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Axial wall conduction in cryogenic fluid microtube

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Abstract. Axial wall conduction plays a crucial role in the thermal performance of micro device. In this background a numerical investigation is carried out to understand the effect of axial wall conduction in a microtube at low temperature. Helium at 100 K enters a microtube of inner diameter of 0.4 mm and length of 60 mm and subjected to constant wall heat flux while the microtube cross-sectional faces are considered insulated. Temperature varying thermo-physical property of helium is considered in the analysis as the value of properties changes appreciably with temperature. Simulations have been carried out for varying flow Re =1, 100, 500, solid wall to fluid conductivity ratio k_{sf} =1.71-2822.3, and microtube wall thickness to inner radius ratio δ_{sf} =1-2. The result shows that conductivity ratio and wall thickness play dominant role in conjugate heat transfer process. It is found that there exist an optimum k_{sf} at which Nu_{avg} is maximum when other parameters are kept constant. Nu_{avg} is found to be lower for higher wall thickness (δ_{sf}). When Helium flow rate is increased, it is found that Nu_{avg} increases.

Keywords: Axial wall conduction, microtube, cryogenic fluid, heat transfer

1 Introduction

Cryogenic heat transfer plays crucial role in many low temperature engineering applications such as cryocoolers and creating superconductors for different applications such as magnetic resonance imaging (MRI) etc. In recent times, there has been surge in microscale applications, even in low temperature range e.g., miniaturized cryocoolers for space applications. This may be attributed to recent developments in micromachining technology [1]. With the increasing power density in many electronic devices, the cooling problem has become a subject of prime interest. The performance of electronic devices is seriously influenced by the maximum operating temperature, and hence lowering the maximum operating temperature of cooling systems is crucial. To increase the maximum power handling of electronic devices, micro-sized devices incorporating coolants are becoming more important.

Heat transfer coefficient in microchannel is directly proportional to channel hydraulic diameter. Lower the hydraulic diameter; the more important the coupling between wall and bulk fluid temperatures becomes since the heat transfer coefficient reaches large values. The consequence is that the axial conductive heat transfer in the wall distorts the actual thermal condition at the solid-fluid interface. Thus negligence of axial conduction effects may lead to erroneous conclusion and/or interprettations.

2 Literature review

The concept of axial wall conduction is not limited to microchannels only. This effect is less important in macro size channels compared to micro size channels. A detailed and updated literature review can be found in Moharana et al. [2]. For brevity, a brief updated review is presented here. Harley et al. [3] did experimental and theoretical study of low Reynolds number, high subsonic Mach number compressible gas stream in channel. Helium, Nitrogen, and argon gasses were utilized. The Knudsen number extended from 0.001 to 0.4 and found that the measured friction factor was in agreement with theoretical predictions considering isothermal, local fully developed, first-order, slip stream. Hsieh et al. [4] both theoretically and experimentally studied gas flow characteristics in a microchannel (D_h = 80 µm) using nitrogen as working fluid (Kn = 0.001 – 0.02) and found that slip effects still exists. Jiao et al. [5] did combined experimental and numerical study to find the heat transfer characteristics of cryogenic helium gas with temperature dependent thermo-physical properties in a miniature tube. They calculated the temperature distribution and velocity profile in a miniature tube, and proposed a correlation for predicting temperature effect on Nu.

Based on their combined experimental and numerical study, Moharana et al. [6] indicated that depending on the geometry and flow conditions (i.e. parameter M), conjugate heat transfer effects become predominant. Moharana et al. [2] had numerically investigated effect of axial wall conduction in a square microchannel engraved on a solid substrate whose bottom face is subjected to constant wall heat flux. They found that there exists an optimum solid to fluid conductivity ratio k_{sf} at which average Nu is maximum. They also found similar observation in circular microtube. Moharana and Khandekar [7] numerically studied axial wall conduction in a microtube subjected to constant wall temperature on its outer surface.

Moharana and Khandekar [8] studied effect of rectangular microchannel aspect ratio on axial wall conduction in solid substrate and found that average Nu is minimum corresponding to channel aspect ratio slightly lower than 2.0. Kumar and Moharana [9] numerically studied axial wall conduction in partially heated microtube subjected to constant wall temperature along the heated length. Tiwari et al. [10] also numerically studied axial wall conduction in partially heated microtube subjected to constant wall temperature along the heated length.

Recently, Mishra and Moharana [11] studied axial wall conduction in a microtube where flow is sinusoidally varying with time i.e. pulsatile flow in nature. Based on the numerical simulation, they concluded that for a particular pulsation frequency (Wo) there exists an optimum value of k_{sf} at which overall Nusselt number (Nu) is maximum, similar to the observation by Moharana et al. [2].

From the review of the literature it is revealed that though many studies have explored conjugate heat transfer in microchannels, they are mostly limited to using water as working fluid where thermo-physical properties hardly vary with temperature. But in case of cooling of high power density electronic devices water is not suitable especially in low temperature applications. And there are comparatively very less number of studies available considering non water especially cryogenic fluid. Secondly, the literature on conjugate heat transfer under temperature dependent cryogenic fluid in microchannel is sparse. Therefore this study is undertaken to explore it in a systematic manner. Thus, the objective of this work is to find the actual boundary condition experienced at the solid-fluid interface of a microtube subjected to constant wall heat flux at its outer surface considering helium at 100 K at the inlet of the microtube for carrying the heat from the system.

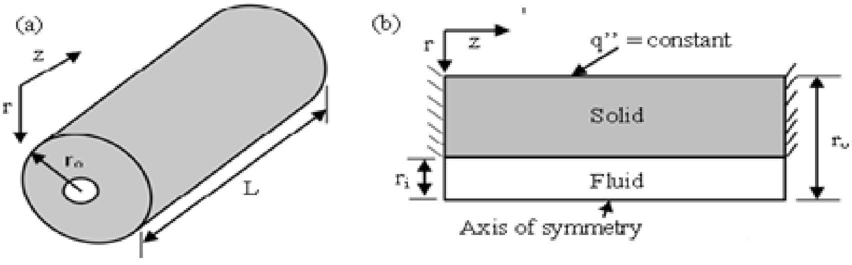

Fig. 1. Microtube and the computational domain considered.

3 Numerical simulation

In this work a two-dimensional numerical study has been undertaken to study the effects of axial wall conduction in conjugate heat transfer situation in simultaneously developing laminar flow and heat transfer in a fully heated microtube subjected to constant heat flux boundary condition on its outer surface. Micro-tube of total length 60 mm is considered for the numerical study. In this work, simultaneously developing single-phase, steady-state laminar fluid flow with constant thermo-physical properties for solid and temperature dependent thermo-physical properties of liquid is considered. Microrotube geometry considered in the study is schematically shown in Fig. 1.

A microtube of total length L = 60 mm and inner radius $r_i = \delta_f = 0.2$ mm is considered in the numerical investigation. While the thickness (δ_s) and conductivity of the tube (k_s) are varied, the other two dimensions of the microtube (L, and δ_f) and the working fluid are kept constant. Helium at 100 K enters the microtube subjected to constant wall heat flux on its outer surface while cross-sectional solid faces are insulated. Ideally, conductivity of fluid is maintained constant as there is no change in working fluid. But thermo-physical properties of helium (including thermal conductivity) change appreciably for a small change in temperature. Therefore, thermo-physical properties as a function of temperature as given in Eq. 1-3 are considered in the simulation process using user defined function/polynomial function.

$$\mu_f(T) = -4.25462\times10^{-7} + 8.25786\times10^{-8}T - 9.43838\times10^{-11}T^2 + 7.6085\times10^{-14}T^3 \quad (1)$$

$$k_f(T) = 0.00793 + 0.000878621T - 2.50172\times10^{-6}T^2 + 3.92\times10^{-10}T^3 \quad (2)$$

$$\rho_f(T) = 0.1186 + 3.38334e^{\left(-\frac{T}{20.41}\right)} + 0.93142e^{\left(-\frac{T}{99.36854}\right)} \quad (3)$$

Helium with a slug velocity of u_{in} and temperature of $T_{f\text{-}in}$ enters the microtube. Thus, the flow is hydro dynamically and thermally developing in nature at the tube inlet. For the purpose of numerical study it is assumed that the working fluid is incompressible and the flow is in steady state; single phase laminar flow; and heat loss by natural convection or radiation to ambient is negligible. Considering angular symmetry a two dimensional computational domain is considered for the analysis as shown in Fig. 1. Thus, the two-dimensional steady Navier–Stokes and energy equations are used to describe the flow and heat transfer in the computational domain. The Knudsen number, which is defined as the ratio of the mean molecular free path of gas molecules to the characteristic dimension of channel, for the present case is in the range of less than 0.01; thus remain in continuum flow regime. Therefore, no slip boundary condition at the inner wall is considered.

The governing differential equations are solved using commercial platform Ansys-Fluent®. The "standard" scheme was used for pressure discretization. For velocity-pressure coupling the SIMPLE algorithm was used in the multi-grid solution procedure. "second-order upwind" scheme was used for solving the momentum and energy equations. An absolute convergence criterion for continuity and momentum equations is taken as 10^{-6} and for energy equation it is 10^{-9}.

Rectangular elements were used for meshing the computational domain and the grid independence test was ensured for all geometry included in the study. For example, local Nusselt number were obtained for a microtube (inner diameter 0.4 mm) with negligible wall thickness, for three different grids of 32×4800, 40×6000, and 50×7500 for checking the mesh independency of the solution. The difference in local Nusselt numbers at the fully developed flow regime between the mesh size of 32×4800 to 40×6000 and mesh size of 40×6000 to 50×7500 was found to be about 0.68% and 0.55%, respectively. Hence, no appreciable change was observed. So, the middle grid (40×6000) was selected.

3.1 Data reduction

The parameters of interest are (a) peripheral averaged local heat flux (b) local bulk fluid temperature (c) peripheral averaged local wall temperature. These parameters allow us to determine the effect of axial conduction on the local Nusselt number. The conductivity ratio (k_{sf}) is defined as the ratio of thermal conductivity of the microtube wall (k_s) to that of the working fluid (k_f). The wall thickness (δ_s) to inner radius (δ_f) ratio is (δ_{sf}). The dimensionless axial coordinate, z, is

$$z^* = \frac{z}{\mathrm{Re.Pr.D_h}} \tag{4}$$

The non-dimensional local heat flux at the fluid-solid interface is given by

$$\phi = \frac{q''}{q''_w} \tag{5}$$

where, q'' is the local heat flux transferred at the solid-fluid interface along the micro-tube length and is heat flux on the outer surface of the micro-tube due to constant wall heat flux boundary condition at outer surface along the micro-tube length. The dimensionless bulk fluid and tube inner wall temperatures are represented

$$\theta_w = \frac{(T_w - T_{fi})}{(T_{fo} - T_{fi})} \tag{6}$$

$$\theta_f = \frac{(T_f - T_{fi})}{(T_{fo} - T_{fi})} \tag{7}$$

where, T_{fi} and T_{fo} are the bulk fluid temperature (averaged over the cross-section) at the channel inlet and outlet. T_f is the average bulk fluid temperature at any location and T_w is the wall temperature at that position. The local Nusselt number is given by

$$Nu_z = \frac{h_z.D}{k_f} \tag{8}$$

where, the local heat transfer coefficient is given as

$$h_z = \frac{q''_z}{(T_w - T_f)} \tag{9}$$

For calculating the average Nusselt number over the full length of the microtube the following equation used

$$\overline{Nu} = \frac{1}{L}\int_0^L Nu_z dz \tag{10}$$

4 Results and discussion

In this work inner radius of the microtube is kept constant, the thickness of the tube wall (refer to Fig. 1) is varied to understand the effect of wall thickness on the heat transfer behavior. Secondly, the wall conductivity and flow rate is also varied. Thus, the parametric variations include δ_{sf} (= 1, 2) k_{sf} (= 1.7105 – 2822) and Re (=1, 100, 500). As the thickness of the solid wall increases, the boundary on which constant heat flux applied moves away from the actual solid-fluid interface. For flow through a tube, maximum heat transfer coefficient will occur if constant heat flux is experienced at the solid-fluid interface of the microtube. Under ideal condition of zero wall thickness, if constant wall heat flux is applied to a circular tube, it will lead to maximum

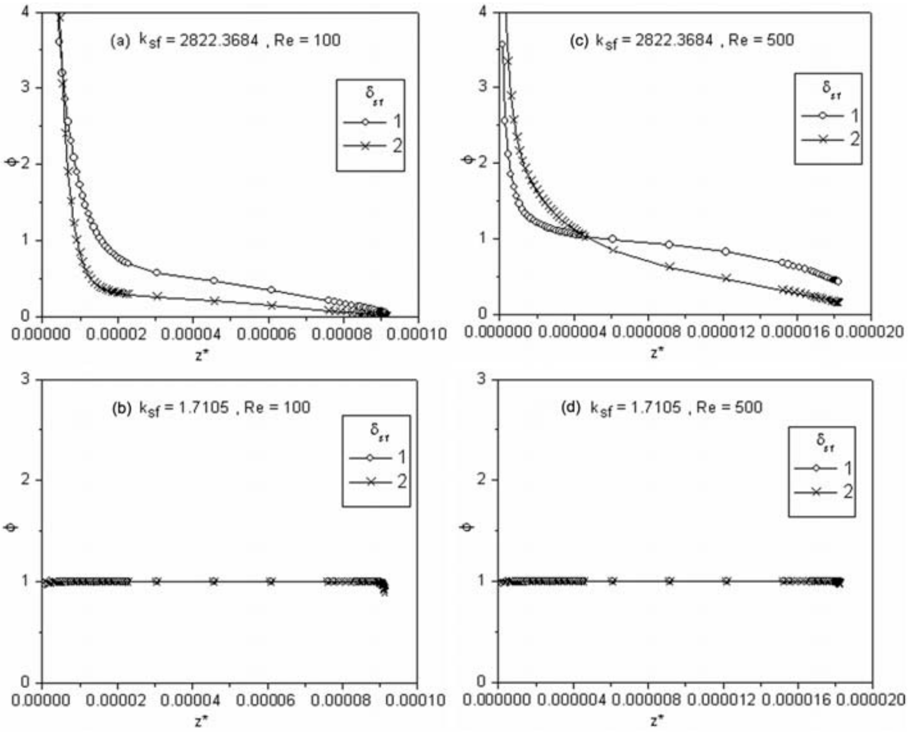

Fig. 2. Axial variation of dimensionless local heat flux at the solid-fluid interface.

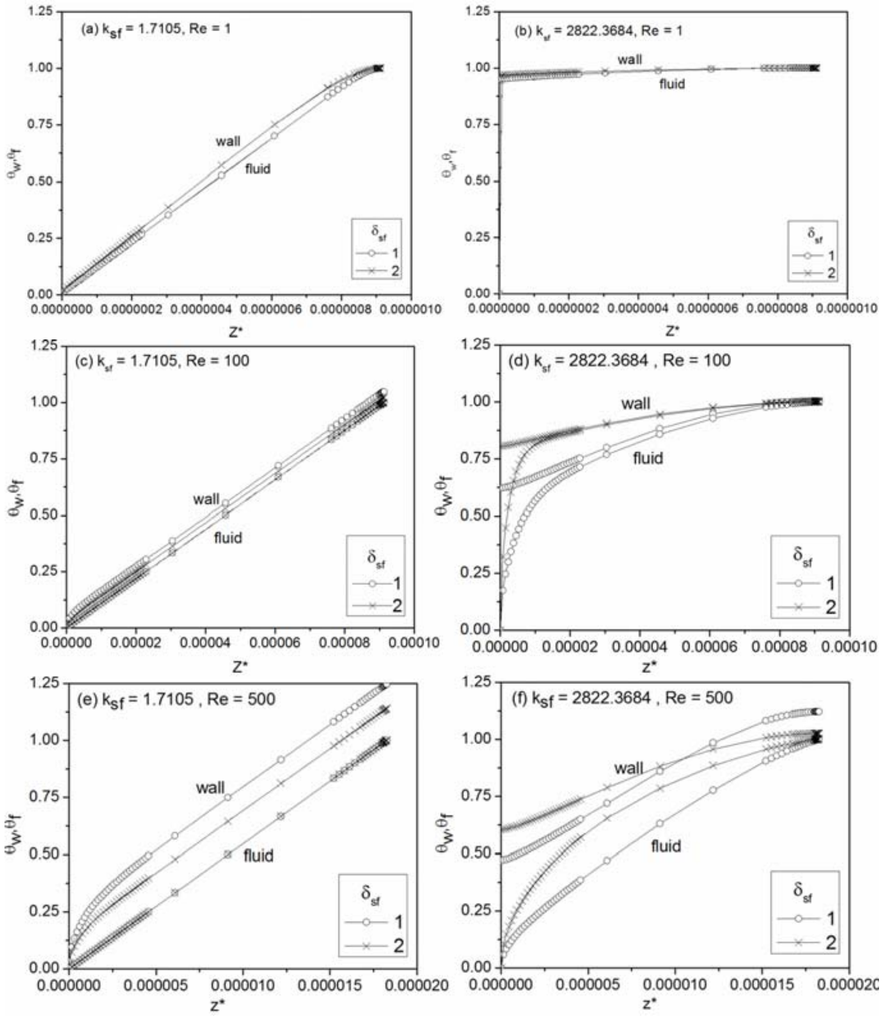

Fig. 3. Axial variation of dimensionless wall and bulk fluid temperature.

value of Nusselt number for fully developed laminar flow i.e. Nu = 4.36.

In actual condition, finite wall thickness exists and thus leads to conjugate heat transfer situation. In such case, the thermal boundary condition experienced at the solid-fluid interface control the heat transfer process instead of the thermal condition imposed on the outer surface. The parameters such as axial variation of wall heat flux, wall temperature, bulk fluid temperature, and local Nusselt number will indicate influence of axial wall conduction. Thus the axial variation of dimensionless local heat flux is presented in Fig. 2. It is found that for lower k_{sf}, the axial variation of dimensionless heat flux at the solid-fluid interface is uniform almost throughout the length of the microtube except near the inlet and also equal to unity. With increasing k_{sf}, it is found to be deviating from the ideal value. Secondly, for thicker walls also i.e. for higher value of δ_{sf}, there is appreciable deviation from ideal value. This indicates dominance of axial wall condition at these situations.

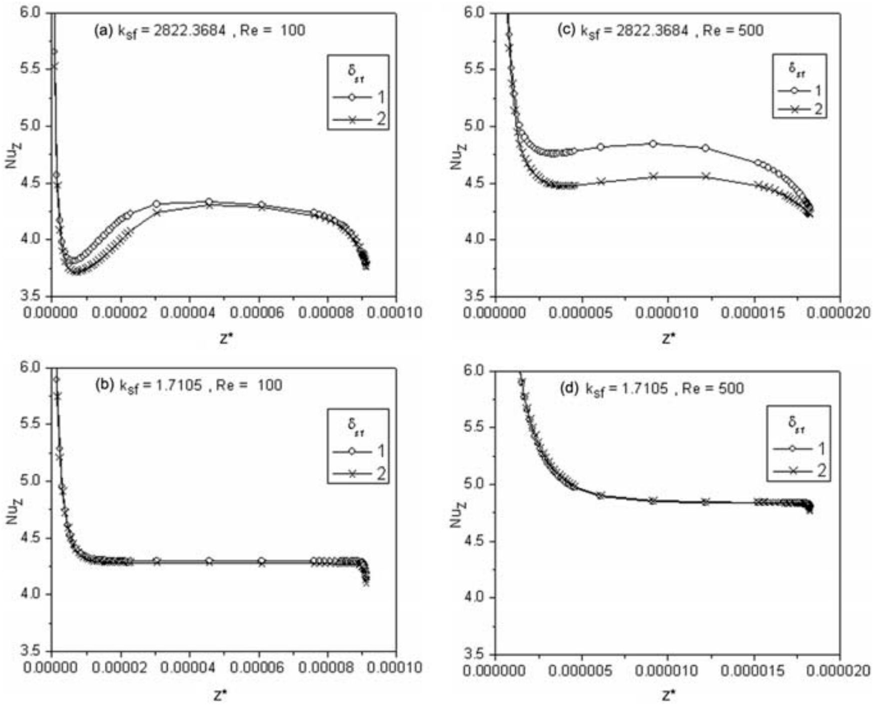

Fig. 4. Axial variation of local Nusselt number as a function of δ_{sf}, k_{sf} and Re.

Fig. 3 shows the axial variation of dimensionless wall and bulk fluid temperature as a function of, k_{sf}, δ_{sf} and Re. At low conductivity ratio (k_{sf}), irrespective of the thickness ratio (δ_{sf}), the wall and the fluid temperatures rise as per the conventional theory applicable for tube with zero/negligible wall thickness subjected to constant heat flux boundary condition. In the fully developed region, the temperature difference (θ_w-θ_f) attains a constant value. The temperature difference increases with increase in flow Re and/or decreasing wall thickness (See Fig. 3(a,c,e)). At higher k_{sf}, the difference between the solid-fluid interface wall temperature and bulk fluid temperature decreases continuously along the length of the microtube and they are no longer linearly varying (See Fig. 4(b,d,f)). This pattern is more towards isothermal condition on the outer surface instead constant wall heat flux. Thus from Fig. 3 it is again clear that the axial wall conduction distorted the thermal boundary condition as in Fig. 3(b) i.e. higher k_{sf} leads to axial wall conduction. Secondly, thicker wall at higher k_{sf} leads to higher axial wall conduction compared to thinner wall thickness. But at lower k_{sf}, wall thickness has negligible effect on axial wall conduction.

Fig. 4 shows the axial variation of local Nu. As discussed above, if the boundary condition experienced at the solid-fluid interface is close to constant wall heat flux, then the local Nusselt number in the fully developed zone will converge close to Nu_z = 4.36. For lower k_{sf} and low Re (see Fig. 4 (b)), the fully developed Nu is found to be close to this value. And as the flow Re increases at same low k_{sf} value, the fully developed Nu is increases for all value of δ_{sf}. Secondly, higher k_{sf} and flow Re =500, fully developed Nu is increases with increasing the value of δ_{sf} and also the fully developed Nu no more remains constant axially shown in Fig. 4 (d). From above discussion we can say the value of local Nusselt number is function of Re, k_{sf}, and δ_{sf}. The axial variation of wall temperature at the solid-fluid interface drifts more towards the trend of constant heat flux in the thicker wall. This generates higher Nusselt number compared to thinner tube wall thickness.

It is very clear from Fig. 5 that the value of average Nusselt number is minimum at very high k_{sf} and with decrease in k_{sf}, Nu_{avg} attains its highest value for all set of δ_{sf} and flow Re. After that the slope goes downward when moving towards lowest value of k_{sf}. The value of average Nusselt number is maximum at an optimum value of k_{sf}. So we can say, for an optimum value of Nusselt number, heat transfer will be maximum as well as almost independent of thickness ratio (δ_{sf}).

5 Conclusions

A numerical study has been carried out for internal convective cryogenic fluid flows in a microtube subjected to conjugate heat transfer situation. This study has been carried out to understand the effect of axial wall conduction in cryogenic fluid for developing laminar flow and heat transfer in a circular microtube subjected to constant heat flux boundary condition imposed on its outer surface. Simulations have been carried out for a wide range of tube wall to fluid conductivity ratio (k_{sf}: 1.7105-2822.3684), tube wall thickness to inner radius ratio (δ_{sf}: 1, 2), and flow Re (1,100, 500). The main outcomes of this study is that the value of Nu_{avg} is increasing with decreasing value of k_{sf} and the rate of increase of Nu_{avg} is higher for smaller values of k_{sf} ($k_{sf} < 30$). Secondly, when other parameters are remaining same, for smaller δ_{sf}, Nu_{avg} is higher than higher δ_{sf}. The difference between the Nu_{avg} values (corresponding to $\delta_{sf} = 1$ to 2) at lower k_{sf} is higher compared to higher k_{sf} values. Finally, the value of Nu_{avg} increases with increasing fluid flow Re while other parameters are constant. Therefore, an optimum k_{sf} exists for temperature dependent cryogenic fluid at which Nu_{avg} is maximum.

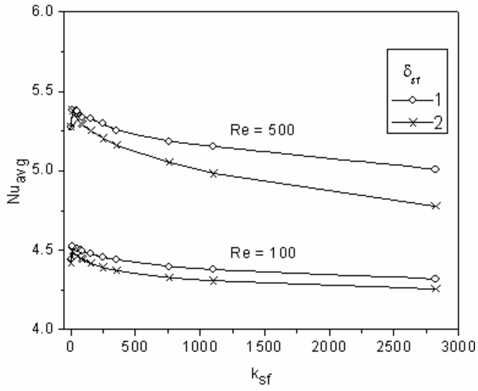

Fig. 4. Average Nusselt number varying with k_{sf} at Re = 100, 500, and $\delta_{sf} = 1, 2$.

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