Using the subsection average velocity, the subsection discharge is estimated to develop the stage discharge relationship in compound channels. The UK Flood Channel Facility (UK-FCF) measured data have been used to verify the model.

2. THEORETICAL ANALYSIS

2.1 Force balance analysis in compound channel sub sections

Following the work of Yang et al. (2014) and Huthoff et al. (2008), a step has been taken to present a simplified and modified approach for flow prediction in compound open channel flow. The analysis considered compound channel geometry as depicted in Fig. 1, consisting of a main channel with either two identical floodplains. When the water in the main channel over flows and inundates its floodplains, the momentum transfer takes place which is expressed in the form of apparent shear stresses acting on the horizontal interfaces between the upper and lower main channels considered as horizontal apparent shear stress and on the vertical interface between the main channel and the floodplains as vertical apparent shear stresses. Hence, in this present work, in dealing flow interaction phenomenon, the compound channel is considered to be divided into four sub sections, as shown in Fig.1.

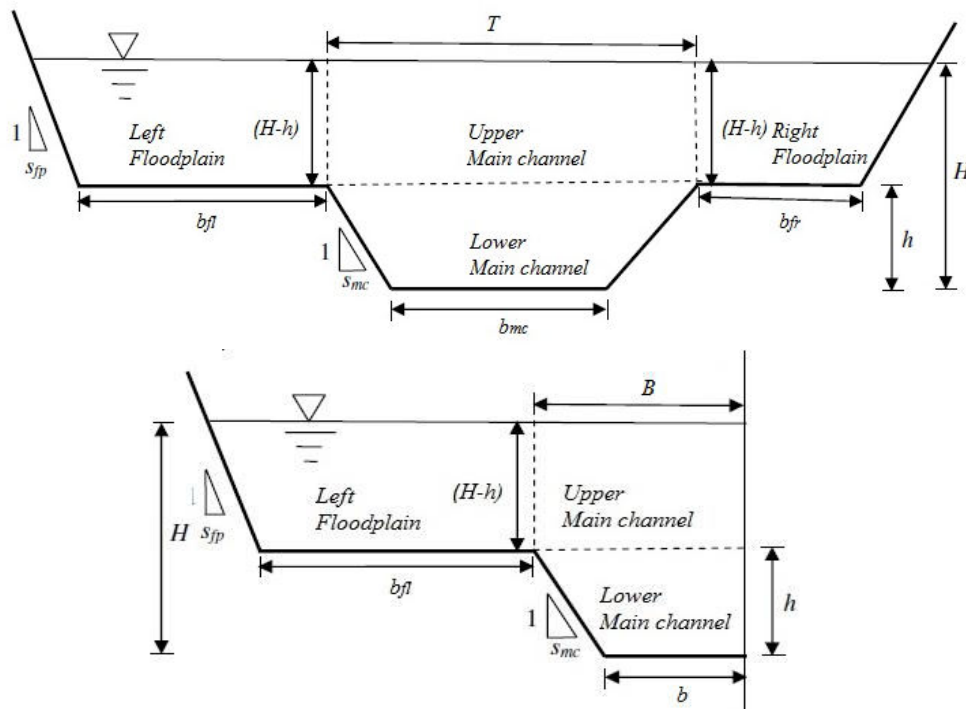


Figure.1 Cross section of compound channel (a) Symmetric with two identical floodplain (b_{fl} & b_{fr}) (b) Asymmetric with one floodplain (b_{fl})

The conceptual horizontal and vertical interfaces among the lower main channel, upper main channel and floodplain are assumed and denoted by broken lines. The present compound channel with subsections are

analyzed based on the concept that for uniform flows, the force balance of water for a control volume with a unit length is the weight component equal to the sum of the total bed shear and the total apparent shear:

$$W_{fl}\sin\theta = P_{fl}\tau_{fl} - (H - h)\tau_{\alpha flmu} \quad (1)$$

$$W_{mu}\sin\theta = (H - h)\tau_{\alpha flmu} + (H - h)\tau_{\alpha frm} + T\tau_{\alpha mlmu} \quad (2)$$

$$W_{fr}\sin\theta = P_{fr}\tau_{fr} - (H - h)\tau_{\alpha frm} \quad (3)$$

$$W_{ml}\sin\theta = P_{ml}\tau_{ml} - T\tau_{\alpha mlmu} \quad (4)$$

where W is the weight per unit length of fluid and equals to $A\rho g$, A is the area of subsection, P is the wetted perimeter, ρ is the water density; g is the gravitational acceleration; $\sin\theta$ denoted as S_0 is the bed slope; T is the width of the horizontal interface between the upper and lower main channels; τ_{α} is the apparent shear stress, $(H-h)$ is flow depths on vertical interfaces between the left and right floodplains and the upper main channel, the subscripts f_l, f_r, m_l and m_u refer to the left flood plain, the right floodplain, lower and upper the main channel respectively. τ can be calculated with the Chezy equation for uniform flows.

2.2 Determination of the Sub sectional Solid Boundary Shear Stress

For uniform flows, the conventional flow resistance equations, such as the Chezy equation, are based on an assumption that the solid boundary shear stress is proportional to the square of the flow velocity which can be simplified by Manning's formula that the Chezy's constant (C) is equal to $C_i = \frac{1}{n_i} R_i^{1/6}$; where, n_i and R_i = Manning coefficient and hydraulic radius in Subsections i , respectively.

Then, the average shear stresses on each boundary are

$$\tau_{fl} = \frac{\rho g v_{fl}^2}{\left(\frac{1}{n_{fl}} R_{fl}^{1/6}\right)^2} \tau_{mc} = \frac{\rho g v_{mc}^2}{\left(\frac{1}{n_{mc}} R_{mc}^{1/6}\right)^2} \tau_{fr} = \frac{\rho g v_{fr}^2}{\left(\frac{1}{n_{fr}} R_{fr}^{1/6}\right)^2} \quad (5)$$

where n_{fl} , n_{mc} , and n_{fr} are Manning's coefficients

2.3 Determination of Apparent Shear Stress

The apparent shear stresses on the vertical and horizontal interfaces are derived from the governing equations (1),(2),(3) &(4) as

$$\tau_{\alpha flmu} = \frac{P_{fl}\tau_{fl} - \rho g A_{fl} S_0}{(H-h)} \quad (6)$$

$$\tau_{\alpha frm} = \frac{P_{fr}\tau_{fr} - \rho g A_{fr} S_0}{(H-h)} \quad (7)$$

$$\tau_{\alpha mlmu} = \frac{P_{ml}\tau_{ml} - \rho g A_{ml} S_0}{(H-h)} \quad (8)$$

Where $\tau_{\alpha flmu}$, $\tau_{\alpha frm}$ and $\tau_{\alpha mlmu}$ are known as apparent shear stress in left vertical interface, right vertical interface and horizontal interface respectively.

2.4 Expression for Horizontal and Vertical Momentum Transfer Coefficients

Many investigators such as Knight and Demetriou (1983), and Wormleaton et al. (1982), conclude from a number of experiments that generally in compound channels, due to faster moving water in the upper main channel and the slower moving water on the floodplain, the lateral momentum transfer takes place in the vertical interfaces meanwhile the momentum transfer also occurs in the horizontal interface that appears generally between the faster moving water in the upper main channel and the slower moving water in the lower main channel. As a result, the vertical apparent shear exists on the interface between the upper main channel and the floodplain, which generally accelerates the flow on the floodplain and resists the flow in the upper main channel. The horizontal apparent stress occurs on the interface between the upper and lower main channels, which generally accelerates the flow in the lower one and resists the flow in the upper one. According to Prandtl's momentum transfer theory and the similarity of velocity distribution, τ_{af1} and ε can be described as follows:

$$\tau_{af1} = \rho \varepsilon_{f1} \frac{\partial u}{\partial z} \quad (9)$$

Where, ε_{f1} = turbulent eddy viscosity coefficient; and z = vertical direction. Because the eddy viscosity is related to appropriate velocity and length scales, it is assumed that it may be expressed as

$$\varepsilon_{f1} = \alpha_0 u h \quad (10)$$

where α_0 = certain dimensionless coefficient, and h = characteristic height in the vertical shear region between the left floodplain and upper main channels.

It is assumed that

$$h \propto h_z, \quad u \propto \frac{v_{mu} + v_{fl}}{2} \frac{\partial u}{\partial z} \propto \frac{v_{mu} - v_{fl}}{h_z} \quad (11)$$

h_z = height of the vertical shear layer between the left floodplain and upper main channel. The apparent shear stress is derived from (9) and presented as:

$$\tau_{af1} = \rho \chi_{f1} \frac{v_{mu} - v_{fl}}{2} \quad (12.a)$$

and

$$\tau_{afr} = \rho \chi_{fr} \frac{v_{mu} - v_{fr}}{2} \quad (12.b)$$

in which χ_{f1} and χ_{fr} are defined as the dimensionless momentum transfer coefficient for the corresponding vertical interfaces, respectively.

Similarly, another apparent shear stress in horizontal interface is given by

$$\tau_{aml} = \rho \chi_{ml} \frac{v_{mu} - v_{ml}}{2} \quad (13)$$

in which χ_{ml} is defined as the dimensionless momentum transfer coefficient for the horizontal interface.

3. MODELLING OF MOMENTUM TRANSFER COEFFICIENTS

Figure 1(a&b) shows sketches of asymmetrical/ symmetrical compound sections, where H , h and $(H-h)$ are the main channel, bankfull and floodplain flow depths, b_{mc} , T and bf are the main channel bottom, main channel top and floodplain widths and S_{mc} the main channel bank slope. The best large-scale measurements for overbank flow in straight two-stage channels are considered those of FCF Phase- A of five series data (www.flowdata.bham.ac.uk). In Table 1, the geometrical characteristics of the FCF experiments are provided. All data series in Table 1 were obtained under steady uniform flow conditions.

Figure 1(b) shows the similar dimensions of the symmetrical compound sections for the wider channel series.

Table-1Details of FCF Series

Series No.	Manning's Roughness		B(m)	h(m)	b(m)	S _o	(B/b)	SS
	Main channel(mc)	Flood plain(fp)						
FCF Series 01	0.0099	0.0099	10.0	0.15	0.75	0.001027	6.67	1:1
FCF Series 02	0.0099	0.0099	6.3	0.15	0.75	0.001027	4.2	1:1
FCF Series 03	0.0099	0.0099	3.3	0.15	0.75	0.001027	2.2	1:1
FCF Series 08	0.0099	0.0099	6.6	0.15	0.75	0.001027	4.0	1:1
FCF Series 10	0.0099	0.0099	3.3	0.15	0.75	0.001027	4.4	1:1

*B is the total width of channel, h is the main channel depth, b is the half width of main channel, B /b is the width ratio, S_o is the bed slope of the channel and SS is the side slope of main channel

Relationship between momentum transfer coefficient and Relative flow depth

From the above discussed equations the momentum transfer coefficient for vertical and horizontal (χ_v and χ_h) interface simplified as

$$\chi_v = \frac{2(P_{fl}\tau_{fl} - \rho g A_{fl} S_o)}{\rho(v_{mu} - v_{fl})(H-h)} \text{ or } \chi_v = \frac{2(P_{fl}\tau_{fl} - \rho g A_{fl} S_o)}{\rho(v_{mu} - v_{fl})(H-h)} \quad (14.a)$$

$$\chi_h = \frac{2(P_{ml}\tau_{ml} - \rho g A_{ml} S_o)}{\rho(v_{mu} - v_{ml})(H-h)} \quad (14.b)$$

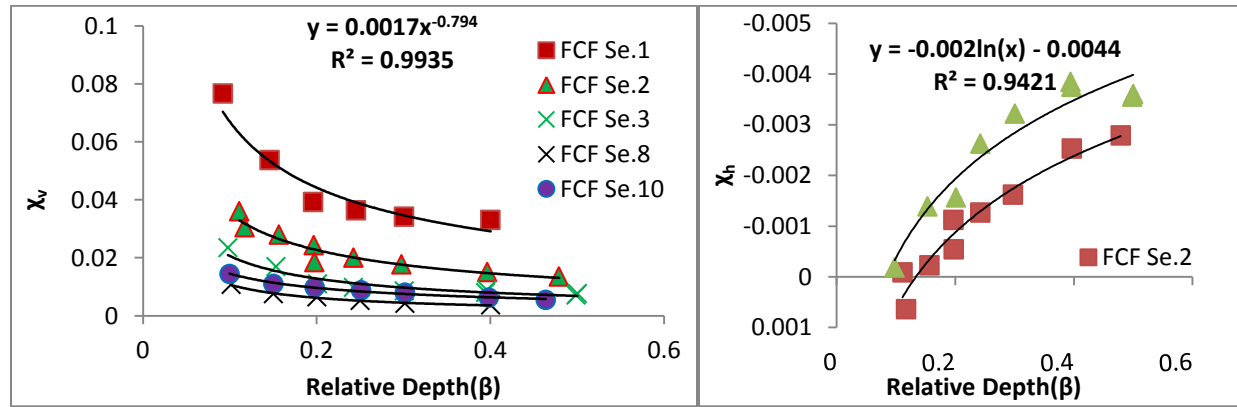


Figure. 2(a&b) variation of vertical and horizontal momentum transfer coefficient with different width ratio channel of SERC-FCF.

In the above Fig. 2(a), it is shown that the varying trends of the vertical momentum transfer coefficient (χ_v) with relative depth (β) for FCF Series 01-03, 08 and 10 are similar. For all the series, the momentum transfer coefficient generally decreases with increasing relative depth even if in case of horizontal momentum transfer coefficient (χ_h) shown in Fig.2(b).Hence, it is supposed that the momentum transfer

coefficient to the relative depth, follows certain relationship. Figure 2 testifies that this is true for the SERC-FCF data which leads to the expressions given as

$$\chi_v = 0.0017\beta^{0.794} \tag{15(a)}$$

$$\chi_h = -0.002 \ln(\beta) - 0.0044 \tag{15(b)}$$

The above two expressions are modeled to finding out the vertical and horizontal momentum transfer coefficient for Large scale compound channels.

4. RESULT AND DISSCUSSION

Stage Discharge Relationship

For the experiments of FCF, Series 01, 02, and 03 with the same main channel width of 1.5 m and different floodplain widths of 4.1, 2.25, and 0.75 m, when derived expressions in equation 15(a&b) for χ_v and χ_h are applied to in turn, the method gives good predictions of stage discharge curve as compare to other methods, as shown in Fig. 3(a). For Series 08 and 10, although they have different main channel side slope, they have the same floodplain and main channel widths as Series 02. Hence, when they are taken as the same momentum transfer coefficient as Series 01,02 and 03, they also obtain very good predictions of the discharge, as shown in Fig.3 (b).

To obtain much more information on the effect of momentum transfer coefficients, χ , on the discharge, Q , numerical tests were further undertaken based on different series of FCF are considered, as shown in Fig.3 (a&b).

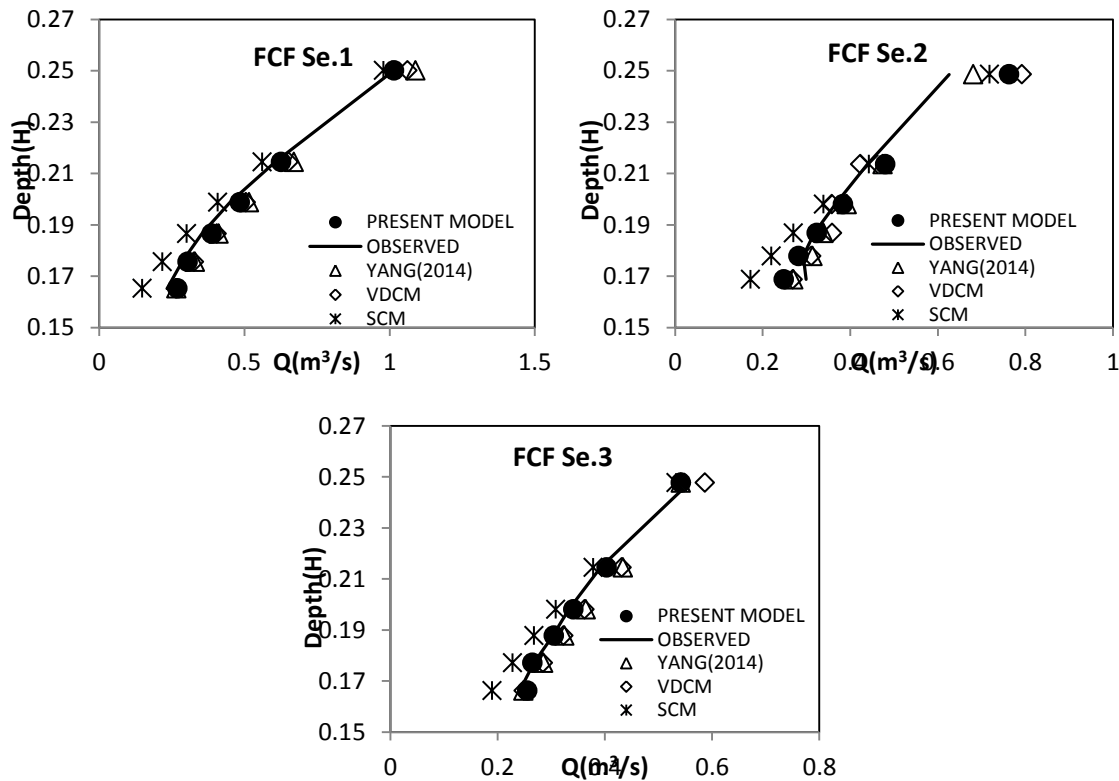


Figure.3 (a) Stage discharge curve for FCF Series 01, 02 and 03

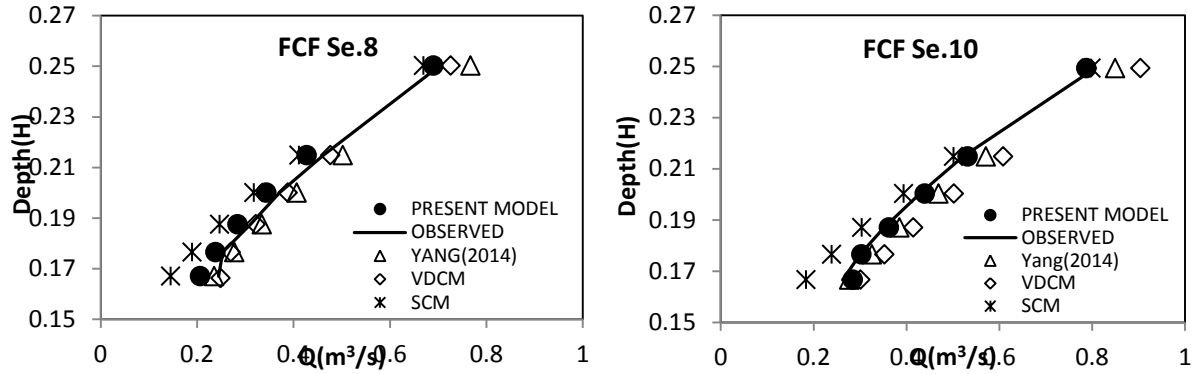


Figure.3 (b) Stage discharge curve for FCF Series 08 and 10

Table-2 Errors in computation of discharge by different methods

	Present Model	<i>Mean Relative Error (%)</i>		
		Yang (2014)	VDCM	SCM
FCF Series 01	6.733646	13.1283	10.79343	15.87662
FCF Series 02	5.497665	6.545004	8.851232	15.75275
FCF Series 03	2.658443	6.639609	7.366103	10.54624
FCF Series 08	8.659591	15.82955	11.22988	12.90225
FCF Series 10	3.256716	8.004849	9.113598	12.81958

The computed discharge is well matched with the observed discharge by the present model shown in Fig. 3. However, the effect of momentum transfer is distinct for the prediction of discharge. From Table-2, the mean relative error is found lower in present method for the entire proposed Series of Large FCF channel. The minimum mean relative error for the present method is only 2.65% for (B/b)=2.2 of Series 03 whereas maximum is found 6.73% for (B/b)=6.67 of Series 01. The present approach has the smallest errors (see Table 2).

5.CONCLUSION

- The presented method is developed by using the momentum transfer coefficients which take account of the effect of the lateral momentum transfer between the floodplains and the upper main channel and of the vertical momentum transfer between the upper and lower main channels.
- The expressions are derived for vertical and horizontal momentum transfer coefficients by analyzing the experimental data of the SERC-FCF. For all the series considered, the momentum transfer coefficient generally decreases with increasing relative depth with a definite trend.
- Using the developed mathematical modelling for the momentum transfer coefficients (χ_v & χ_h) the presented method has the advantage of being applicable to well deal with the apparent shear forces on the interfaces between the adjoining subareas by comparing with the other conventional divided channel methods. The discharge distribution and stage discharge relationship predictions

of the method are verified against the large-scale compound channel data from the FCF at HR Wallingford. The results by the presented method give good agreement with experimental data.

- From the error analysis of the conventional methods, the accuracy of the present approach has been successfully examined to the experimental large channel FCF data sets with a minimum relative error of 2.65%.
- The expression for evaluating the momentum transfer coefficient should be validated in respect of accuracy by a vast number of experimental data and as well as natural river data. Again the problem is not resolved and to be studied further for an asymmetric compound channels.

REFERENCES:

1. Chow, V.T.(1959). Open Channel Hydraulics.1st edn. McGraw-Hill Book Co., New York.
2. Huthoff, F., Roos, P. C., Augustijn, D. C. M., and Hulscher, S. J. M. H. (2008).Interacting divided channel method for compound channel flow. *J. Hydraul. Eng.*, 10.1061/ (ASCE) 0733-9429(2008)134:8 (1158), 1158–1165.
3. Knight D. W. (1999). Flow mechanics and sediment transport in compound channels [J]. *International Journal of Sediment Research*, 14(2): 217-236.
4. Knight, D.W., and Demetriou, J. D. (1983). Flood plain and main channel flow interaction. *J. Hydraul. Eng.*, 10.1061/(ASCE)0733-9429(1983)109:8(1073), 1073–1092.
5. Knight, D.W., Hamed, M.E. (1984). Boundary shears in symmetrical compound channels. *Journal of Hydraulic Eng.*, ASCE, 110(10), 1412–1430.
6. Myers, W.R.C. and Brennan, E.K.(1990). Flow Resistance in Compound Channel. *Journal of Hydraulic Research* 28(2): 141–155.
7. Myers, W.R.C., Brennan, E.K. (1990). Flow resistance in compound channel. *J. Hydraulic Res.*, IAHR, 28(2), 141–155.
8. Rameshwaran P. and Naden P. S. (2003). Three-dimensional numerical simulation of compound channel flows [J]. *Journal of Hydraulic Engineering, ASCE*, 129(8): 645-652.
9. Shiono, K. and Knight, D.W.(1991) .Turbulent open-channel flows with variable depth across the channel. *Journal of Fluid Mechanics* 222, 617–646.
10. Wormleaton, P. R., Allen, J., and Hadjipanous, P. (1982). “Discharge assessment in compound channel flow.” *J. Hydraul. Div.*, 108(9), 975–994.
11. Wormleaton, P. R., and Hadjipanous, P. (1985). Flow distribution in compound channels. *J. Hydraul. Eng.*, 10.1061/ (ASCE) 0733-9429(1985) 111:2(357), 357–361.
12. Yang, K.Y., Cao, S.Y., Liu, X. N. (2014). Stage-Discharge Prediction in Compound Channels. *J. Hydraul. Eng.*10.1061/(ASCE)HY.1943-7900.0000834.21(4): 353–361.



