Distributed Incremental Leaky LMS

M. Sowjanya, A.K. sahoo and Sananda Kumar

***Abstract*— Adaptive algorithms are applied to distributed networks to endow the network with adaptation capabilities. Incremental Strategy is the simplest mode of cooperation as it needs less amount of communication between the nodes. The adaptive incremental strategy which is developed for distributed networks using LMS algorithm, suffers from drift problem, where the parameter estimate will go unbounded in non ideal or practical implementations. Drift problem or divergence of the parameter estimate occurs due to continuous accumulation of quantization errors, finite precision errors and insufficient spectral excitation or ill conditioning of input sequence. They result in overflow and near singular auto correlation matrix, which provokes slow escape of parameter estimate to go unbound. The proposed method uses the Leaky LMS algorithm, which introduces a leakage factor in the update equation, and so prevents the weights to go unbounded by leaking energy out.**

***Index Terms*—Distributed processing, Finite precision effects, Incremental Leaky LMS, Numerical Stability, Parameter Drift.**

I. Introduction

WIRELESS Sensor networks play a very important role in day to day life due to its wide applications. Distributed processing deals with extraction of information from the local data received and processed by each sensor nodes that are randomly distributed at diverse locations [1][2]. The applications of distributed processing are measurement of temperature, humidity, pressure, wind direction and speed, illumination intensity, vibration intensity, sound intensity, power-line voltage, chemical concentrations, pollutant levels, monitoring a moving target in a region monitored by a collection of sensors deployed in the field and so on [3][4]. Consider a network with N nodes distributed in a geographic region, where the nodes will receive the noisy local data. Each node has to estimate the parameter of interest from their local data and the information received from the neighborhood nodes. The amount of communication between the nodes depends on the mode of cooperation between them. Various works have been done in this area from time to time. An adaptive distributed strategy is developed based on incremental techniques as in [3]. A distributed LMS algorithm that achieves asymptotically unbiased estimates via diffusion adaptation has been done in [5]. Incremental RLS adaptive networks with noisy links have been analyzed in [6].

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LMS is widely used due to its ease of implementation, but it suffers from drift problem [7] when implemented in finite precision environment [8]. It is vulnerable to ill conditioning or inadequate excitation of the input. The incremental adaptive solution using LMS (ILMS) algorithm [3] suffers from drift problem in non ideal situations [9]. Incremental mode of cooperation is considered in this paper, where each node communicates with its neighbor in a cyclic manner. Information flows in a sequential manner along the nodes and the nodes estimate the parameter of interest by using any adaptive algorithm as in [3][6]. In this paper we propose Incremental Leaky LMS (ILLMS) algorithm to overcome the drift problem of LMS. Leaky LMS is a modified version of LMS where small leakage is allowed in the update equation and it introduces some bias in the parameter estimate [5] [10]. The Eigen spread of Leaky LMS is lesser than LMS algorithm [11]. So leaky LMS converges faster than LMS in case of inputs with high Eigen spread, like colored input [11].

The paper is organized as follows. We introduce what is incremental LMS algorithm and the causes of weight drift problem in Section II. Section III briefly presents the proposed method Incremental Leaky LMS which handles the drift problem. The performance of the proposed method is evaluated in Section IV. Simulation results are discussed in section V, conclusions and future works are given in Section VI.

II. Weight Drift Problem With LMS

Conventional LMS algorithm is widely used due to its simplicity and ease of implementation. The optimization function or the cost function [3][10] is

$$\min_w \left[J(w) \triangleq E \left| d - u w \right|^2 \right] \tag{1}$$

Where E is the expectation operator. Solving the least mean squares criterion results in the weight update equation (2) [10].

$$w(k) = w(k-1) + \mu u(k) e(k) \tag{2}$$

Where $w(k)$ is the filter weight vector, μ is step size,

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$u(k)$ is the input sequence and

$$e(k) = d(k) - u^T(k)w(k-1)$$
$$d(k) = u^T(k)w_0 + v(k)$$

w_0 is the optimum solution.

The convergence and stability of LMS algorithm may be problematic in non-ideal conditions. In such cases the weight estimates don't converge to the optimum value and go unbounded, i.e. they diverge. The difficulties with LMS algorithm are discussed below:

A. Finite Precision Effects

In digital implementation of the adaptive filters, all the inputs and the intermediate values are quantized using an analog to digital converter. All the quantities are stored in registers whose word length is finite, so all the values are truncated to some precision. They result in finite precision errors which get accumulated with time and result in an overflow [7]. A real time system processes the quantization and finite precision errors in a very unstable manner.The noise or the finite precision arithmetic errors will become non zero mean variables due to these errors. We cannot just increase the precision as the cost of implementation is a strong function of the number of bits used in the digital implementation.

B. Ill Conditioning of the input

The ill conditioning of the input refers to wide Eigen spread, where Eigen spread is the ratio of the largest eigen value to least eigen value of the auto correlation matrix of the input sequence. It results in near singular auto correlation matrix, which makes the estimate to escape slowly from the expected value to infinity. Even though all the other signals are finite, the parameter estimate will go unbounded due to inadequacy of excitation [8].

C. Numerical Stability:

An algorithm is said to be numerically stable if it limits the maximum deviation from the infinite precision implementation, whereas a numerically unstable algorithm allows the errors to accumulate with time, which results in divergence of the estimate [7]. Increasing the precision does not affect the numerical stability, which is a strong function of the algorithm. Numerically stable algorithm keeps track of the finite precision errors and corrects itself.

D. Accuracy:

Accuracy of an algorithm implementation is the magnitude of the deviation from the infinite precision performance. The smaller deviation indicates more accuracy [8]. Accuracy is a strong function of the number of bits used for storage i.e. precision.

III. PROPOSED FRAMEWORK

LMS algorithm suffers from numerical instability, accuracy affects, finite precision effects and is sensitive to ill-conditioning of the input sequence [7][8]. The incremental adaptive solution provided for parameter estimation in distributed networks using LMS algorithm will result in unbounded parameter estimate in practical implementation [10]. Leaky LMS solves the drift problem [5] and this paper deals with implementation of Incremental Leaky LMS for parameter estimate in distributed networks. Leaky LMS is modified version of the LMS algorithm and its optimization equation is

$$\min_w \left[J^{\alpha}(w) \triangleq \alpha \|w\|^2 + E|d - uw|^2 \right] \tag{3}$$

Where α positive real number $0<\alpha<1$

The weight update equation for the Leaky LMS algorithm [12] is

$$w(k) = (1 - \alpha\mu)w(k-1) + \mu\alpha u(k)e(k) \tag{4}$$

The subsequent analysis follows the following data model and the assumptions done for the algorithm implementation and performance analyses are listed below:

A. Data Model and Assumptions:

1. The desired unknown vector w_0 relates $\{u_{k,i}, d_k(i)\}$ as $d_k(i) = u_{k,i}w_0 + v_k(i)$.
2. Where $v_k(i)$ is white Gaussian noise with variance $\sigma^2_{v,k}$ and independent of $\{u_{k,i}, d_k(i)\}$ for all i, j.
3. Input sequence $u_{k,i}$ is spatial and time independent.
4. The input is corrupt with white Gaussian noise $n_{k,i}$, with zero mean and $\sigma^2_{n,k}$ variance

$$z_{k,i} = u_{k,i} + n_{k,i}$$

Algorithm for Incremental Leaky LMS Solution

Let $\psi_k^{(i)}$ denote a local estimate of w_0 at node k at time i.
Start with $w_{-1} = 0$
For each time $i \geq 0$, repeat :
Set $\psi_0^{(i)} = w_{i-1}$
  For nodes $k = 1$ to N , repeat :
  Receive $\psi_{k-1}^{(i)}$ from previous node
    $\psi_k^{(i)} = (1 - \mu_k\alpha)\psi_{k-1}^{(i)} + \mu_k\alpha z_{k,i}^*(d_k(i) - u_{k,i}\psi_{k-1}^{(i)})$
    $k = 1, 2, ..., N$
  End
$w_i = \psi_N^{(i)}$
Send w_i to node 1
End

Fig. 1 shows the data flow and the weight updation in Incremental Leaky LMS strategy in a distributed network with N nodes.

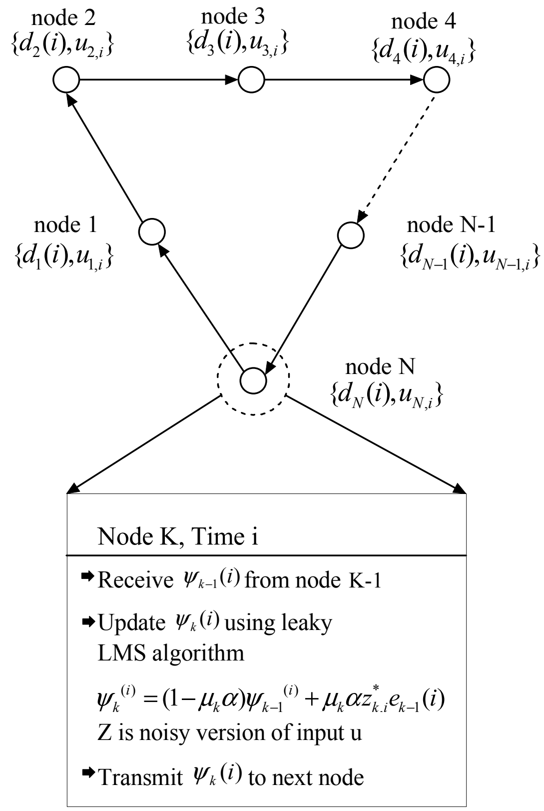


Fig. 1. Data Flow and updation in Incremental leaky LMS

IV. Performance Analysis

A. Solution to Weight Drift Problem:

In this section an example is given to illustrate the weight drift problem occurring in LMS update equation and how leaky LMS solves the problem [13]. Let's consider that the input vector $u(k)$ is orthogonal to weight error vector

$$\tilde{w}(k) = w_0 - w(k-1).$$

Now the LMS update equation is

$$w(k) = w(k-1) + \mu u(k) e(k)$$

Where $e(k) = d(k) - u^T(k) w(k-1)$

$$d(k) = u^T(k) w_0 + v(k)$$

The weight error vector satisfies the equation

$$\tilde{w}(k) = \tilde{w}(k-1) - \mu u(k) v(k)$$

Taking norm on both sides the above equation can be written as

$$\left\|\tilde{w}(k)\right\|^2 = \left\|\tilde{w}(k-1)\right\|^2 + \mu^2 \left\|u(k)\right\|^2 v^2(k)$$

Solving this recursion for $\tilde{w}(N)$ we get

$$\left\|\tilde{w}(N)\right\|^2 = \sum_{K=1}^{N} \mu^2 \left\|u(k)\right\|^2 v^2(k) + \left\|\tilde{w}(0)\right\|^2$$

From above equation,it is obvious that $\left\|\tilde{w}(N)\right\|^2 \to \infty$ with N if $\mu \| u(k) \| v(k)$ is a power sequence or not a finite energy sequence.This situation doesn't occur when Leaky LMS is used, as the weight update equation contains leakage factor. This results in energy leakage and thus assuring stability.

B. Convergence in the Mean

The Leaky LMS update equation is

$$w(k) = (1-\alpha\mu) w(k-1) + \mu\alpha u(k) e(k)$$

The weight error vector satisfies the equation

$$\tilde{w}(k) = (1-\mu\alpha)\tilde{w}(k-1) + \mu\alpha w_0 - \mu\alpha u(k) e(k)$$

Taking expectation on both sides the above equation

$$E\left[\tilde{w}(k)\right] = (1-\mu\alpha) E\left[\tilde{w}(k-1)\right] + \mu\alpha w_0 - \mu\alpha E\left[u(\mathrm{k}) e(\mathrm{k})\right]$$

$$E\left[\tilde{w}(k)\right] = \left[1 - \mu\alpha\left[I + R_u\right]\right] E\left[\tilde{w}(k-1)\right] + \mu\alpha w_0$$

The mean $E\left[\tilde{w}(k)\right]$ converges to zero, and consequently $E\left[w(k)\right]$ converges to w_0 if and only if $0 \langle \mu \langle \frac{2}{\alpha + \alpha\lambda_{\max}}$

Where $\lambda_{\max}$ is the largest Eigen value of the matrix $R_u = E[u(n)u(n)^T]$. The second term $\mu\alpha w_0$ in the equation signifies the bias, which is the cost paid to assure stability by using Leaky LMS. In other words Leaky LMS is convergent in mean, if the stability condition [14] is met.

V. Simulation Results And Discussion

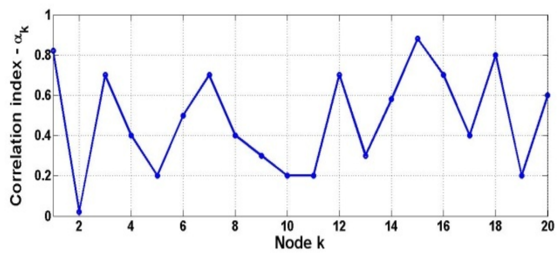
The weight drift problem caused due to non ideal (practical) conditions is solved by applying leakage to the weight update equation in the Incremental LMS algorithm. The sensitivity to inadequate spectral excitation is solved by Leaky LMS due to the presence of leakage factor, which prevents auto correlation matrix to be near singular in worst cases. The stability condition for the convergence of mean of the parameter estimate is derived.

The computer simulation results are provided in this section by considering a case where Incremental LMS fails. As the non ideal conditions cannot be shown in simulations, the step size is considered at which the ILMS fails. A network with 20 nodes that are placed randomly in an area have been considered. All the simulations are carried out using regressors with shift structure to cope up with realistic situations. To generate performance curves 500 independent experiments are performed and averaged.The unknown weight vector to be estimated is an N×1 vector, set as w_0=col {1, 1, 1...M}/$\sqrt{M}$, where the tap size M is taken as 10. Input sequence is time correlated sequence and can be generated as

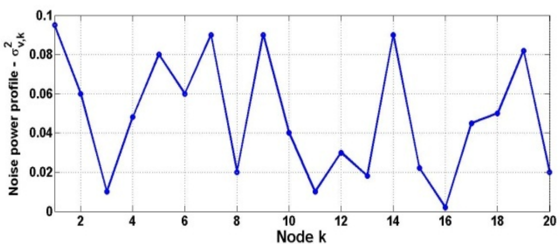
$$u_k(i) = \alpha_k u_k(i-1) + \beta_k Z(i) \quad , i > -\infty$$

Where $\beta_k = \sqrt{\sigma_{u,k}^2 (1-\alpha_k^2)}$

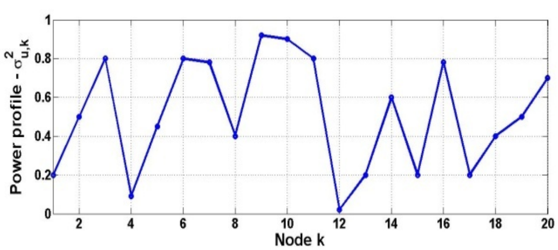
The input is considered to be corrupted with WGN with zero mean and standard deviation of 0.03. Correlation coefficient α_k, noise power $\sigma_{v,k}^2$, regressor power $\sigma_{u,k}^2$ at each node are taken randomly as shown in fig. 2.



(a)



(b)



(c)

Fig. 2. (a) Correlation Index, (b) Noise power profile, (c) Input signal power profile.

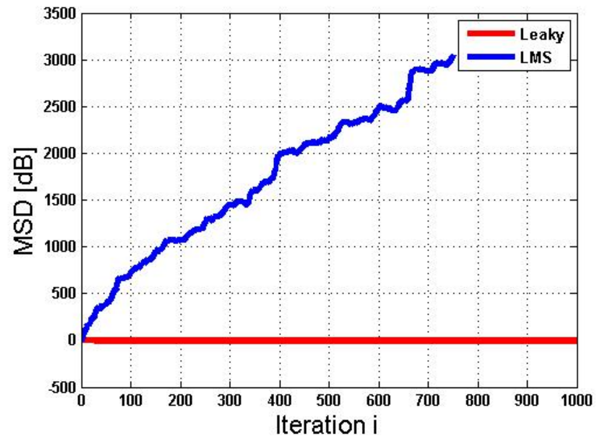
A. Learning curves at individual nodes:

Step size μ is taken 0.3 and leakage factor α is taken 0.001 for all nodes. The Performance metrics considered here are

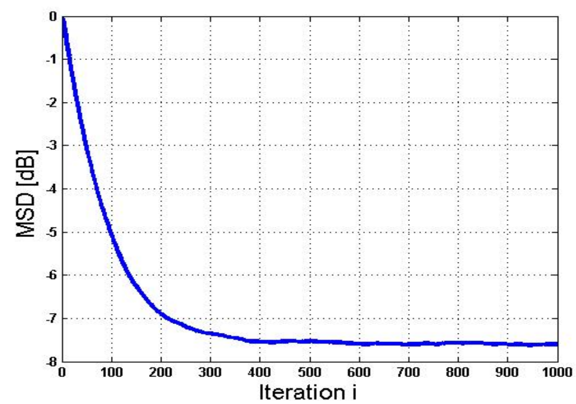
Mean Square Error (MSE): $E\left[e^2(k)\right]$

Mean Square Deviation (MSD): $E\left[\widetilde{w}(k)^2\right]$

In Fig. 3(a) the mean square deviation for ILMS and ILLMS are observed at node 4 in the distributed network that we have considered. At this given step size incremental LMS fails and the weights have gone unbounded, whereas the Incremental Leaky LMS have overcome the drift problem and the parameter converges as shown in Fig. 3(b).



(a)

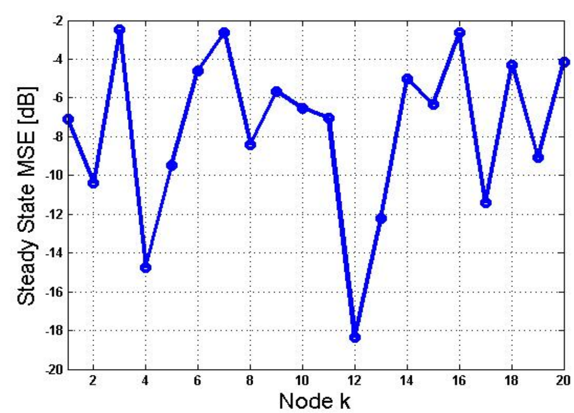


(b)

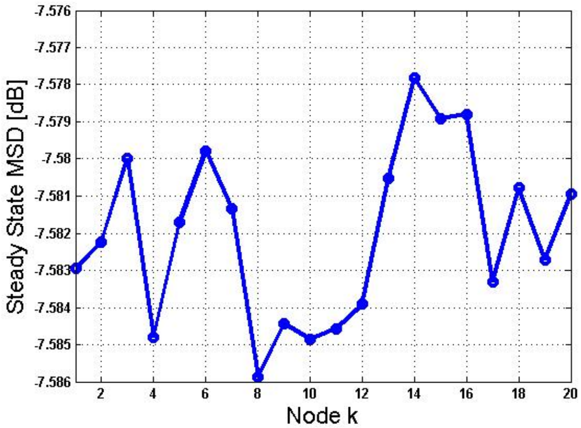
Fig. 3. (a) Comparing MSD for ILMS and ILLMS and ILLMS at node 4, (b) MSD at node 4 for Incremental LLMS.

B. Steady State performance:

The steady state curves are generated by running the learning process for 1000 iterations and averaging the last 500 samples.



(a)



(b)

Fig. 4. Steady state performance (a) Steady State MSE, (b) Steady State MSD

In Fig. 4, the steady state MSE and MSD are observed at each node The MSE roughly reflects background noise at all nodes. Despite of the diverse statistic profile, the MSD is almost uniform at all nodes with a deviation of ± 0.01 dB, which indicates good performance of the network.

VI. CONCLUSION

This paper presents an efficient approach to tackle the drift problem that arises in distributed incremental LMS approach due to finite precision effects, quantization errors, inadequate or ill-conditioned inputs by implementing Leaky LMS algorithm for distributed processing using incremental strategy. Incremental Leaky LMS solves the drift problem, by introducing leakage factor which results in the energy leakage and thus prevent weights to go unbounded. But it introduces bias in the mean value of parameter estimate i.e. it will not converge to optimum value. The proposed method has been compared with existing Incremental LMS, which shows that Incremental Leaky LMS algorithm is numerically stable. The simulation results show that the MSD is even over the network, which indicates good performance of the network.

The work can be extended to diffusion and probabilistic diffusion cooperation strategies where communication burden is more. Block Incremental Leaky LMS can be implemented to distributed networks, which reduces the computational complexity while assure stability for all practical cases.

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