A Novel Fuzzy based Adaptive Control of the Four Tank System

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Abstract- **The paper has analyzed the four tank system (FTS) from the mechanism modeling and has also established the nonlinear and linear mathematical model. The FTS is a typical control system with nonlinear, coupling and time delay characteristics which can be used to test the applications of different control algorithms on complex systems. The aim of the process is to keep the liquid level in the tanks at the desired values. Here, Fuzzy Modified Model Reference Adaptive Control (FMMRAC) is proposed and is applied to the tank system to test its effectiveness. The response of the FMMRAC controller is verified and is compared with other control algorithms through simulation. The performance of the closed loop system is simulated using LabVIEW software. The method used here is validated using the simulated results.**

Keywords: **Four Tank System, Proportional-integral-derivative controller, Fuzzy system, Model Reference Adaptive Control**

I. INTRODUCTION

The four tank system (FTS) is a typical control system with nonlinear, coupling and time delay characteristics. The control of liquid level in tanks and flow between tanks is a basic problem in the process industries. The process industries require liquids to be pumped as well as stored in tanks and then pumped to another tank. Many times, the liquids are processed by chemical or mixing treatment [1 - 2] in the tanks, but always the level of fluid in the tanks must be controlled and the flow between tanks must be regulated.

Several researchers have investigated the problem of controlling the liquid level of a single or multiple tanks in recent years. Johanson [3] has derived the mathematical model of the quadruple tank system for multivariable laboratory process and decentralized proportional integral (PI) controller is applied for both minimum and non-minimum phase systems. A constrained predictive control algorithm based on feedback linearization employed to a coupled tank apparatus has described in [4]. Intelligent controls including fuzzy logic (FL) [5-6], neural network (NN) control [7-8], and genetic algorithms (GA) [9] have also been applied to the coupled tanks system. Zumberge and Passino [10] have reported the comparison between conventional control and intelligent control applied to the process control. The controller must have the capability to adapt to changes in plant dynamics. Different adaptive control methods like Model Reference Adaptive Control (MRAC), Gain Scheduling, and Self Tuning Regulator (STR) can be used to improve the transient performance of a controller which are described in [9], [11]. The drawback of Gain scheduling method is that it is used an open loop compensation technique. The STR determines the parameters of the process to make suitable for the dynamic changes of the controller. The problem of this method is that when the model error changes slightly, it can lead to large changes in parameters which results oscillation in the process variables. Tsai et al. [11] have been controlled the temperature in an oil-cooling machine using MRAC method.

Liu and Hsu [12] have proposed the adaptive back stepping control and MRAC method for improvement of the performance in case of a sensor-less direct torque control synchronous reluctance motor drive system. Miller and Mansouri [13] have described a method where estimation and control is performed for effective performance of noise in MRAC. Linear predictive control has used in this method. Improvement in the transient performance of the conventional MRAC has been the point of research for quite some time. A modified MRAC has been proposed by Datta and Ioannou [14] which show the improved steady-state as well as transient performance.

The methods like NN, FL, GA, their hybrid combinations and other evolutionary algorithms like Ant Colony, Particle Swarm Optimization (PSO), and Bacterial Foraging can also be applied for the control of the parameter of the process. The hybrid tank system is highly a nonlinear system. Mohideen [9] has proposed a controller called Modified MRAC. This method has given very good transient as well as steady-state performance when controlling the liquid level in the four tank system. One of the most effective ways to solve the control problem is to use the technique of intelligent control system or hybrid technology of the conventional and intelligent control techniques. Fuzzy logic controller is one of the intelligent controllers and is a logical model of the human behavior of the process operator [10]. The fuzzy controller gives the better performance than those of the conventional controllers in terms of settling time, response time, overshoot and robustness.

The tank level control is a typical representative of the process control. The control accuracy of the liquid level system is affected by the system status, system parameters and the control algorithm. In this paper, Fuzzy Modified MRAC (FMMRAC) controller is used where the fuzzy controller is used to tune the parameters K_p, K_i and K_d of the PID controller. Fuzzy control is used to make the system fast and stable, but the steady-state error still exists. However, the conventional PID controller is the characteristics of high accuracy and eliminates the steady-state error. Therefore, the fuzzy controller is used to tune the parameters of the PID controller for getting the better performance of the FTS.

This paper is organized as follows: Section II describes the general formulation of the FTS model. In Section III, adaptive controller is described. Section IV shows the simulation results and describes about the responses of the different controllers and compares the characteristics of the process. Finally, the conclusion is given in Section V.

II. GENERAL FORMULATION OF THE SYSTEM MODEL

The schematic of the four tank system has proposed by Johansson [3] as shown in Fig. 1. This system represents dynamics of multivariable interaction, because each pump influences the two outputs. The system has an adjustable multivariable zero that can be set to a right half or left half plane by changing the valve settings of the system. The analysis of FTS uses fluid mechanics theory to analyze the system and establishes the system model based on the nonlinear mechanism. The aim is to control the levels in the lower tanks with two centrifugal pumps. The system has two inputs and two outputs. There is a reservoir under the tanks to accumulate the outgoing water from tank 1 and tank 2. The process inputs are v_1 and v_2 (input voltages to the pumps) and the outputs are y_1 and y_2 (voltages from the level measurement devices). In Fig. 1, h_i is the level of liquid in tank i where i =1, ..., 4.

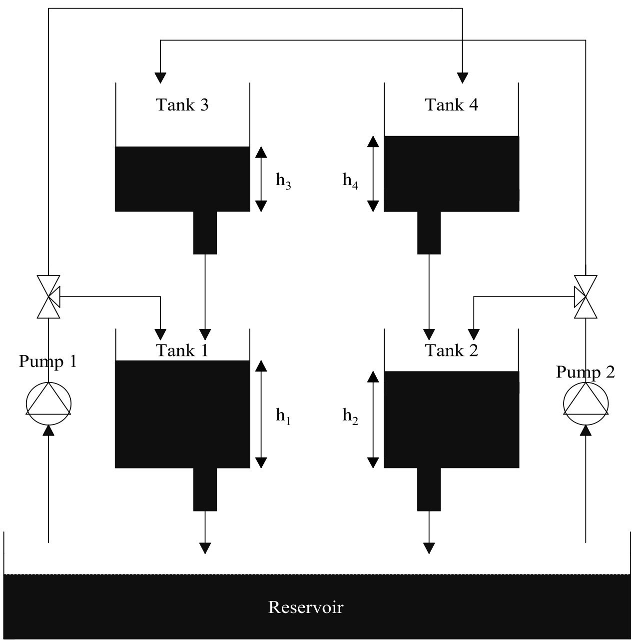


Fig. 1: Block diagram of Four Tank System

Bernoulli's law is used for flows out of the tanks. The nonlinear model of the FTS is obtained by using Mass balance equation and Bernoulli's law which are shown in Eq.s (1) - (4) as

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_a k_1}{A_1}v_1 \qquad (1)$$

$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_b k_2}{A_2}v_2 \qquad (2)$$

$$\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_b)k_2}{A_3}v_2 \qquad (3)$$

$$\frac{dh_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_a)k_1}{A_4}v_1 \qquad (4)$$

where

A_i = Area of cross-section of Tank i, i =1, ..., 4
a_i = Area of cross-section of outlet hole,
h_i = Level of liquid in tanks

The voltage applied to pump i is v_i and the corresponding flow is k_iv_i. The parameters (γ_a, γ_b) are determined from the valve settings of the system. It can be shown that a multivariable right half plane zero will be present when ($\gamma_a + \gamma_b$) < 1 for the nonlinear system. The flow to tank 1 is $\gamma_a k_1 v_1$ and the flow to tank 4 is $(1-\gamma_a)k_1v_1$. Similarly, the flow to tank 2 is $\gamma_b k_2 v_2$ and the flow to tank 3 is $(1-\gamma_b)k_2v_2$. The acceleration of gravity is denoted by g. The parameter values of the process are given in Table I [3].

Table I: Parameter values of the FTS model

Parameter	Value	Parameter	Value
a_1, a_2, a_3, a_4	2.3 cm^2	k_1	5.51 cm^3/s
A_1, A_2, A_3, A_4	730 cm^2	k_2	6.58 cm^3/s
$\bar{v}_1$	60%	γ_a	0.333
$\bar{v}_2$	60%	γ_b	0.307

The model and control of the FTS are studied at minimum-phase characteristics. The variables $H_i = h_i - \bar{h}_i$ and $u_i = v_i - \bar{v}_i$ are the deviation variables where $\bar{h}_i$ and $\bar{v}_i$ are the steady-state values of h_i and v_i, respectively. The linearized model equations for the FTS are

$$\frac{dH}{dt} = \begin{bmatrix} -\frac{C_1}{A_1} & 0 & \frac{C_3}{A_1} & 0 \\ 0 & -\frac{C_2}{A_2} & 0 & \frac{C_4}{A_2} \\ 0 & 0 & -\frac{C_3}{A_3} & 0 \\ 0 & 0 & 0 & -\frac{C_4}{A_4} \end{bmatrix} H + \begin{bmatrix} \frac{\gamma_a k_1}{A_1} & 0 \\ 0 & \frac{\gamma_b k_2}{A_2} \\ 0 & \frac{(1-\gamma_b)k_2}{A_3} \\ \frac{(1-\gamma_a)k_1}{A_4} & 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} H \tag{5}$$

where

$$C_i = a_i \sqrt{\frac{g}{2\bar{h}_i}}, \qquad i = 1, ..., 4$$

The Eq. (5) can be written as

$$\frac{dH}{dt} = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1 T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2 T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} H + \begin{bmatrix} \frac{\gamma_a k_1}{A_1} & 0 \\ 0 & \frac{\gamma_b k_2}{A_2} \\ 0 & \frac{(1-\gamma_b) k_2}{A_3} \\ \frac{(1-\gamma_a) k_1}{A_4} & 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} H \tag{6}$$

where

$$T_i = \frac{A_i}{a_i} \sqrt{\frac{2\bar{h}_i}{g}}, \qquad i = 1, ..., 4$$

The inputs are pseudorandom binary sequences (PRBS) with low amplitudes, so that the dynamics are captured by a linear model. The model outputs match with the responses of the real process. The four tanks in the FTS are of Acrylic type. It has also four numbers of smart level transmitters (DPT) to sense the level of each tank. Two numbers of control valves are mounted in the mechanical rigid frame to control the flow rate of the water. The storage tank has the capacity of 75 liters. Centrifugal pumps are provided to circulate the water from the storage tank. Four numbers of rotameters are connected in the inlet of the process tank to visualize the flow rate which is (10-100) liters per hour (LPH). For simulating the FTS, its mathematical model [1] is necessary and has developed using Mass balance equation and Bernoulli's law which are shown in Eq.s (1) – (4). The system is designed according to the mathematical model. For developing the mathematical model for FTS, the density of liquid in the inlet, in the outlet and in the tank is assumed to be same.

III. ADAPTIVE CONTROLLER DESIGN

The use of adaptive controllers such as Model Reference Adaptive Controller (MRAC) and Self-Tuning Regulator (STR) are due to the nonlinear and non-stationary nature of the system. In this paper, MRAC strategy is employed due to the nonlinear nature of the level process. To design an MRAC with equally good transient as well as steady-state performance is a challenging task. The aim is to design an MRAC with very good steady-state and transient performance for a nonlinear process such as the hybrid tank process. In this case, it is assumed that the response of the reference model represents the set point of the process in a standard feedback loop. In MRAC, a reference model is used to adjust the regulator parameters. The reference model is a part of a control system. Adjustment of system parameters in an MRAC can be obtained in two ways:

a. Gradient Method (MIT Rule)
b. Lyapunov Stability Theory

A MRAC, which employs the famous MIT rule for tuning of the parameters is developed and applied is shown in Fig. 2. The controller has two adjustable parameters θ_1 and θ_2. The controller output u is calculated from these parameters, the command signal u_c and the output y of the process as

$$u = \theta_1 u_c - \theta_2 y \tag{7}$$

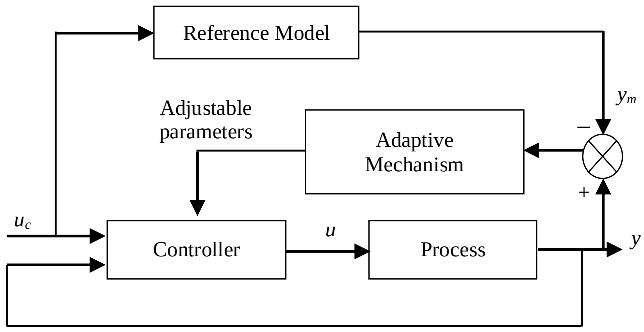


Fig. 2: Block diagram of Model Reference Adaptive Control

The change in the value of the adjustable parameters of the controller with respect to time as per the MIT rule is given as

$$\frac{d\theta_1}{dt} = -\gamma_1 e \frac{\partial e}{\partial \theta_1}$$

and

$$\frac{d\theta_2}{dt} = -\gamma_2 e \frac{\partial e}{\partial \theta_2} \tag{8}$$

where γ_1 and γ_2 are the adaptation gains for θ_1 and θ_2 respectively. The speed of convergence relies on the value of the adaptation gain. If the adaptation gain is too small, then it gives a stable response, but it requires a long time for the output to converge with the reference model. When the adaptation gain is too large, the output oscillates. Hence, there is always a trade-off needed between the stability and the speed of convergence while choosing the value of adaptation gain.

For the improvement of transient performance of the system, a modification can be done in the MRAC method. A PID controller is used in the MRAC method and it is called as Modified MRAC which is shown in Fig. 3. Thus, the output of the controller u in Modified MRAC controller is given as

$$u = \theta_1 u_c - \theta_2 y - \left(K_p e + K_i \int edt + K_d \frac{de}{dt} \right) \tag{9}$$

For getting the better performance in terms of settling time, rise time and mean square error (MSE), the fuzzy controller is used to tune the parameters K_p, K_i and K_d of the PID controller which are present in the Modified MRAC. The controller is called as Fuzzy Modified MRAC (FMMRAC) as shown in Fig. 4.

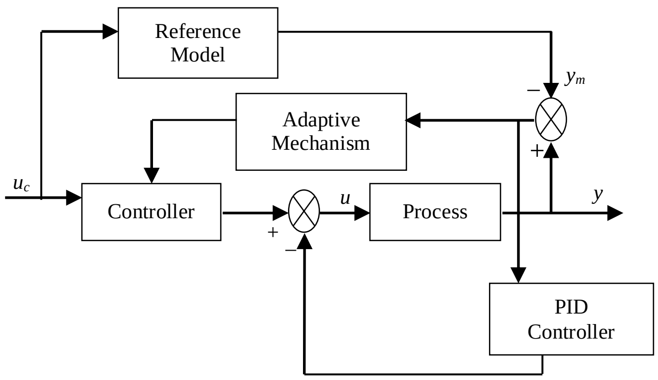


Fig. 3: Block diagram of Modified MRAC

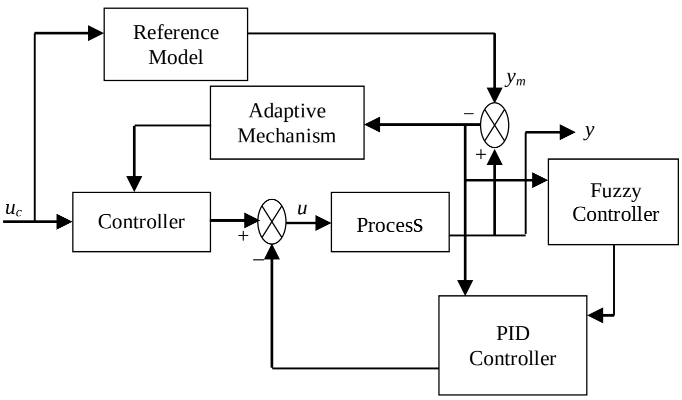


Fig. 4: Block diagram of Fuzzy Modified MRAC

Fuzzy parameters of the membership functions have been determined by using fuzzy system designer (FSD) in LabVIEW. In fuzzy algorithm, triangular-shaped built-in membership functions have been used. The fuzzy controller has two input variables. The error $e(t)$ is the first input variable and the second input variable is the differential of $e(t)$. The output (y) of fuzzy controller is the control signal of the actuator. The fuzzy linguistic variables with FSD tools described by five membership functions are shown in Fig. 5. The fuzzy logic controller (FLC) accepts the input variables, matches them up with the linguistic variables and determines the appropriate output corresponding to the input variables. The fuzzy rule base consists of a collection of fuzzy IF-THEN rules.

Membership functions with linguistic values are described as Negative Big (NB), Negative Medium (NM), Negative Small (NS), Zero (ZE), Positive Small (PS), Positive Medium (PM), Positive Big (PB) respectively. Linear type output membership functions have been used in fuzzy rule base which has 49 fuzzy rules.

The transfer function of the coupled tank system is given by

$$G_p(s) = \frac{0.0001626}{s^2 + 0.0446s + 0.0004834} \tag{10}$$

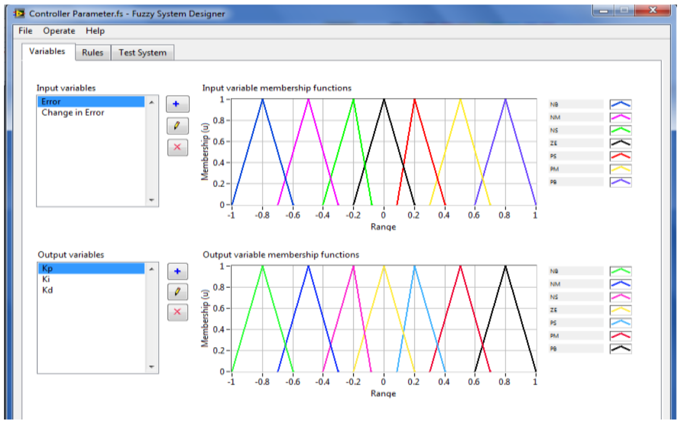


Fig. 5: Membership functions linguistic variables *e*, de/dt and *y*

The FTS setup in the coupled tank process is estimated as an over-damped second order system with zero delay. The damping factor is 1.0143 and the time constant of the process is 45.483 sec. The coupled tank process is more sluggish due to the interaction between the two tanks. Initially, the reference model of the system can be selected as the model of the FTS system without interaction. The transfer function of the reference models m_1 and m_2 are taken as

$$G_{m_1}(s) = \frac{0.00087}{s^2 + 0.477s + 0.000461} \tag{11}$$

$$G_{m_2}(s) = \frac{0.000153}{s^2 + 1.051s + 0.00051} \tag{12}$$

The adaptation rules for the MRAC parameters θ_1 and θ_2 are calculated as

$$\frac{d\theta_1}{dt} = -\gamma_1 e \frac{0.477s + 0.000461}{\left(s^2 + 0.477s + 0.000461\right)} u_c \tag{13}$$

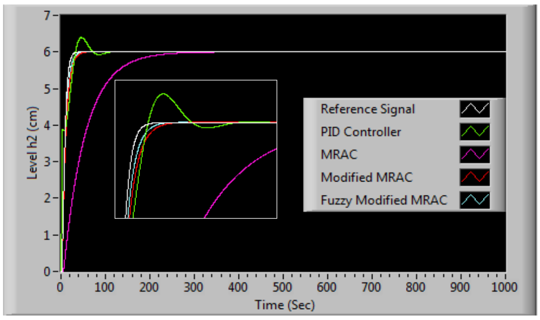
$$\frac{d\theta_2}{dt} = -\gamma_2 e \frac{1.051s + 0.00051}{\left(s^2 + 1.051s + 0.00051\right)} y \tag{14}$$

where γ_1 and γ_2 are the adaptation gains for θ_1 and θ_2 respectively.

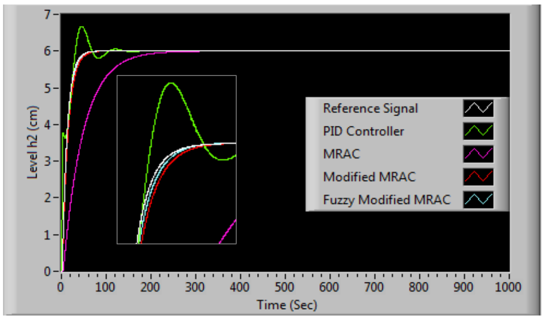
The adaptation gains are either too small or too large. The adaptation gains are chosen in such a way that the system is stable and the output response tracks the desired set point value. The values of adaptation gains used here are 0.00015 and 0.0038, respectively. When the adaptation gains are large, the output responses of the system have oscillations and overshoot. Hence, small values are used for the adaptation gains to get the zero overshoot.

IV. SIMULATIONS

The FTS can be simulated using LabVIEW software. The performance analysis of different controllers is examined by applying a step input of amplitude 6 to the command signal u_c and the responses are presented in Fig. 6. The step response of different controllers when G_{m1} is used as the reference model is shown in Fig. 6 (a). Similarly, Fig. 6 (b) shows the step response of different controllers when G_{m2} is used as the reference model. It consists of the response of modified MRAC and FMMRAC along with the responses of the reference model, PID controller and MRAC. The response of PID controller has overshoot. The FMMRAC controller performs better than the other controllers.



(a)



(b)

Fig. 6: Step responses of different controller: (a) Reference model m_1, (b) Reference model m_2

The performance criteria for all the four controllers using reference model m_1 are calculated and shown in Table II. It makes a comparison of the performance indices such as settling time (t_s), rise time (t_r) and MSE obtained from step response analysis of all the models. Here, the aim of the process is to track the reference model in an optimal manner. The relative values of t_r and t_s have to be considered rather than the absolute values in order to understand the effect of all the four controllers. It is because the output of the process has to be compared with the output of the reference model rather than the input signal, u_c. In order to obtain the relative values, the values of the reference model are subtracted from the values of the respective controller models. The comparison of performance indices with respect to that of reference model m_1 is described in Table III. The FMMRAC controller has superior performance than that of the PID controller, MRAC and modified MRAC controllers.

Table II. Comparison of performance indices of different controllers using reference model m_1

Type	Reference Model	PID Controller	MRAC	Modified MRAC	Fuzzy Modified MRAC
t_r (sec)	15.9849	22.2688	105.92	21.3996	19.3072
t_s (sec)	23.7534	31.0929	175.18	27.3326	24.7002
MSE	0	0.0810	2.2472	0.0032	0.0014

Table III. Comparison of performance indices with respect to that of reference model m_1

Type	PID Controller	MRAC	Modified MRAC	Fuzzy Modified MRAC
t_r (sec)	6.2839	9.9339	5.4147	3.3223
t_s (sec)	7.3395	151.43	3.5792	0.9468
MSE	0.0810	2.2472	0.0032	0.0014

The performance criterion for the four controllers using reference model m_2 is shown in Table IV. From the Table, it is observed that the proposed FMMRAC method has given the superior performance than the other three controllers in terms of settling time, rise time and MSE. The comparison of performance indices of the four controllers with respect to that of reference model m_2 is explained in Table V.

Table IV. Comparison of performance indices of different controllers using reference model m_2

Type	Reference Model	PID Controller	MRAC	Modified MRAC	Fuzzy Modified MRAC
t_r (sec)	23.9804	23.9815	110.44	25.8439	23.9933
t_s (sec)	34.7306	98.5468	165.58	39.1428	35.62
MSE	0	0.2759	2.1117	0.0108	0.0094

Table V. Comparison of performance indices with respect to that of reference model m_2

Type	PID Controller	MRAC	Modified MRAC	Fuzzy Modified MRAC
t_r (sec)	0.0011	86.4595	1.8635	0.0129
t_s (sec)	63.8162	130.8398	4.4122	0.8894
MSE	0.2759	2.1117	0.0108	0.0094

V. CONCLUSION

This paper presents the control of the level in FTS model using Fuzzy Modified MRAC method. The FTS is analyzed and modeled using Mass balance and Bernoulli's law. The linearized model of the FTS is derived. The simulations are carried out using the linearized model of the FTS. Numerical simulation indicates that the FMMRAC controller has more advantages than the PID controller, MRAC and modified MRAC controller. The FMMRAC controller has fast response, good robustness and low settling time. Also, it has a strong ability to adapt to the changes of the system parameters and anti-disturbance performance. The FMMRAC controller has performed very well even when the reference model order and parameters are different from the process model parameters which indicate the robustness of the design.

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