CORPORATE BOND YIELD APPROXIMATION USING PIECE WISE CUBIC SPLINE AND RESIDUALS ESTIMATION WITH LIQUIDITY FACTORS

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ABSTRACT

Corporate bond yields are not easy to approximate on account of their limited trading frequency and irregularity in issuance. The corporate bond market in India demonstrates poor investor participation. The purpose of this paper is to present an account of piece wise cubic spline methodology of yield estimation that can be feasibly implemented by practitioners. This paper attempts to approximate the corporate bond yield using residual maturity for listed and traded bonds using traded data from the period January 2001 to October 2014. We use a two stage process, where in the first stage, we model the yield behavior of corporate bonds against their residual maturity nodes of 3 years, 5 years and 20 years, respectively. In the second stage, we identify the significant factors of bond liquidity that impacts the cubic spline residual to improve accuracy in estimation. The first step of piece wise cubic spline estimates the 3 node parameters to predict the logarithmic yield to maturities against their remaining maturity in years. The second step intends to analyze the prediction errors and provides for correction in accuracy incorporating factors of bond liquidity such as issuer, issue and liquidity factors, respectively. The dual approach explains the variation in yields and could be used for identifying quality papers and the reasons for lower investor participation in bond markets and for arriving at term structure in related markets. This does not include pure central govt. loans, state loans, treasury bills, dated securities, bond of departmental undertaking or india development bonds.

Key Word: *Yield, Cubic Spline, Liquidity, Residual*

1. INTRODUCTION

Bond markets provide the benefits of diversifying credit risks as an alternative to conventional bank lending. Bond markets help supply long-term funds for the growth of the infrastructure or other sectors to fulfill long-term investment needs. Bonds provide diversity in financial products with the flexibility to meet the specific needs of investors and borrowers. The corporate debt market in India has historically demonstrated poor investor participation unlike the Govt. Securities market. The bond market comprises a smaller fraction of the total turnover where the major share of trading belongs to government securities. The seven major groups of issuer include large private sector firms, public sector entities, financial institutions, private banks, public sector banks and foreign banks. The types of instruments include partly convertible debentures (PCDs), fully convertible debentures (FCDs), deep discount bonds (DDBs), zero coupon bonds (ZCBs), bonds with warrants, floating rate notes and secured premium notes (SPNs), CPs (Commercial Papers), CDs (Certificates of Deposits). In the wholesale debt market (WDM) segment of NSE, the share of corporate bonds is modest at 2.5% of all securities. This paper provides piecewise cubic spline interpolation of bond yields and its application to the term structure during January 2001 to October 2014. The term structure of interest rates forms the basis for the valuation of all fixed income instruments. The yields are the return on the current price of purchase of a bond over the present discounted value of the stream of cash flows due in future. The author intends to provide a practical and accessible approach to cubic spline interpolation for implementation by practitioners. This technique enables users to fit a yield curve to observed rates (Libor or bond yields) reasonably accurately and can produce a satisfactory zero coupon curve under most circumstances. Using a two step process, we model the yield behavior of corporate bonds against their residual maturity nodes of 3 years, 5 years and 20 years, respectively. The first step includes cubic spline fitting and the second step analyzes the prediction errors and provides for correction in accuracy incorporating factors of liquidity. The yield changes vary from % in 2001 to % in 2014 and for each of the years the spreads are over % per annum. The yield also changes with respect to the residual maturity and is seen to increase with falling residual maturity.

2. METHODOLOGY

Basic Cubic spline method and its variations have been used in the literature including its applications to term structure estimation in USA and European bond markets and for instruments such as spot and forward rates. Splines are a non-parametric polynomial interpolation method simplest of which is least squares regression spline which fails to control compounding errors. McCulloch (1975) and Beim (1992) used cubic splines and found it satisfactory, including Vasicek and Fong (1982) who suggested improvement on the approach using exponential splines for the spot rate curve. Adams and Van Deventer (1994) illustrated the technique to obtain maximum smoothness for forward curves and extended to quadratic splines. The basic technique has been improved by Fisher, Nychka and Zervos (1995), Waggoner (1997) and Anderson and Sleath (1999). Piecewise interpolation is the fitting of curve that gives accuracy at the nodes when each piece touches a node. The set of spines, which touch at the nodes can produce a continuous curve, joining observed rates as smoothly as possible, which is the most straightforward means by which we can deduce meaningful data on the correct interest rate term.

For example, three maturing bonds, with maturity dates 14/06/2015, 17/08/2016 and 28/08/2017, respectively (current date = 01/01/2010). The time to maturity for each of these is 4.875, 6.05 and 7.08 years, respectively. In cubic spline method, the zero-rate between 4.875 and 6.05 years is described by one set of parameters and between 6.05 years and 7.08 years is described by another set of parameters.

Suppose that $\{ (x_k , y_k) \}$ N k = 0 are N + 1 points,

where a = x 0 < x 1 < ··· < x N = b .

The function S (x) is called a cubic spline if there exist N cubic polynomials S_k (x) with coefficients s_k,0, s_k,1,s_k,2,and s_k,3 that satisfy the properties of that the second derivative of the resulting function is continuous and cubic are smooth functions.

HYPOTHESES

We propose to test the following Null Hypotheses;

A spline could of various forms and the nature would be simple to more complex.

H1: A cubic polynomial of the form $y = ax^3 +bx^2 +cx+d$, a set of parameters a, b and c exists that will provide reasonable accuracy.

The number of nodes determine the complexity of behavior.

H2: There exists more than one node of *ax*'s which gives the piece wise cubic polynomial to provide better approximations.

The residual errors are estimated as the difference between actual yield and estimated yield.

H3: The residuals of 2 piece wise cubic polynomials are significantly related to bond liquidity factors.

3. DATA

The data includes daily trading data from January 2001 to October 2014 of all listed and traded corporate bonds in NSE. The yield to maturity is the ones provided by NSE for traded scrips during the same period. Their reported prices other than the bond characteristics, credit rating, includes No of trades, traded value, low price/rate, high price/rate, last trading price, yield to maturity (YTM), etc. This does not include pure central govt. loans, state loans, treasury bills, dated securities, bond of departmental undertaking or india development bonds.

Table 1 provides the descriptive statistics of few variables of interest, where the maturity in months vary from a mean of 30.71 to 300 and No. of trades from 1.1 to maximum 6.0.

Table 1: Descriptive Statistics

Variable	Minimum	Mean	Maximum	STD
Time_Maturity	1	30.71096	300	37.40981
Trading_Interval	1	24.44767	256	31.55464
No_of_Trades	1	1.184385	6	0.537123
Traded_Value	0.01	8.250623	100	7.188011
Low_Price_Rate	74.6269	102.1773	348.84	8.408798
High_Price_Rate	74.6269	102.1903	348.84	8.409225

Tabl2 provides the trend of changes in YTM from 2001 to 2014, where the year wise spread in YTM has increased from 6.1 to 15.3 to 0 and 22.6, which means the spline estimation is all the more challenging with recent time period.

Table 2: YTM Changes during 2001 to 2014

Year	Mean	Std Dev	Minimum	Maximum
2001	9.4	1.1	6.1	15.3
2002	7.5	1.1	4.8	15.8
2003	6.0	0.9	3.9	13.2
2004	5.9	0.8	3.6	11.4
2005	6.7	0.7	4.4	11.6
2006	7.6	0.7	4.5	10.9
2007	8.4	0.8	4.8	11.5

2008	8.8	1.4	0.0	15.5
2010	7.6	1.3	0.0	16.2
2011	9.1	1.3	0.0	28.1
2012	9.4	4.5	0.0	22.6
2013	8.8	1.6	0.0	23.6
2014	9.0	1.4	0.0	22.6

4. RESULTS

The results of estimation are provided in Table3, Table 4 and Table 5, respectively. Table 3 gives the Log-linear Spline Order Estimates reported below. We find higher nodes beyond third order such as RM^4 , RM^5 and RM^6 are insignificant.

Table 3: Log-linear Spline Order Estimates

Dependent Variable= Log (YTM)	Beta	t-Value	P-Value
Intercept	2.29141	114.62	<.0001
RM	-0.24983	-4.58	<.0001
RM^2	0.14602	3.76	0.0002
RM^3	-0.02703	-3.41	0.0007
RM^4	0.03283	2.98	0.0029
RM^5	-0.00585	-1.79	0.0735
RM^6	0.00122	1.28	0.2022
R-Square	0.0128	Adj R^2	0.011

Hypothesis 1: The Null Hypothesis is accepted at 1 % level of significance which states that a cubic polynomial does exist.

Table 4 provides the descriptive Statistics of Logarithmic YTM for Cubic nodes. The estimated Cubic LYTM is close to actual LYTM with the mean of actual at 2.17 and that of predicted at 2.16.

Table 4: Descriptive Statistics of Logarithmic YTM for Cubic nodes

Variable	N	Mean	Std	Minimum	Maximum
LYTM *actual*	4704	2.17	0.168829	0.579138	3.005504
CubLYTM *predicted*	4704	2.16	0.022801	2.139176	2.431571

Hypothesis 2: The Null hypotheses is accepted since the piece wise cubic spline exists for 3 significant nodes at 1% level of significance for 3, 5 and 20 years. The MSE reduces and so does adjusted R^2.

Table 4 is an estimation of the residuals of the previous step reported in Table 3 here. We find the ratio of Trading Interval to Time Maturity which is the trading interval adjusted for remaining maturity as a significant liquidity characteristics that is related to the residuals. This implies the possibility of improving the predicted cubic LYTM by using bond characteristics.

Table 4: Residual Spline Error Estimates

Dependent Variable= Log (YTM)	Beta	t-Value	P-Value
Intercept	0.09792	5.87	<.0001
Trading Interval /Time Maturity	-0.08283	-4.1	<.0001
R-Square	0.0076	Adj R^2	0.0071
MSE	0.21201		

Hypothesis 3: The null hypotheses is accepted at 1 % level of significance since the residual errors are significantly impacted by the liquid factors of coupon frequency (negatively related), Trade volume (positively related) and interval of trading (positively related) with adjusted R^2=0.77 and parameters significant at 1% level, respectively. The liquidity factors are critical to explain the convexity property of bond yield, where the slope of yield ($\Delta p/\Delta y$) towards maturity turns flatter and the change in yield ($\Delta p/\Delta y$) is lower. Conclusion

This paper devised a two stage estimation process of bond yield. This approximated the yield to maturity for varying residual maturities in the Indian bond market since the period January 2001 to October 2014. We tested the hypotheses of existence of piece wise polynomials and also the factors of liquidity. The results concluded that there exists spline methods that mimicked variations in yield curve, the form of which could be improved by piece wise method for three major empirical nodes (3, 5 and 20). Further, we also determined the liquidity factors of bond trading such as the ratio of Trading Interval to Time Maturity which is the trading interval adjusted for remaining maturity as a significant liquidity characteristics that is related to the residuals. This implies the possibility of improving the predicted cubic LYTM by using bond characteristics.

Thus there may exist alternatives to spline such as simulation or ANN which are used to forecast yields. This provides a practical, feasible, transparent and simple method of yield determination. Further, a practitioner can tweek the parameters to alter the yield curve in different situations of maturity or bond characteristics. Therefore, one can employ the approach to implement constructing spot yield curves from discount rates. This enables practitioners to fit a yield curve to observed rates (Libor or bond yields) reasonably accurately and produces a satisfactory spread curve under most circumstances.

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