Two Stage spectrum Sensing for Cognitive Radio using CMME

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***Abstract*—Spectrum sensing is fundamental functionality in Cognitive Radio (CR) to identify spectral white spaces for opportunistic communication. Around last decade thorough study of spectrum sensing suggests that traditional energy detection (ED) performance is better and fast at high SNR and worst in low SNR, whereas cyclo-stationary and Eigenvalue based methods perform better at low SNR, but implementation complexity is high. As suggested in IEEE 802.22 standard for CR, we propose in this paper, novel two stage spectrum sensing based on energy detection as coarse sensing first stage and combination of maximum-minimum eigen value based detection technique (CMME) as fine sensing second stage. The dual stage threshold parameter is designed to maximize the probability of detection for given constraints on the probability of false alarm. Comparative analysis of single stage ED, Cyclostationary detection (CSD), Combination of maximum-minimum eigen value detection (CMME), Combination of Cyclostationary detection and ED detection and Combination of CMME and ED detection is performed using probability detection versus SNR curve.**

Keywords- Cognitive radio, CMME, energy detection, probability of detection, Two-stage spectrum sensing

I. INTRODUCTION

The inefficient usage of the electromagnetic spectrum and the ever increasing demand for more and more spectrum for wireless communication has led to an extreme scarcity of available spectrum [1]. Recent measurement analysis of spectrum occupancy by various regulatory organisations in different countries suggests that licensed spectrum is underutilized in different time and geographical locations [2]. To overcome the problem of low spectrum utilization, Cognitive Radio (CR) concept given by Dr. Mitola has emerged as a good solution. CR by sensing and adapting to environmental conditions is able to detect or sniff the free spectrum availability in licensed band of primary user (PU) and intelligently allocate unlicensed or secondary users(SU) without interfering to PUs [1]. The critical functionality of CR is to sense the wide spectral bandwidth continuously. Much research is carried on spectrum sensing techniques, since various wireless technologies use different modulation techniques, data rate and bandwidth [3], so difficult to detect.

Many spectrum sensing techniques have been proposed in literature, that are included in emerging IEEE 802.22 Wireless Regional Area Network standard for CR are energy detection(ED), covariance based detection as blind technique, cylostationary detection technique (CSD) as semi-blind technique and Matched filter as non-blind technique [4] [5] [6]. ED is simple and fast technique, which do not require prior information about PU and works better in high SNR. But ED is not robust at low SNR and unable to differentiate between noise and signal. CSD is robust to low SNR, but requires that signal have statistical cyclostationary property that are inherent in all modulated signals and its mathematical complexity does not support hardware implementation [4]. Also sensing time that is critical for CR is high in CSD. Covariance based techniques depend on the presence of correlation between detected signals and these are blind techniques among them max-min eigenvalue based detection is more explored [7]. CMME requires large matrix manipulation, so sensing time required is bit high compared to ED. So for efficient sensing IEEE 802.22 standard prefers two stage sensing that is coarse sensing which covers large bandwidth and sensing time is low and fine sensing that concentrates on lower bandwidth and using very robust sensing techniques like CSD or Eigenvalue based techniques [6] [8].

In [9] discuss the two stage spectrum sensing in which, both stages uses the ED and divides bandwidth into coarse resolution and fine resolution that depends on mean sensing time. But increasing sensing time in ED does not have any effect under low SNR conditions. And [10] uses two stage sensing technique that uses ED in coarse sensing and CSD in fine sensing stage. In this paper, we propose a two-stage sensing approach based on energy detection and eigen value based detection. For the given channel; energy detection is performed in the first stage . If the energy is greater than the threshold λ, the channel is declared as occupied. Otherwise, in the second stage CMME detection is performed. If the decision metric exceeds the threshold γ_2 in the second stage, the channel is declared as occupied. Otherwise, it is declared as empty and available for secondary use. We are comparing with [10] two-stage spectrum sensing using ED and cyclostationay approach based on the probability of detection versus SNR. Also optimal thresholds λ and γ_2 for two stage plot based on mathematical analysis shown and analysed.

The rest of the paper is organized as follows. Following this introduction the remaining part of the paper is organized as follows. Section II describes the two stage spectrum sensing by explaining energy detection as coarse sensing and CMME as

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fine sensing. Section III presents the mathematical analysis for two stage sensing. Simulation results are presented in Section IV. The observation from Section IV is shown in Section V. Finally section VI concludes this paper.

II. TWO STAGE SPECTRUM SENSING SCHEME

Analysis for two stage sensing taken quite similar to [10] with assuming that, there are L channels to be sensed. In this scheme, the first stage i.e., coarse sensing stage the channel is tested by using energy detection technique. If the decision in coarse sensing (D^c) is greater than the threshold λ, then the channel is declared as occupied. Else the received signal is sensed by using the second stage i.e., Fine sensing stage by using CMME technique. If the decision in fine sensing (D^f) is greater than the threshold γ_2, then the channel is considered as occupied else it is empty. Proposed two stage sensing shown in Fig1:

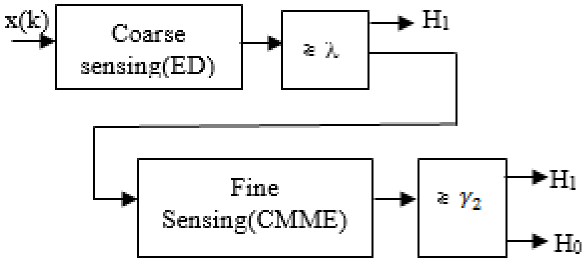

Fig. 1: Two-stage spectrum sensing scheme (ED and CMME detection technique)

A. Coarse Sensing

In coarse sensing stage, the signal is sensed by using energy detection technique [10]. The energy detector accumulates the energy (2) of M^c samples and compares it with the threshold λ to decide the presence or absence of primary user. Probability of presence of a primary user is denoted by H_1 and absence is denoted by H_0. The received signal in the first stage is given as, $x\left(k\right)$ where $k = 1, 2, ..., M^c$, the primary users signal $s\left(k\right)$ and receiver noise $n\left(k\right)$. The noise is assumed as an i.i.d. Random Gaussian process with variance σ_n^2 and mean zero, while the signal is assumed to be an i.i.d. Random process with variance and mean zero. OFDM is a key technology in most of the emerging technologies like WRAN, WiMAX, Long Term Evolution (LTE), Digital Video Broadcasting (DVB) etc. [11]. So in our analysis we use DVB-T 2K mode OFDM signal as primary signal [11].

$$x(k) = \begin{cases} n(k) & under\ H_0 \\ s(k) + n(k) & under\ H_1 \end{cases} \tag{1}$$

The decision rule for the energy detector is given as,

$$D^c = \sum_{k=1}^{M^c} x(k)^2 \begin{array}{c} H_1 \\ > \\ < \\ H_0 \end{array} \lambda \tag{2}$$

The test statistic D^c for large M^c can be modeled by a Gaussian distribution as follows [12] [10],

$$D^c = \begin{cases} N(M^c\sigma_n^2, 2M^c\sigma_n^4) & under\ H_0 \\ N(M^c(\sigma_n^2+\sigma_s^2), 2M^c(\sigma_n^2+\sigma_s^2)^2) & under\ H_1 \end{cases} \tag{3}$$

The probability of false alarm, P_f^c, and probability of detection, P_d^c, for the energy detector (first stage) are:

$$P_f^c = Q\left(\frac{\lambda - M^c\sigma_n^2}{\sqrt{2M^c\sigma_n^4}}\right) \tag{4}$$

$$P_d^c = Q\left(\frac{\lambda - M^c(\sigma_n^2+\sigma_s^2)}{\sqrt{2M^c(\sigma_n^2+\sigma_s^2)^2}}\right) \tag{5}$$

where $Q\left(a\right)$ is the Q-function.

B. Fine Sensing

If the condition $D^c > \lambda$ fails in the first stage i.e., ED stage then the signal is again sensed by using CMME detection technique. Since as mentioned previously due to SNR limiting condition, ED performance is very poor irrespective of increasing sensing time as preferred in IEEE 802.22 WRAN standard [6] [8]. Whereas CMME works better in low SNR with correlated signals without any prior information about primary signal and channel noise [7] [13] [14]. Assume that there are $K \geq 1$ secondary users. Then the received signal at the i_{th} secondary user is denoted by $x_i(k)(i = 1, 2, ..., K)$. Then the statistical matrix can be defined as:

$$\begin{aligned} x(k) &= [x_1(k), x_2(k), ..., x_K(k)]^T \\ s(k) &= [s_1(k), s_2(k), ..., s_K(k)]^T \\ n(k) &= [n_1(k), n_2(k), ..., n_K(k)]^T \end{aligned} \tag{6}$$

Where the received signal is given by $x\left(k\right)$, $(k = 1, 2, ..., N)$ where N is the number of samples in CMME technique. $s\left(k\right)$ is the transmitted signal passed through a wireless channel and $n\left(k\right)$ is the additive white Gaussian noise (AWGN) with mean zero and variance σ_n^2 .

According to the above definitions, (1) can be written as,

$$x = s + n \tag{7}$$

Considering the statistical covariance of the received signal, transmitted signal and noise signal as,

$$\begin{aligned} R_x &= E\left(xx^T\right) \\ R_s &= E\left(ss^T\right) \\ R_n &= E\left(nn^T\right) \end{aligned} \tag{8}$$

Let us consider N consecutive samples, then the statistical covariance matrices of the received signal, transmitted signal and noise signal becomes,

$$\begin{aligned} R_x(N) &= \tfrac{1}{N}xx^T \\ R_s(N) &= \tfrac{1}{N}ss^T \\ R_n(N) &= \tfrac{1}{N}nn^T \end{aligned} \tag{9}$$

Assuming that the noise is real. Let $A(N) = \frac{N}{\sigma_n^2}\ R_n(N)$, $\mu = \left(\sqrt{N-1}+\sqrt{K}\right)^2$, $\nu = \left(\sqrt{N-1}+\sqrt{K}\right)\left(\frac{1}{\sqrt{N-1}}+\frac{1}{\sqrt{K}}\right)^{1/3}$. Assume that $\lim_{N\to\infty}\frac{K}{N} = \alpha\,(0<\alpha<1)$. Then $\frac{\lambda_{max}(A(N))-\mu}{\nu}$ converges (with probability one) to the Tracy-Widom distribution of order 1 as mentioned in [7] [13] [15]. Assuming that the noise is complex. Let $A(N) = \frac{N}{\sigma_n^2}\ R_n(N)$, $\mu^{'} = \left(\sqrt{N}+\sqrt{K}\right)^2$, $\nu^{'} = \left(\sqrt{N}+\sqrt{K}\right)\left(\frac{1}{\sqrt{N}}+\frac{1}{\sqrt{K}}\right)^{1/3}$. Assume that $\lim_{N\to\infty}\frac{K}{N} = \alpha\,(0<\alpha<1)$. Then $\frac{\lambda_{max}(A(N))-\mu^{'}}{\nu^{'}}$ converges (with probability one) to the Tracy-Widom distribution of order 2 [13].

When the parameter N is large then, μ and $\mu^{'}$, ν and $\nu^{'}$ are nearly same, but their limit distribution is different [7]. From [14] which provides details about the tables for Tracy-Widom distribution function that are calculated by numerical computation. For example $F_1^{-1}(0.9) = 0.45, F_1^{-1}(0.5) = 0.98, F_1^{-1}(0.99) = 2.02$

λ_{max} and λ_{min} are the maximum and minimum eigen values of the received statistical covariance matrix $(R_x(N))$.

Algorithm [13] [6]

$$P_f^f = P\left(\lambda_{max} > \gamma^{'}\left(\lambda_{max}-\lambda_{min}\right)\right) \tag{10}$$

From above assumptions, we can get:

$$P_f^f = P\left(\frac{\sigma^2}{N}\lambda_{max}(A(N)) > \gamma^{'}\left(\lambda_{max}-\lambda_{min}\right)\right) \tag{11}$$

$$\approx P\left(\lambda_{max}(A(N)) > \gamma^{'}\left(\sqrt{N}-\sqrt{K}\right)^2\right) \tag{12}$$

$$= 1 - F_1\left(\frac{\gamma^{'}\left(\sqrt{N}-\sqrt{K}\right)^2-\mu}{\nu}\right) \tag{13}$$

$$P_d^f = Q\left(\frac{\gamma_2-\left(\frac{\lambda_{max}}{\lambda_{max}-\lambda_{min}}\right)}{\sqrt{4N\left(\frac{\lambda_{max}}{\lambda_{max}-\lambda_{min}}\right)}}\right) \tag{14}$$

Let

$$\gamma^{'} = \frac{\left(\sqrt{N}+\sqrt{K}\right)^2}{\left(\sqrt{N}-\sqrt{K}\right)^2}\left(1+\frac{\left(\sqrt{N}+\sqrt{K}\right)^{-2/3}}{(NK)^{1/6}}\right)\star \left(F_1^{-1}\left(1-P_f^f\right)\right) \tag{15}$$

The threshold 2 is: $\gamma_2 = \frac{\gamma^{'}+1}{\gamma^{'}} = 1+\frac{1}{\gamma^{'}}$.

Therefore, the judgment rule for CMME detection technique is:

$$D^f = \frac{\lambda_{max}}{\lambda_{max}-\lambda_{min}} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_2 \tag{16}$$

III. MATHEMATICAL ANALYSIS

The overall probabilities of false alarm and detection for a single channel are given by [10],

$$P_f = P_f^c\left(1-P_f^c\right)P_f^f \tag{17}$$

$$P_d = P_d^c\left(1-P_d^c\right)P_d^f \tag{18}$$

The objective is to design a decision strategy (determination of λ and γ_2) in order to maximize the probability of detection of each channel subject to a false alarm rate constraint. Therefore the corresponding problem is given by,

$$\max_{(\lambda,\gamma_2)} P_d(\lambda,\gamma_2) \qquad s.t. P_f \le \beta \tag{19}$$

The inequality constraint in the problem (19) can be reduced to an equality constraint by using the theorem in [10].

Theorem: The optimal value of the probability of detection in (19) is attained by $P_f = \beta$. Proof for the following is analysed similar to the [10] since P_d consists of Tracy-widom distribution function which is similar to Gaussian function (differentiable) and Q-function (differentiable). Hence P_d is differentiable and decreasing function of the thresholds λ and γ_2. Thus, the first derivative of P_d w.r.t λ and γ_2 is negative. Hence, the maximum P_d is attained for the lowest possible λ and γ_2. Same analysis holds for P_f. Assuming (λ^*, γ_2^*) to be the optimal solution of (19) corresponding to $P_f < \beta$. With keeping either of them λ^* or γ_2^* constant and varying one of them produce the better solution than as assumed. Therefore, (λ^*, γ_2^*) cannot be the optimal solution of the problem. Thus, the optimal P_d is attained by $P_f = \beta$. Hence, it is rewritten as similar in [10],

$$\max_{(\lambda,\gamma_2)} P_d(\lambda,\gamma_2) \qquad s.t. P_f = \beta \tag{20}$$

Solving (17),

$$\beta = P_f^c\left(1-P_f^c\right)P_f^f \tag{21}$$

Substituting (4) and (13) in (21), we get,

$$\lambda = f\left(\gamma_2\right) = Q^{-1}\left(\frac{\beta-\left(1-F_1\left(\frac{\gamma^{'}\left(\sqrt{N}-\sqrt{K}\right)^2-\mu}{\nu}\right)\right)}{1-\left(1-F_1\left(\frac{\gamma^{'}\left(\sqrt{N}-\sqrt{K}\right)^2-\mu}{\nu}\right)\right)}\right)\star \sqrt{2M^c\sigma_n^4}+M^c\sigma_n^2 \tag{22}$$

since $\gamma^{'} = \frac{1}{\gamma_2-1}$.

Therefore, the problem (20) can be simplified to an unconstrained problem as follows :

$$\max_{\gamma_2} P_d(f(\gamma_2), \gamma_2) \tag{23}$$

The optimal γ_2 and $\lambda = f(\gamma_2)$ can be obtained from (23) and (22). So the problem is unimodal in γ_2 and the optimal value of γ_2 is calculated at each SNR which gives the maximum probability of detection (P_d). The final optimal equation (P_d) which is the function of γ_2 can be evaluated in MATLAB.

IV. SIMULATION RESULTS

In these simulations, OFDM signal 2k mode of DVB-T standard is taken for the analysis. The transmitted OFDM signal is organised in frames. Each frame consists of 68 OFDM symbols with 1705 sub-carriers in 2k mode and transmitted with a symbol duration of T_s =244 μsec [16].

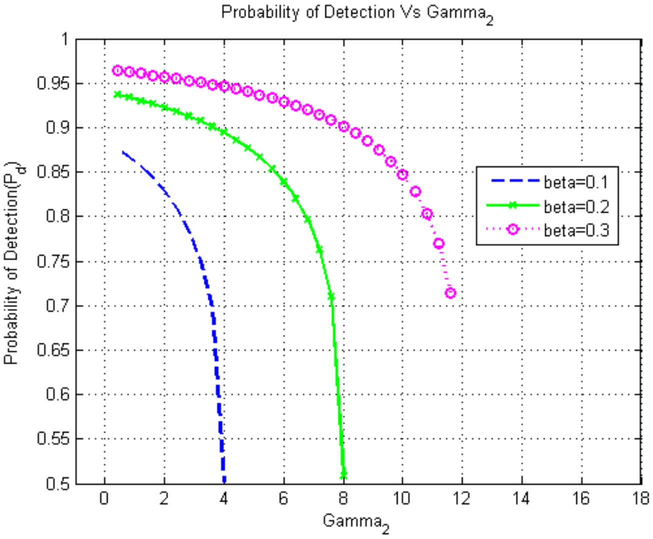

Fig. 2: Probability of Detection versus γ_2

From Fig. 2 we observe that probability of detection variation w.r.t γ_2 for different values of β for two-stage sensing. It is clear from the Fig. 2 that the maximum probability of detection is attained when the probability of false alarm satisfies constraint (19) with equality as shown in Theorem.

Fig. 3 shows the performance of probability of detection(P_d) vs SNR of all the three spectrum sensing techniques. From Fig. 3, we observe that for the SNR less than -24dB the two-stage spectrum sensing scheme performs better than the Energy detection technique or CMME detection technique.

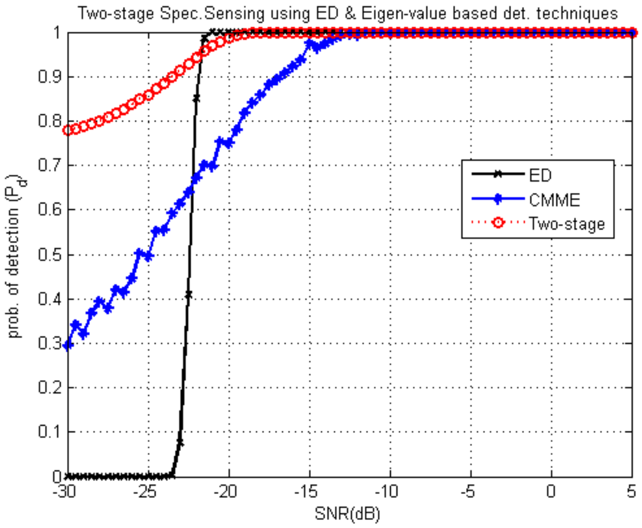

Fig. 3: Probability of Detection versus SNR

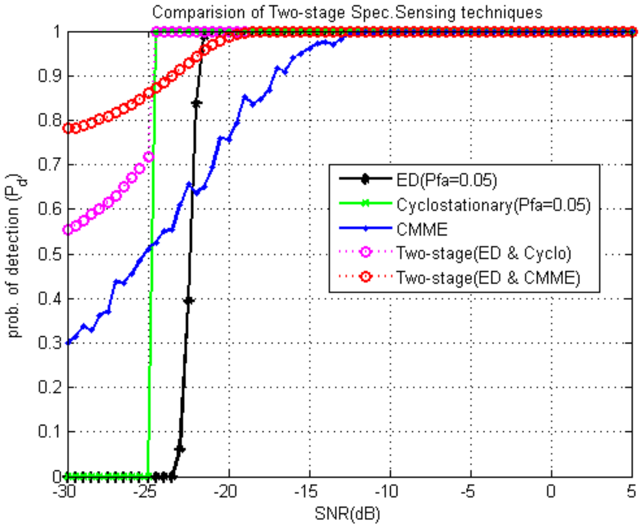

Fig. 4: Probability of Detection versus SNR

From Fig.4 , we can observe the probability of detection (P_d) vs SNR of the spectrum sensing techniques (Energy Detection (ED), Cyclostationary based detection technique, CMME detection, Two-stage spectrum sensing using ED and Cyclostationary based detection technique [10], Two-stage spectrum sensing using ED and CMME). As we can see that the probability of detection (P_d) of Two-stage spectrum sensing using ED and CMME giving better performance compared to Two-stage spectrum sensing using ED and Cyclostationary based detection technique.

On all the three sensing schemes it is assumed that the Probability of false alarm constraint, β and SNR is same for all the channels.

V. OBSERVATION

From TABLE I we can observe that the probability of detection (P_d) is better for Two-stage spectrum sensing schemes compared to individual detection techniques in both the cases. The probability of detection (P_d) performance of two-stage spectrum sensing using ED and CMME is better compared to two-stage spectrum sensing using ED and CSD [10].

SNR (dB)	ED ($P_d{}^c$)	CSD ($P_d{}^f$)	CMME ($P_d{}^f$)	Two-stage spec.sen. (ED-Cyclo) (P_d)	Two-stage spec.sen. (ED-CMME) (P_d)
-30	0	0	0.3	0.57	0.79
-23	0.1	1	0.62	1	0.92
-20	1	1	0.78	1	1

TABLE I: Comparison of Prob. of detection P_d of spec.sen. tech's with SNR

VI. CONCLUSION

Two-stage sensing schemes are analysed in terms of its detection performance. In particular, we designed optimum thresholds for the energy detection, cyclostationary based detection and combination of maximum-minimum eigen value detection (CMME) stages so as to maximize the probability of detection with constraints on the probability of false alarm. We considered DVB-T 2k mode OFDM signal for our analysis and compared our two-stage sensing schemes and observed the probability of detection performance (P_d). We observed that at low SNR, where the energy detector's performance is poor, the two-stage sensing schemes provides better detection. Performance analysis of two stage spectrum sensing technique based on timing need to be carried out and further CMME can be used to estimate the noise variance and fed it back to ED to enhance the performance of coarse sensing. Indirectly making the dual stage as full blind and self-adaptive. Also, detection performance needs to be checked using real-time measured data.

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