Grating lobe and Sidelobe Suppression using Multi-Objective Optimization Techniques

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***Abstract*—The range resolution of RADAR signal depends on the bandwidth of the signal. Linear Frequency Modulation (LFM) signal is used in RADAR because of its good range resolution. The range resolution can be further improved by introducing a frequency step between consecutive LFM pulses. The resultant signal is known as Stepped Frequency Pulse Train (SFPT). When the product of pulse duration and frequency step become more than one, the Autocorrelation Function (ACF) of SFPT shows ambiguous peaks, known as grating lobes. These grating lobes along with sidelobes present in the vicinity of the mainlobe are not desirable. In literatures, there are analytical methods available to find the parameter of SFPT for completely nullify or suppress these grating lobes to a desired level but not much attention is given to sidelobes. In this paper, we use a Multi-Objective Optimization Technique (Multi-Objective Particle Swarm Optimization (MOPSO)) to find the optimized parameter of SFPT. The problem was formulated by taking the objective of minimization of grating lobe and minimization of peak sidelobe. The efficacy of the proposed method is shown through exhaustive simulation results.**

***Keywords*—Frequency stepped pulse train, Grating lobe suppression, Multi-objective optimization, RADAR signal processing, Sidelobe suppression**

I. Introduction

In RADAR, for high range resolution we required signal with high bandwidth. Generation of such type of signal increase the overall cost and system complexity. The conventional hardware used in RADAR may not be able to sustain this instantaneous large amount of bandwidth. To overcome this limitation, wide-band signal is splitted into narrow-band signals. The narrow-band signals are transmitted and received separately and added coherently at receiver end to get the effect of the wide-band signal. Such type of signal is known as Stepped Frequency Pulse Train (SFPT) or Synthetic Wide-band Waveform (SWW) or Frequency Jumped Train (FJT).

A SFPT of N pulses, each of duration t_p and pulse repetition frequency t_r is shown in Fig. 1. Each pulse has a bandwidth B and the frequency step between pulses is Δf. The advantage of using SFPT is that the duration between pulses can be used to adjust the center frequency of other narrow-band pulses. The matched filter response of SFPT suffers from ambiguous peaks known as grating lobes when the product of pulse duration and frequency step become more than one ($t_p \Delta f > 1$). These grating lobes along with sidelobes present in the vicinity of the mainlobe are undesirable and can hide the small target or can cause a false alarm detection.

Different methods are given in literatures to nullify grating lobes or to suppress it to an acceptable level. In [1], [2] the pulse width is varied to reduce the grating lobes but varying pulse width destroys the periodicity of the pulse train. In [3] the energy of pulse train is distributed non-uniformly over the desired frequency band to get lower range sidelobes, higher range resolution and reduced grating lobe but spectral weighting applied for non-uniform distribution of energy, introduces additional losses in sensitivity. Levanon and Mozeson [4] have proposed an analytical technique to establish a relation between the parameter of SFPT. The grating lobes are nullified by placing the null of the Autocorrelation Function (ACF) of single LFM pulse exactly at the location of grating lobe. In this approach number of pulses N has to be large for a significant increase in bandwidth. In [5] the grating lobes are reduced to an acceptable level by forcing the amplitude of ACF of a single frequency LFM pulse below a predefined level at the location of grating lobe. The above mentioned approach does not suppress range sidelobe that occur near the mainlobe of matched filter response to SFPT. Non-Linear Frequency Modulated (NLFM) is used instead of LFM pulse for suppression of range sidelobe but NLFM waveforms are not Doppler tolerant. A Multi-Objective Optimization (MOO) technique (Nondominated Sorting Genetic Algorithm (NSGA-II) [6]) is used by Sahoo and Panda [7] to find the parameter of SFPT for sidelobes and grating lobes suppression. The complexity of the algorithm is $O\left(MN^2\right)$, where M is the number of objective function and N is the population size. In this work, to achieve better Pareto front with less computation complexity, a new MOO technique (Multi Objective Particle Swarm Optimization(MOPSO) [8]) is used.

The paper is structured as follows: Section II gives description of SFPT. In Section III, we formulate the problem used for optimization. Section IV describes the MOO algorithm. Simulation results are shown in Section V and conclusion is presented in Section VI.

II. Stepped Frequency Pulse Train

The SFPT consists of N pulses, each of duration t_p and the time separation between two pulses is t_r. The envelope of unmodulated pulse is given by [4]

$$u(t) = \frac{1}{\sqrt{t_p}} rect\left(\frac{t}{t_p}\right) \tag{1}$$

Frequency modulation is applied to unmodulated pulse to get an LFM signal.

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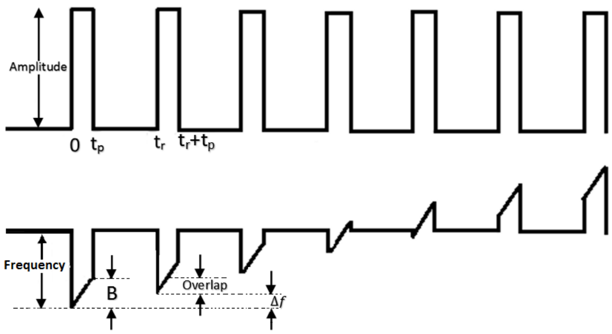

Fig. 1. Stepped frequency pulse train

The complex envelope of LFM pulse is given by

$$u_1(t) = \frac{1}{\sqrt{t_p}} rect\left(\frac{t}{t_p}\right)\exp\left(j\pi k t^2\right) \tag{2}$$

Where k is the frequency slope of LFM signal and is defined as

$$k = \pm\frac{B}{t_p} \tag{3}$$

B is the bandwidth of single LFM pulse. $+$ and $-$ signs correspond to positive and negative frequency slope respectively. In this paper, we have used the positive value of k but the analysis is equally valid for the negative value of k. The instantaneous frequency of LFM signal is given by

$$f\left(t\right) = \frac{1}{2\pi}\frac{d\left(\pi k t^2\right)}{dt} \tag{4}$$

A uniform pulse train having N number of LFM pulses separated by $t_r \geq 2t_p$ is expressed as

$$u_N\left(t\right) = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} u_1\left(t - nt_r\right) \tag{5}$$

To maintain unit energy the multiplication factor $\frac{1}{\sqrt{N}}$ is included in the expression.

Another slope of k_s is added to entire LFM pulse train, and the complex envelope is represented as

$$u_s\left(t\right) = u_N\left(t\right)\exp\left(j\pi k_s t^2\right) \tag{6}$$

or

$$u_s\left(t\right) = \frac{1}{\sqrt{N}}\exp\left(j\pi k_s t^2\right)\sum_{n=0}^{N-1} u_1\left(t - nt_r\right) \tag{7}$$

where

$$k_s = \pm\frac{\Delta f}{t_r} \qquad \Delta f > 0 \tag{8}$$

$+$ and $-$ signs stand for positive frequency and negative frequency slope respectively. The overall bandwidth of SFPT is expressed as

$$B_T = \left(k + k_s\right) t_p \Delta f \tag{9}$$

The ACF of $u_s(t)$ is given by [4]

$$\left|R\left(\tau\right)\right| = \left|\left(1 - \frac{|\tau|}{t_p}\right)\sin c\left(B\tau\left(1 - \frac{|\tau|}{t_p}\right)\right)\right|\left|\frac{\sin\left(N\pi\tau\Delta f\right)}{N\sin\left(\pi\tau\Delta f\right)}\right| \tag{10}$$

The expression for $\left|R\left(\tau\right)\right|$ is the product of two terms. First one is the ACF of single LFM pulse given by

$$\left|R_1\left(\tau\right)\right| = \left|\left(1 - \frac{|\tau|}{t_p}\right)\sin c\left(B\tau\left(1 - \frac{|\tau|}{t_p}\right)\right)\right| \tag{11}$$

The second term describes the grating lobes

$$\left|R_2\left(\tau\right)\right| = \left|\frac{\sin\left(N\pi\tau\Delta f\right)}{N\sin\left(\pi\tau\Delta f\right)}\right| \quad \tau \leq t_p \tag{12}$$

$\left|R_2\left(\tau\right)\right|$ produces the grating lobe at $\tau_g = \frac{g}{\Delta f}$ where $g = 1, 2, ... \lfloor t_p \Delta f \rfloor$. These grating lobes can hide the weak target. In [4] a relation is given between the pulse duration of single LFM pulse t_p, its bandwidth B and frequency step Δf to nullify first two grating lobes. It is also shown that in some cases nullifying first two grating lobes leads to removal of all the grating lobes.

III. Problem Formulation

In [7] some problems were formulated for sidelodes and grating lobes suppression. We use problem 1 for our MOO approach. For grating lobes suppression we want the amplitude of $\left|R_1\left(\tau\right)\right|$ at position of grating lobe to be very low. For sidelobe suppression we use Peak to Sidelobe Ratio (PSR) as a performance measure. PSR is defined in [7] as

$$PSR = \frac{Maximum\, sidelobe\, level\, in\, ACF}{mainlobe\, level} \tag{13}$$

For a constant value of N, ACF, $\left|R_1\left(\tau\right)\right|$ and $\left|R_2\left(\tau\right)\right|$ are functions of $t_p\Delta f$ and t_pB. By choosing the appropriate value of $t_p\Delta f$ and t_pB we can suppress grating lobes and sidelobes. The constraint for this problem is that increase in bandwidth should be significant i.e. $N\Delta f > B$. The value of t_pB is chosen such that $t_pB = \left(c+1\right)t_p\Delta f$ (c is any positive number) to ensure $B > \Delta f$, such that there is some frequency overlap between the pulses. MOPSO algorithm [8] is used to search for the values of $t_p\Delta f$ and t_pB which results in low sidelobes and low grating lobes amplitude. The objective functions which are to simultaneously optimized, defined in [7], are as follow:

$minimize\, F_1 = \max\left[\left|R_1\left(\tau_g\right)\right|\right]$ where $g = 1, 2, ... \lfloor t_p\Delta f \rfloor$
$minimize\, F_2 = PSR$ in dB
$Subjected\ to\ the\ constraints\ Nt_p\Delta f > t_pB$

IV. Multi Objective Optimization Algorithm

In practice, there might exist a set of conflicting objectives for which we want to find the best solution. The objectives may not have a single best solution with respect to all problems. Here a set of solutions might exists in search space that are superior to the rest of solutions with respect to all objective functions but are inferior among themselves with respect to one or more objectives. These solutions are called Nondominated solutions or Pareto Optimal solutions. None of the Nondominated solution is better than others. The superiority of one solution over other depend upon the extra knowledge about the problem.

In the proposed work, we uses MOPSO algorithm to optimize the objective functions explained in Section III. The pseudo-code for this algorithm is shown in Algorithm 1. In this algorithm w is Inertia weight. Inertia weight affects the convergence and exploitation. R_1 and R_2 are two random values. c_1 and c_2 are personal and global learning coefficient respectively. $PBEST$ is the personal best position of the solution attains so far. REP is External Repository, which store the nondominated solutions. External repository has two component, the Archive Controller and the Grid. Work of the archive controller is to decide which solution stays in the repository. Archive controller works until number of solutions in repository are less than the maximum length of the repository. As the repository reaches its maximum size, repository becomes a grid. If a new solution is inserted into the grid then, a solution must be deleted from more populated area. If the new solution goes beyond the boundary of the grid then, the grid has to be recalculated and individuals have to be relocated. A special mutation operator is used in this approach that explore all the particle at the beginning of the search and decrease rapidly with respect to the number of iteration.

V. SIMULATION RESULTS

Simulations are carried out to optimize F_1 and F_2 simultaneously using the MOPSO algorithm. The population size and the number of generations each of 100 are used. Inertia weight is selected as 0.73. Personal Learning Coefficient and Global Learning Coefficient both are selected as 1.5. The Pareto fronts for MOPSO and NSGA-II based approach are shown in Figs. 2, 3. From Figs. 2, 3 it is clear that the Pareto

Algorithm 1 Multi Objective Particle Swarm Optimization Algorithm

initialize *POP()* to random value
initialize *VEL()*
evaluate each particle in *POP()*
store nondominated vectors in the *REP*
generate hypercube
PBEST=POP
generation=0
while (generation $<$ maxGenerations) **do**
  for Each particle i **do**
    $VEL(i) = w * VEL(i) + R_1 * c_1(PBEST(i) - POP(i)) + R_2 * c_2(REP(i) - POP(i))$
    $POP(i) = POP(i) + VEL(i)$
    maintain *POP[i]* within search space
    evaluate particle i
    update *REP*
    update *PBEST(i)*
  end for
  increase *generation* by one
end while

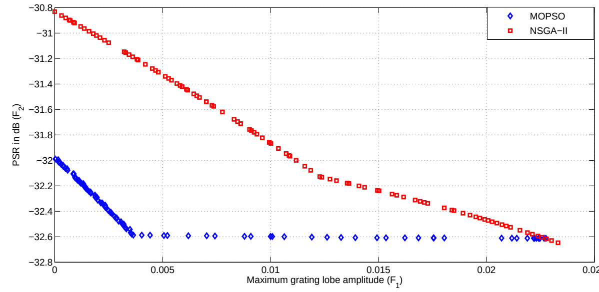

Fig. 2. Pareto front obtained using MOPSO and NSGA-II for $t_p\Delta f \in [2,\ 10]$, $c \in [2,\ 10]$ and $N = 8$

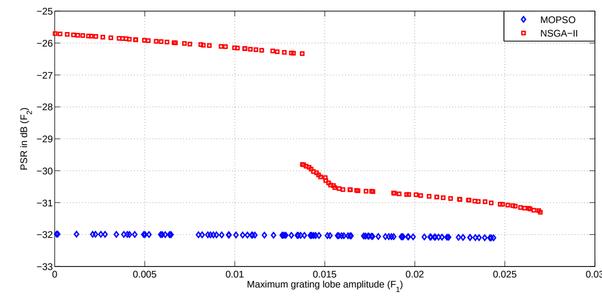

Fig. 3. Pareto front obtained using MOPSO and NSGA-II for $t_p\Delta f \in [2,\ 10]$, $c \in [2,\ 5]$ and $N = 8$

front obtained from MOPSO based approach is better than that of NSGA-II approach. Fig. 2 illustrates the Pareto front obtained from MOPSO and from NSGA-II approach for $t_p\Delta f \in [2,\ 10]\,, c \in [2,\ 10]$ and $N = 8$. From Fig. 2 it is evident that in MOPSO approach, for zero grating lobe amplitude ($F_1 = 0$), maximum sidelobe level is 32 dB below the mainlobe level as compare to 30.8 dB in NSGA-II algorithm. From Fig. 2, if the maximum grating lobe amplitude to be not more than 0.02, then the result obtained by the proposed MOPSO based approach is better than the result obtained by NSGA-II.

For $t_p\Delta f \in [2,\ 10]\,, c \in [2,\ 5]$ and $N = 8$ the value of objectives F_1 and F_2 obtain from MOPSO approach and from NSGA-II are shown in Fig. 3. From Figs. 3 it is clear that MOPSO algorithm gives better result for both the objective functions. Figs. 4- 7 display the plots of $|R_1(\tau)|\,, |R_2(\tau)|$ and ACF for different values of $t_p\Delta f$ and t_pB, obtained form NSGA-II and MOPSO algorithm. In Fig. 4 the null of $|R_1(\tau)|$ is located on the position of grating lobe of $|R_2(\tau)|$, which means $F_1 = 0$. The value of $t_p\Delta f$ and t_pB are given by NSGA-II algorithm. PSR for this case is 30.8315 dB below the mainlobe level. Fig. 5 depicts $|R_1(\tau)|\,, |R_2(\tau)|$ and ACF for $F_1 = 0$. The value of $t_p\Delta f$ and t_pB are obtain by MOPSO algorithm. In this case the PSR is 31.9889 dB below the mainlobe level. So there is an improvement of 1.1574 dB in PSR. Figs. 6, 7 depicts the plot of $|R_1(\tau)|\,, |R_2(\tau)|$ and ACF for $F_1 = 0.01$, parameters are chosen by NSGA-II and MOPSO respectively. In NSGA-II $F_1 = 0.01$ ($40dB$ below

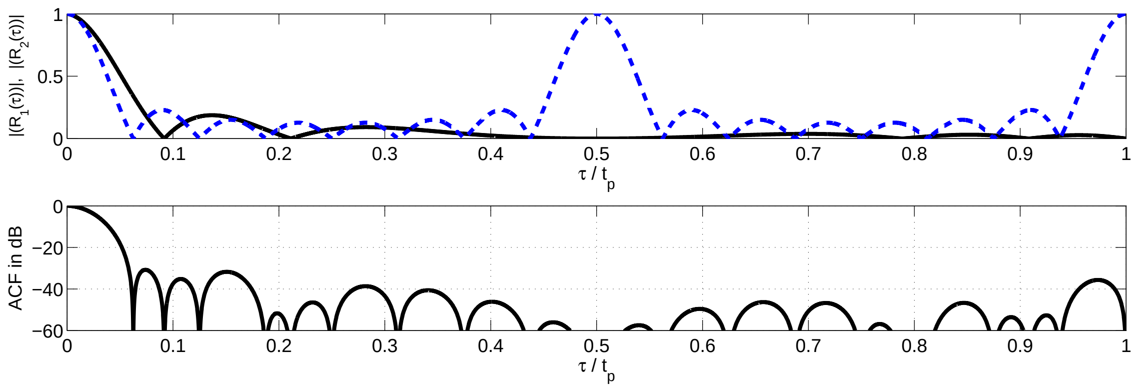

Fig. 4. SFPT for $F_1 = 0$. Parameter obtained from NSGA-II. $t_p \Delta f = 2$, $c = 5$ and $t_p B = 12$. Top $|R_1(\tau)|$ (solid) and $|R_2(\tau)|$ (dash). Bottom ACF (in dB)

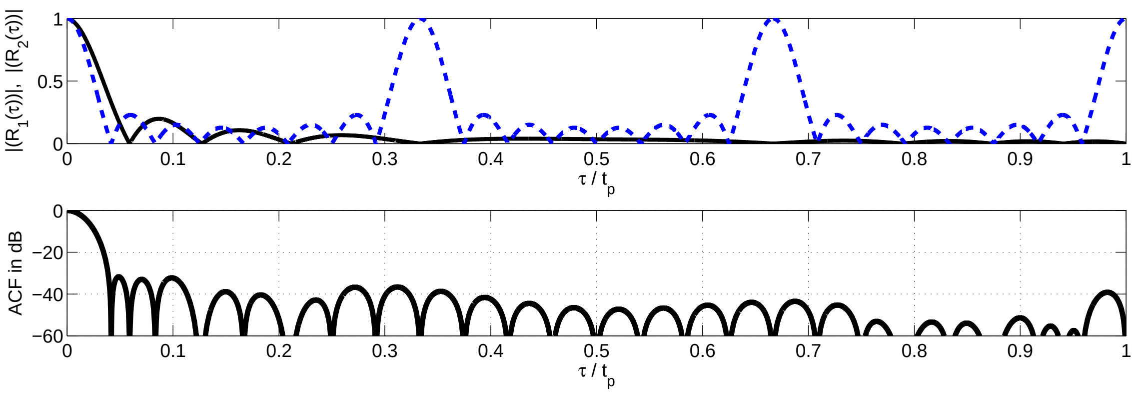

Fig. 5. SFPT for $F_1 = 0$. Parameter obtained from MOPSO. $t_p \Delta f = 3$, $c = 5$ and $t_p B = 18$. Top $|R_1(\tau)|$ (solid) and $|R_2(\tau)|$ (dash). Bottom ACF (in dB)

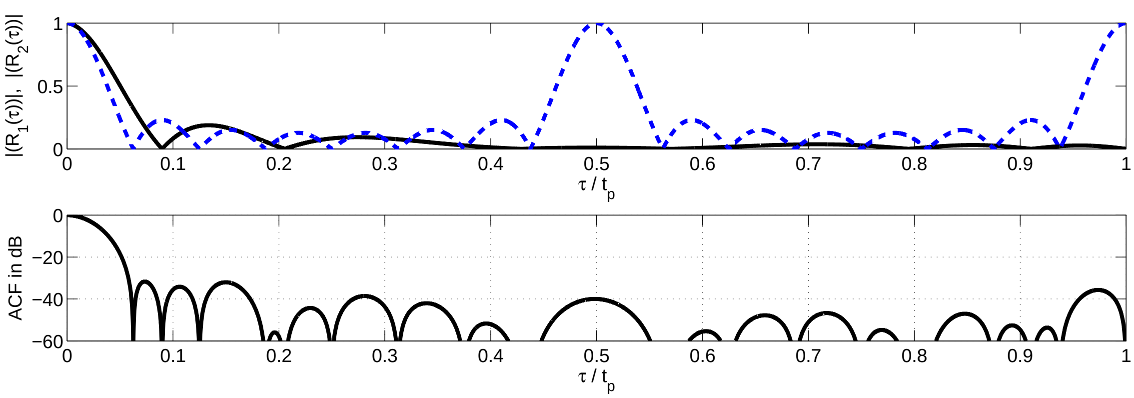

Fig. 6. SFPT for $F_1 = 0.01$. Parameter obtained from NSGA-II. $t_p \Delta f = 2$, $c = 5.12$ and $t_p B = 12.24$. Top $|R_1(\tau)|$ (solid) and $|R_2(\tau)|$ (dash). Bottom ACF (in dB)

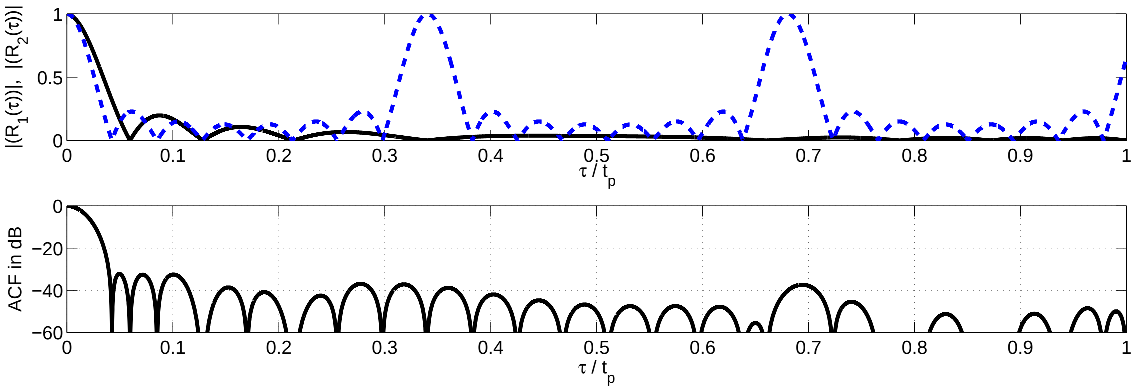

Fig. 7. SFPT for $F_1 = 0.01$. Parameter obtained from MOPSO. $t_p\Delta f = 2.93$, $c = 5.06$ and $t_pB = 17.75$. Top $|R_1(\tau)|$ (solid) and $|R_2(\tau)|$ (dash). Bottom ACF (in dB)

the mainlobe level) can be observed around $\tau/t_p = 0.5$.

For MOPSO $F_1 = 0.01$ can be found near around $\tau/t_p = 0.36$. The PSR for NSGA-II and MOPSO is 31.6869 and $32.1971\,dB$ below the mainlobe level respectively. For $F_1 = 0.01$ there is an improvement of $0.5102\,dB$ in sidelobe level as compare to NSGA-II.

VI. Conclusion

In this paper, a multi-objective optimization approach based on NSGA-II and MOPSO is investigated to find the parameter of stepped frequency pulse train for low sidelobes and reduced grating lobes at the matched filter output. The proposed work facilitates us to choose waveform parameter from the Pareto front according to the requirements of stepped frequency pulse train. From the simulation results, we can conclude that MOPSO based approach provide a better trade-off solutions between the objective functions through Pareto front than that of NSGA-II.

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