Cellular Capacity Maximization via Robust Downlink Beamforming

Venkatesh Chebolu and Siddharth Deshmukh, *Member*, *IEEE*

***Abstract*—In this paper we propose an optimized multi-cell downlink beamforming solution in which our objective is to maximize capacity of a cellular network. We first formulate an optimization problem, maximizing the received signal power of every active user in a cell, subjected to limiting the overall interference observed by other users below a specified level. In addition, we also put constraint on maximum transmit power of the serving base station. Next, in order to compute robust beamforming vector we accommodate channel estimation error in our formulation. We consider channel imperfection as error between true and estimated channel coefficients and we assume error is bounded with in an ellipsoidal set. The resulting formulation is a non-convex optimization problem and to get a tractable solution we exploit linear matrix inequality based S-procedure. The final reformulation is solved by using semi definite relaxation. The efficacy of proposed solution in improving cellular capacity and efficient power transmission is shown by simulations.**

Keywords- Downlink beamforming, Semi definite programing, Cellular capacity maximization.

I. Introduction

Beamforming is a powerful and efficient technique to transmit and receive the desired signal in presence of co-channel interference by exploiting spatial diversity [1]. In cellular networks downlink beam forming is used to transmit the data from the base station (BS) to desired user by deploying multiple antennas at the serving BS. In addition to co-channel interference mitigation, efficient beamformer design can improve downlink system capacity and minimize total transmit power [2]. One way to achieve efficient beam- former design is coordination between neighbouring BSs [3]. This technique is also called co-ordinated multipoint transmission (CoMP).

CoMP has been widely recognized as a spectrally efficient technique which gives high data rates to user terminals in a cellular network. In a perfect CoMP central unit plays an important role for the purpose of coordination between the BSs. The central unit does all signal processing tasks with BSs sharing their data and global CSI [4]. However such a centralized processing requires an additional resources of ideal black haul which is prohibited in practical system. Distributed or decentralized CoMP systems have become a recent research interest as we have practical limitation in centralized CoMP [5]. Acquisition of CSI at the transmitter i.e. BS (CSIT) in downlink beamforming in COMP system is very essential for effective downlink beamforming towards the desired user terminal. But even with sophisticated training, estimated CSI cannot be perfect in the practical systems due to several reasons like estimation error, delay and quantization errors. Hence in addition to design of decentralized CoMP system it is important to find the robust design methodologies against imperfect channel knowledge for accomplishing the effective gains of CoMP in practical scenarios. In robust formulation of cellular beamforming problem the true CSIT is considered to be confined within an uncertainty region and beamforming vectors are designed such that they remain feasible despite the imperfections in estimated channel [6]. Such problems typically lead to optimization problems with infinite number of constraints and reformulating them to tractable equivalent forms is a challenging task.

Recently many robust methodologies against channel imperfections have been developed for cellular networks. For example, authors in [7] have proposed multicellular downlink beam former design with the objective of minimizing the sum power used by each BS to transmit data to local users. In addition CSI imperfection is considered and signal to interference plus noise ratio (SINR) is maintained above the desired level. Solution for the above problem has been obtained with the help of S-procedure and semi definite relaxation (SDR). In [8] authors studied multi-cell wireless networks and especially focussed on joint robust transmission optimization over the cells. They aimed at maximizing the minimum worst case rate of the network with imperfect CSI. Optimized solution has been achieved by converting the problem in to convex problem and solving it by employing convex optimization problem solvers. In [9] authors minimize the total transmit power at BSs by taking equivocation rates and individual SINR as constraints. In [10] authors minimize the total downlink power at BSs by restricting the SINR outage probability below a threshold.

In this paper, we considers a decentralized CoMP approach in which each BS with in a cell only sends data to its local user. This technique helps in reducing the signalling overhead compared with centralized CoMP. Here we formulate a robust distributed optimization problem that maximizes the received signal power of each user present in a particular cell subject to total transmitted power at the base station remains below the required level and overall interference on the other cell users due to the transmission of BS in given cell should be maintained below a desired level in the presence of imperfect CSI. We considers the imperfections in CSI between true and estimated channel coefficients is confined within a spherical uncertainty set. The worst case solution of the above proposed

Venkatesh Chebolu is with the Department of Electronics and Communication Engineering, National Institute of Technology Rourkela, India. (Email ID: venki.chebolu@gmail.com)

Siddharth Deshmukh is with the Department of Electronics and Communication Engineering, National Institute of Technology Rourkela, India. (Email ID: deshmukhs@nitrkl.ac.in)

problem can be obtained by reformulating non convex problem as semi definite relaxation and constraints as linear matrix inequalities. The rest of the paper is structured as follows. In section-II we provide the system model and problem formulation where non convex formulation is presented. Section-III develops the robust downlink beamforming formulation in the presence of imperfect CSI and later we reformulate the non- convex problem in to convex problem by employing semi definite relaxation. Simulation results are shown in the section IV. Finally conclusion is presented in section V.

Notations: In this paper, R^n, C^n and H^n are used to denote the sets of n dimensional real, complex vectors and complex Hermitian matrices, respectively. $\mathbf{W}$ and w (or W) are employed to describe a matrix and vector. The notation $(.)^H, (.)^T$ and $Tr(\mathbf{A})$ indicate the conjugate transpose ,the transpose and trace of matrix $\mathbf{A}$. E(.) and $\|.\|$ provides the expectation operation and Euclidean norm respectively.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a cellular network with N cells. We assume that each cell in cellular network consists of one base station and each base station in the network is equipped with M no of antennas. Further we consider L active single antenna users in each cell of the network. In the cellular network each BS directs the beam to its corresponding users. Objective of the directed beams is to increase the received signal power of desired user and to restrict the inter-cell interference to the other users below a particular value. In addition, it also maintain the total signal power transmitted by the BS below a particular threshold. Let, $S_l = \{1, 2, ...U\}$ is the set of locally active users in particular cell q and $S_o = \{1, 2, ...R\}$ the set of user in adjacent cells which are subjected to inter-cell interference due to the transmission by the q-th BS with in the network. Here we define, $h_i \in C^{M\times 1}$ as the channel vector which consists of channel coefficients between the BS in cell q and active users $i \in S_l, g_t \in \ C^{M\times 1}$ as channel vector which consists of channel coefficients between the q-th BS and users $t \in S_o$. The received signal at user i is given by,

$$y_i = h_i^H w_i s_i + \sum_{j=1, j\neq i}^{U} h_i^H w_j s_j + v_i + n_i, \tag{1}$$

where, $w_i \in C^{M\times 1}$ is the beamforming vector corresponding to the i-th user and s_i is the complex scalar which denotes the data symbol for user i. The resultant inter-cell interference observed at user i because of the transmission by BSs in the cells other than q is denoted by v_i. n_i denotes the circularly symmetric complex Gaussian random variable at user i which has zero mean and variance $\sigma^2, n_i \sim CN(0, \sigma^2)$. We also assume that the average energy of the transmitting symbol s_i is normalized to unity i.e $E_{s_i}(|s_i|^2) = 1$. The expression for the signal to interference plus noise ratio observed by user i can be written as

$$SINR_i = \frac{\left|h_i^H w_i\right|^2}{\sum_{j\neq i}\left|h_i^H w_i\right|^2 + \xi_i + \sigma^2}, \tag{2}$$

where, ξ_i in above expression is total inter-cell interference experienced by user i due to the BSs in other cells i.e $\xi_i = E[|v_i|^2]$. Next, to maximize the capacity for the desired user, we formulate an optimization problem maximizing the SINR while satisfying the transmit power constraint. Since maximizing SINR is non convex we maximize the received signal power while restricting the undesired interference power to other users. Thus, if every BS restricts interference power to some threshold, the optimization problem to calculate the optimum downlink beamforming vectors at any given BS q , can be expressed as

$$\begin{aligned} &\max_{w_i} \quad \left|h_i^H w_i\right|^2 \\ &s.t \ \ \textstyle\sum_{t\in S_o}\sum_{i\in S_l} \mu_t w_i^H g_t g_t^H w_i \leq K \\ &\textstyle\sum_{i\in S_l} \eta_i w_i^H w_i \leq p_q, \end{aligned} \tag{3}$$

where, K is the maximum allowed interference power at active users $t \in S_o$ due to the transmission of BS q. p_q is the maximum allowed transmitted signal power by the BS q. The weighting factor needed for the active user $i \in S_l$ is represented by η_i and μ_t required for the adjacent outer-cell users $t \in S_o$ by the BS q. These coefficients are used by the scheduler for setting up the priority levels which depends on the quality (cost) of requested services by different users or to proportionally maintain fairness among the users. The objective function in (3) denotes the received signal power at user i, i.e. $i \in S_l$ and left side term of inequality in constraint one in (3) is the overall interference power on the other cell users except users in S_l due to the transmission of BS q. Finally left side term of inequality in constraint 2 in (3) is the total signal power transmitted by the BS q. In next section, we modify our formulation for robust solution by considering imperfect channel state information .

III. ROBUST DOWNLINK BEAMFORMING FORMULATION IN THE PRESENCE OF CSI

Let us assume, $\hat{g}_t \in C^{M\times 1}$ and $\hat{h}_i \in C^{M\times 1}$ be the estimated CSIs at the BSs. Then the true CSI can be expressed as

$$\begin{aligned} g_t &= \hat{g}_t + e_t, \qquad \forall t \in S_o, \\ h_i &= \hat{h}_i + e_i, \qquad \forall i \in S_l, \end{aligned} \tag{4}$$

where, $e_t \in C^{M\times 1}$ and $e_i \in C^{M\times 1}$ denotes the CSI error vectors. We considered that the vectors e_t and e_t are bound with in a ellipsoidal sets defined as

$$e_t^H Q_t e_t \leq 1, \qquad \forall t \in S_o, \tag{5}$$

$$e_i^H Q_i e_i \leq 1, \qquad \forall i \in S_l, \tag{6}$$

where, $Q_t \in C^{M\times M}$ and $Q_i \in C^{M\times M}$ are a positive semi definite matrix which characterizes the shape and size of the ellipsoid. Substituting for g_t and h_i from (4) in (3), we will get

$$\begin{aligned} &\max_{w_i} \ \min_{e_i^H Q_i e_i \leq 1} \ \left|(\hat{h}_i^H + e_i^H) w_i\right|^2 \\ &s.t \ \min_{e_i^H Q_i e_i \leq 1} \ \textstyle\sum_{t\in S_o}\sum_{i\in S_l} \mu_t w_i^H \\ &\qquad (\hat{g}_t + e_t)(\hat{g}_t^H + e_t^H) w_i \leq K \\ &\textstyle\sum_{i\in S_l} \eta_i w_i^H w_i \leq p_q, \ \ \forall i \in S_l \end{aligned} \tag{7}$$

By introducing slack variable z, we can write (7) as

$$\begin{aligned} & max \quad z \\ s.t \quad & \left|(\hat{h}_i^H + e_i^H)w_i\right|^2 \geq z \\ & e_i^H Q_i e_i \leq 1, \quad \forall i \in S_l, \\ \min_{e_i^H Q_i e_i \leq 1} \quad & \textstyle\sum_{t \in S_o} \sum_{i \in S_l} \mu_t w_i^H \\ & (\hat{g}_t + e_t)(\hat{g}_t^H + e_t^H) w_i \leq K \\ & e_t^H Q_t e_t \leq 1, \quad \forall t \in S_o, \\ & \textstyle\sum_{i \in S_l} \eta_i w_i^H w_i \leq p_q, \quad \forall i \in S_l \end{aligned} \tag{8}$$

The problem in (8) and its constraints can be rewritten as

$$\begin{aligned} & min \quad -z \\ s.t \quad & (\hat{h}_i^H + e_i^H)\mathbf{W}_i(\hat{h}_i + e_i) - z \geq 0, \\ & e_i^H Q_i e_i \leq 1, \quad \forall i \in S_l, \\ & -\textstyle\sum_{t \in S_o} \sum_{i \in S_l} \mu_t (\hat{g}_t^H + e_t^H)\mathbf{W}_i \\ & \quad (\hat{g}_t + e_t) + K \geq 0, \\ & e_t^H Q_t e_t \leq 1, \quad \forall t \in S_o, \\ & Rank(\mathbf{W}_i) = 1 \quad \forall i \in S_l, \\ & \textstyle\sum_{i \in S_l} \sum_{j \in 1}^{n} \eta_i r_j^H \mathbf{W}_i r_j - p_q \leq 0, \end{aligned} \tag{9}$$

where, $\mathbf{W}_i = w_i w_i^H$ is a positive semi definite matrix with unit rank. Here r_j is the unit identity vector of order n×1 which has 1 at the j-th position and 0 everywhere with n as beamforming vector length. Next we discuss, linear matrix inequality based s-procedure to formulate (9) to SDP form.

Lemma1 (s-procedure [4]): Let $\phi_i(e)$, for i=0, 1, be defined as

$$\phi_i(e) = e^H \mathbf{A}_i e + b_i^H e + e^H b_i + c_i, \tag{10}$$

where, $\mathbf{A}_i \in H^{M \times M}, b_i \in C^M$ and $c_i \in R$ Suppose, there exists an $\hat{e} \in C^M$ such that $\phi_1(\hat{e}) < 0$. Then the following two conditions are equivalent:

1) $\phi_0(e) \geq 0$ and $\phi_1(e) \leq 0$ are satisfied for all e
2) There exists $\lambda \geq 0$ such that

$$\begin{bmatrix} \mathbf{A}_0 & b_0 \\ b_0^H & c_0 \end{bmatrix} + \lambda \begin{bmatrix} \mathbf{A}_1 & b_1 \\ b_1^H & c_1 \end{bmatrix} \geq 0. \tag{11}$$

The first four constraints of problem (9) can be written as

$$\hat{h}_i^H \mathbf{W}_i \hat{h}_i + \hat{h}_i^H \mathbf{W}_i e_i + e_i^H \mathbf{W}_i \hat{h}_i + e_i^H \mathbf{W}_i e_i - z \geq 0, \tag{12}$$
$$e_i^H Q_i e_i - 1 \leq 0, \quad \forall i \in S_l,$$

$$\begin{aligned} & -\textstyle\sum_{t \in S_o} \sum_{i \in S_l} \; [\mu_t \hat{g}_t^H \mathbf{W}_i \hat{g}_t + \hat{g}_t^H \mathbf{W}_i e_t + \\ & \qquad e_t^H \mathbf{W}_i \hat{g}_t + e_t^H \mathbf{W}_i e_t] + K \geq 0, \\ & e_t^H Q_t e_t - 1 \leq 0, \quad \forall t \in S_o, \end{aligned} \tag{13}$$

As per Lemma1, the pairs of inequalities in (12) and (13) hold if and only if there exist $\lambda_i \geq 0$ and $\lambda_t \geq 0$ such that the matrix inequalities in (16), indicated on the top of the following page hold. From the second inequalities in (12) and (13) one can understand that the condition $\phi_1(\hat{e}) < 0$ is trivially satisfied. Hence the optimization problem in (9) can be equivalently rewritten as

$$\begin{aligned} & min \quad -z \\ s.t \quad & \Upsilon_i \geq 0, \quad \forall i \in S_l, \\ & \Upsilon_t \geq 0, \quad \forall t \in S_o \\ & \lambda_i \geq 0, \quad \forall_i \\ & \lambda_t \geq 0, \quad \forall_t \\ & Rank(\mathbf{W}_i) = 1, \quad \forall i \in S_l, \\ & \textstyle\sum_{i \in S_l} \sum_{j \in 1}^{n} \eta_i r_j^H \mathbf{W}_i r_j - p_q \leq 0. \quad \forall i \in S_l \end{aligned} \tag{14}$$

Here we can observe that the rank constraint in (14) is not convex. The problem in (14) can be solved by removing the fifth nonconvex constraint and solving the remaining convex problem using numerical optimization packages, e.g., CVX solver, and finally keeping only the rank one solutions for $\mathbf{W}_i$.

IV. SIMULATION RESULTS

A. Simulation set up

In this section, we evaluate performance of cellular network in terms of improvement in capacity of desired user by using the solution of the problem in (14). In the first step, we generate the 7-cell cluster and users, in every cell of the cluster. Fig.1 shows the example of one user distribution with 7 users. Monte Carlo simulations have been performed over the distribution of one user and we produce 100 uncertain channel realization per user satisfying $\|e_t\|^2 \leq \varepsilon^2$ with different values. Here we employ channel model used in [4] which is given as

$$g_t = 10^{-(128.1+37.6\log_{10}(1))/20}.\Psi_t.\phi_t.(\hat{g}_t + e_t) \tag{15}$$

where, the distance between the BS and the user is denoted by l, Ψ_t and ϕ_t are the shadowing and antenna gain respectively. $\hat{g}_t$ and e_t indicate the estimated CSI and CSI error corresponding to i th user. Here we choose a spherical uncertainty set with error radius i.e $Q_b = \varepsilon^{-2} I_M$ for all $b = t, i$.

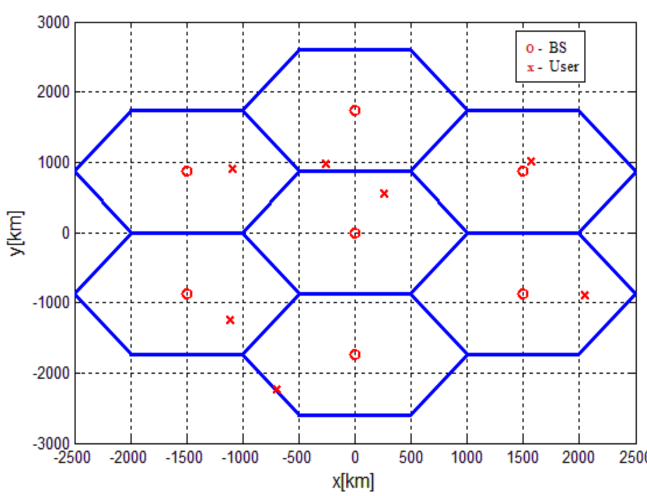

Fig. 1. An example of user distributions used in monte-carlo simulations

B. Performance evaluation

In this sub-section, we examine the performance of robust downlink beamforming for the maximization of capacity of

$$\Upsilon_i = \begin{bmatrix} \mathbf{W}_i + \lambda_i Q_i & \mathbf{W}_i \hat{h}_i \\ \hat{h}_i^H \mathbf{W}_i & \hat{h}_i^H \mathbf{W}_i \hat{h}_i - z - \lambda_i \end{bmatrix} \succeq 0, \qquad \forall i \in S_l,$$
$$\Upsilon_t = \begin{bmatrix} -\sum_{t \in S_o} \sum_{i \in S_l} \mu_t \mathbf{W}_i + \lambda_t Q_t & -\sum_{t \in S_o} \sum_{i \in S_l} \mu_t \mathbf{W}_i \, \hat{g}_t \\ -\sum_{t \in S_o} \sum_{i \in S_l} \mu_t \, \hat{g}_t^H \mathbf{W}_i & -\sum_{t \in S_o} \sum_{i \in S_l} \mu_t \, \hat{g}_t^H \mathbf{W}_i \hat{g}_t + K - \lambda_t \end{bmatrix} \succeq 0, \quad \forall t \in S_0, \tag{16}$$

the desired user in the presence of imperfect CSI. Fig.2, Fig.3 and Fig.4 illustrates the plot of target power level at BS versus total capacity of the desired user.

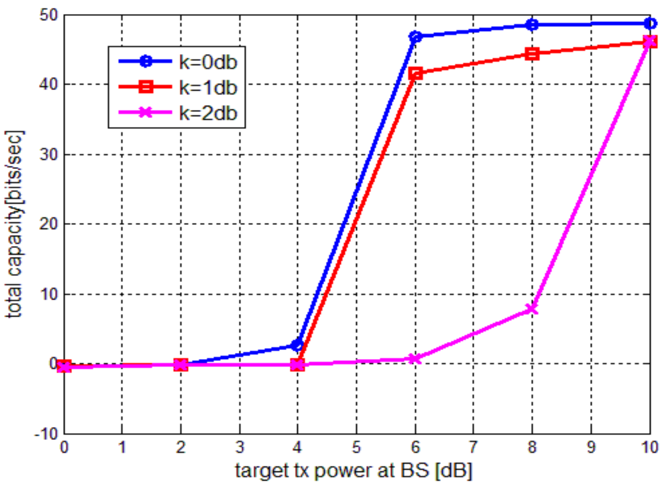

Fig. 2. Total capacity versus targeted transmit power values in 7-cell scenario with one user per cell and 4 antenna elements per BS for error radius, ε=0.05

In Fig.2 we compare the capacity of desired user with different upper bounds for interference power (K) induced on the other user by varying the total power at BS in seven cell scenario with one user per cell and an error radius ε=0.05. In Fig.3 and Fig.4, we perform similar analysis which has

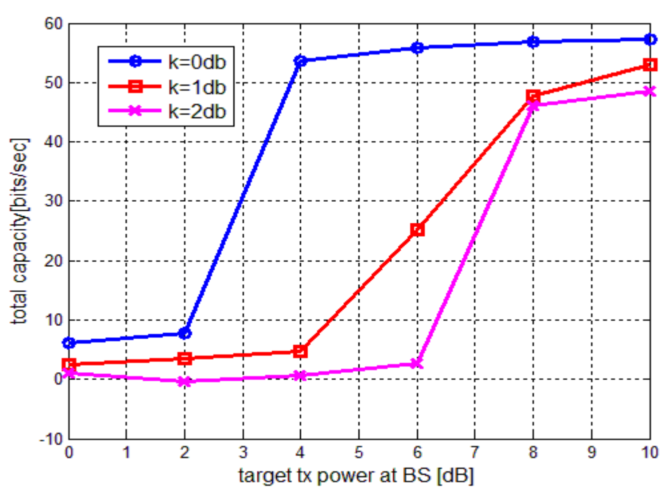

Fig. 3. Total capacity versus targeted transmit power values in 7-cell scenario with one user per cell and 4 antenna elements per BS for error radius, ε=0.1

been done in Fig.2, but with different error radius ε=0.1 and ε =0.5 respectively. From Fig.2, Fig.3 and Fig.4, it can be observed as upper bound for interference power increases total capacity decreases. Hence the upper bound for the interference power must be as minimum as possible. But for particular minimum upper bound for interference power, the capacity will be saturated. Hence from the results it can be observed

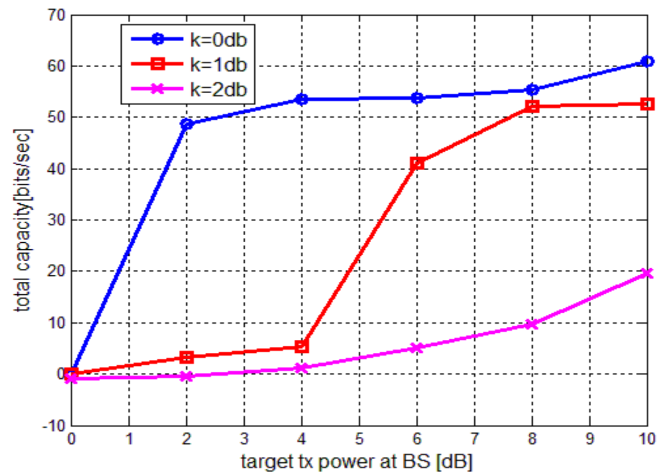

Fig. 4. Total capacity versus targeted transmit power values in 7-cell scenario with one user per cell and 4 antenna elements per BS for error radius, ε=0.5

that solution to the proposed problem gives high capacity and power efficiency for a cellular network.

V. Conclusion

This paper Maximizes the capacity of the cellular network by increasing the received signal power and reducing the inter-cell interference induced from other cell BSs at every user in the network while satisfying the transmit power constraint in the presence of imperfect CSI. We formulated an optimization problem for improving the capacity of the desired user and later we modify our original non convex problem in to a tractable formulation with convex constraints and LMI. We showed that reformulated problem can be solved easily by using semi definite relaxation. Simulation results have shown that minimum upper bound for interference power gives more capacity for the cellular network and at some minimum upper bound capacity will be saturated. We also observed that higher value for error radius demands more robust system.

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