

# BER PERFORMANCE IMPROVEMENT OF AN FNN BASED EQUALISER USING FUZZY TUNED SIGMOIDAL ACTIVATION FUNCTION

Jitentriya K Satapathy\* and Susmita Das\*

\*Department of Electrical Engineering,  
National Institute of Technology, Rourkela-769 008, Orissa (India)

## ABSTRACT

Adaptive equalisers are characterised in general by their structures, the learning algorithms and the use of training sequences. This paper presents a novel technique of improving the performance of conventional multilayer perceptron (MLP) based decision feedback equaliser (DFE) of reduced structural complexity by adapting the slope of the sigmoidal activation function using fuzzy logic control technique. The adaptation of the slope parameter increases the degrees of freedom in the weight space of the conventional Feedforward Neural Network (CFNN) configuration. Application of this technique reduces the structural complexity of a conventional FNN equaliser, provides faster learning and significant performance gain.

## 1. INTRODUCTION

The distortions introduced in the communication channel cause the transmitted symbols to spread and overlap over successive time intervals, resulting in a phenomenon, known as Inter Symbol Interference (ISI). In addition to ISI, the transmitted symbols are subjected to other impairments such as thermal noise, impulse noise and non-linear distortions arising from the modulation and demodulation process, cross talk interference, the use of amplifiers and converters etc. Neural network based equalisers have been proposed in recent past which are very efficient and provide significant performance improvements over the conventional ones. A multilayer FNN architecture consists of a number of processing neurons organised in layers and is capable of providing complex the non-linear decision boundary associated with the optimal Bayesian equaliser. The performance of the FNN equaliser can be enhanced by incorporating decision feedback. It is shown that the FNNDFE as shown in Fig. 1 using the Back Propagation (BP) algorithm [2] gives a significant improvement in performance.

The slope parameter of the sigmoidal activation function of individual neuron in a multilayer

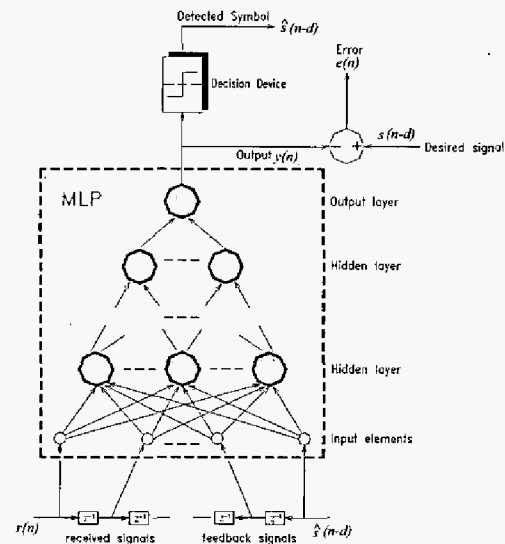


Fig. 1 A conventional FNN equaliser

neural network paradigm plays an important role as it decides decision-making ability of that node. Performance of conventional MLP neural equaliser can be improved by tuning the slope of the activation function along with weight updation and in the proposed work this parameter has been adapted using a fuzzy logic controller technique [3]. While the BP algorithm takes control by recursively updating the network weights and threshold values, the fuzzy controller approach adjusts the slope of the sigmoid activation function of all the nodes of the network at the same time, thus making the proposed structure a hybrid one for adapting the network parameters. Further, the training time generally depends upon the complexity of the underlying process. Hence, the proposed work is aimed at reduction of the training time for a given neural network topology. The improved performance in the simulation results is a clear indication of the efficacy of the proposed technique.

## 2. DESCRIPTION OF THE PROPOSED METHOD

The MLPDFE can be trained in a supervised manner using the Back Propagation algorithm.

At time index  $n$ , the  $m \times 1$  received signal vector  $\mathbf{r}(n)=[r(n), r(n-1), \dots, r(n-m+1)]$  and  $n_b \times 1$  decision signal vector  $[\hat{s}(n-d-1), \hat{s}(n-d-2), \dots, \hat{s}(n-d-n_b)]$  are fed into the feedforward filter and feedback filter of the decision feedback equaliser respectively. The signal at the input layer of the decision feedback equaliser can be represented by a  $(m+n_b) \times 1$  vector as

$$\mathbf{x}(n) = [r(n), r(n-1), \dots, r(n-m+1); \hat{s}(n-d-1), \dots, \hat{s}(n-d-n_b)]^T \quad (2.1)$$

The final estimated output signal  $y(n)$  at time index  $n$ , can be calculated as follows [1].

$$y(n) = F_o \left( \sum_{k=1}^{N_2} w_{ko}^{(3)}(n) F_k \left( \sum_{j=1}^{N_1} w_{jk}^{(2)}(n) F_j \left( \sum_{i=0}^{m-1} w_{ij}^{(1)}(n) r(n-i) + \sum_{p=1}^{n_b} w_{pj}^{(1)}(n) \hat{s}(n-d-p) + th_j^{(1)}(n) \right) + th_k^{(2)}(n) \right) + th_o^{(3)} \right) \quad (2.2)$$

where all  $F$ 's denote sigmoidal activation functions in the neurons.  $N_1$  and  $N_2$  are the number of neurons in the two hidden layers respectively. The output of the decision device can be defined as

$$\hat{s}(n-d) = \begin{cases} 1 & \text{if } y(n) \geq 0 \\ -1 & \text{otherwise} \end{cases} \quad (2.3)$$

The  $w$ s (weights) and  $th$ s (threshold levels) in Equation (2.2) are values specified by the training algorithm, so that after training is completed the equaliser will self-adapt to the changes in the channel characteristics occurring during transmission (decision directed mode).

The idea behind this work is that for a fully trained network, the error at each node is to be minimized. Under such circumstances there will be no further change in the synaptic weights or the threshold values of the network. It confirms the basic concept embedded in BP algorithm that the change in synaptic weights and thresholds is only possible if an error term of  $j^{\text{th}}$  neuron in  $l^{\text{th}}$  layer  $\delta_j^{(l)}(n)$  exists in the nodes because the mathematical equations governing the updation of the above parameters are expressed as

$$\Delta w_{ij}^{(l)}(n+1) = \eta \delta_j^{(l)}(n) y_j^{(l-1)}(n) + \alpha \Delta w_{ij}^{(l)}(n) \quad (2.4)$$

and

$$\Delta th_j^{(l)}(n+1) = \beta \delta_j^{(l)}(n) \quad (2.5)$$

where  $\eta$  is the learning-rate parameter,  $\alpha$  is the momentum parameter,  $\beta$  is the threshold level adaptation gain and layer  $l \in [1, 2, \dots, L]$ .

Hence the error term at individual (neuron) of a neural structure is to be minimised to get a pseudo-optimal solution. In a neural network paradigm the synaptic weights and threshold values are generally considered as free parameters in conventional sense, which are adapted using appropriate learning algorithms

in order to train the network. However, there are many other parameters like slope of the sigmoidal activation function, learning-rate parameter for synaptic weights, thresholds and momentums etc., which can also be tuned to enhance the adaptability of the network. In the present research work attempt has been made to adapt the slope of the sigmoid activation function only using the fuzzy logic controller approach to design a fuzzy tuned FNN (FZTUNFNN) equaliser. The fuzzy logic controller technique [3,4] is applied to determine the amount of correction needed for the slope of the sigmoidal activation function at each node of the network. Basically a fuzzy controller evaluates the change in the control action based on the information regarding error and rate of change of error at the process output. The same concept is adopted in the proposed work.

The node error term known as  $\delta_j^{(l)}(n)$  and its rate of change

$$\Delta \delta_j^{(l)}(n) = \delta_j^{(l)}(n) - \delta_j^{(l)}(n-1) \quad (2.6)$$

are fed into the fuzzy controller block as shown in Fig. 2. The output generated from the control block  $\Delta \phi(n)$  is used to update the slope of  $\phi(n)$  of the sigmoidal activation function using the relation

$$\phi(n+1) = \phi(n) + \Delta \phi(n) \quad (2.7)$$

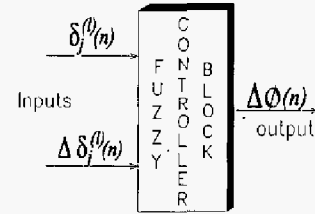


Fig. 2 Sigmoid slope change by fuzzy logic approach

## 3. SIMULATION STUDY

The simulation model of an adaptive equaliser considered is illustrated in Fig. 3. In the simulation study the channel under investigation is excited with a 2-PAM signal, where the symbols are extracted from uniformly distributed bipolar random numbers  $\{-1, 1\}$ . The channel output is then contaminated by an AWGN (Additive White Gaussian Noise). The pseudo-random input and noise sequences are generated with different seeds for the random number generators. The power of additive noise has been taken as 0.01, representing a SNR of 20dB. The BER performances for each SNR are evaluated based on  $10^7$  more received symbols (test samples) and averaged over 20 independent realizations, after training is completed with sequences of 1000 samples. The bit error rate is obtained with detected symbols being fed back, as this technique presents a more realistic scenario in comparison with correct symbol feedback. Further, in order to prove the

robustness and consistency in performance of the proposed neural structure, equalisation of two typical channels is simulated. The advantage gained in terms of performance enhancement and faster training by the proposed fuzzy tuned FNN equaliser can be clearly demonstrated by comparing its BER performance with a CFNN structure trained with more number of samples. The proposed structures in the FNN framework reported yield superior result in terms of BER performance, provide of faster learning (i.e., exactly half the number of training samples in comparison to a conventional FNN equaliser) and is of reduced structure.

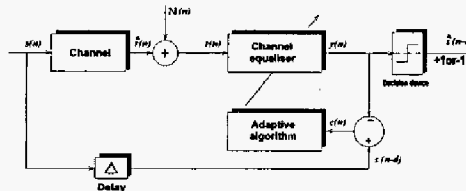


Fig. 3 Simulation model of an adaptive equaliser

The first example used is two-tap simple minimum-phase channel [1] defined by its transfer function  $H_1(z)=1+0.5z^{-1}$  (2.8)

Fig. 4 illustrates the BER performance comparison of the proposed structures {a two layer (1,1) structure} with a conventional FNN {a two layer (2,1) structure configuration} and the optimal Bayesian DFE in terms of BER performance. The configuration of a conventional FNN DFE is set to  $m=2$  (two samples in the feedforward section),  $n_b=1$  (one sample in the feedback section) and  $d=1$  (transmitted sequence delayed by one sample). The proposed FZTUNFNN equaliser is able to provide a performance gain of 2 dB at BER level of  $10^{-4}$  over conventional FNN one and it is close to the optimal Bayesian Equaliser [5] performance. Even increasing the training samples to 2000 for a conventional FNN equaliser, there is no significant improvement in performance.

Another example studied is a five-tap deep-null communication channel [6], which is characterised by the following transfer function.

$$H_2(z) = 0.9413 + 0.3841z^{-1} + 0.5684z^{-2} + 0.4201z^{-3} + z^{-4} \quad (2.9)$$

Fig.5 depicts the significant BER performance enhancement by all the proposed equaliser {a two layer (1,1) structure} when compared with a CFNN one {a two layer (5,1) structure} with parameters chosen as  $m=5$ ,  $n_b=4$  and  $d=4$  after being trained with 1000 samples. The proposed FZTUNFNN equaliser is able to provide better performance in terms of the minimum SNR to get a prefixed error probability level (18.5 dB against 20 dB to obtain  $BER=10^{-4}$ ) and is close to the optimal Bayesian performance in comparison to the

conventional FNN equaliser. It is also observed that the performance of a conventional FNN equaliser trained with 2000 samples is still poor in comparison to the proposed equaliser.

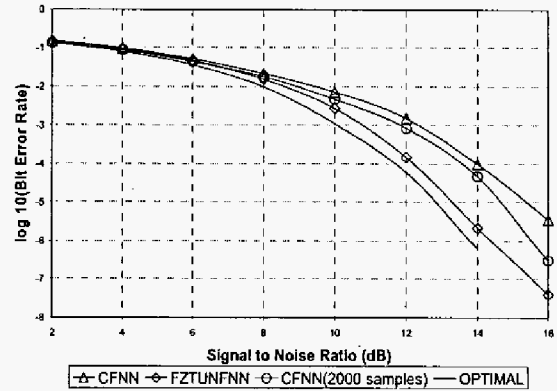


Fig. 4 BER performance comparison of the proposed equaliser with conventional FNN for channel  $H_1(z)$

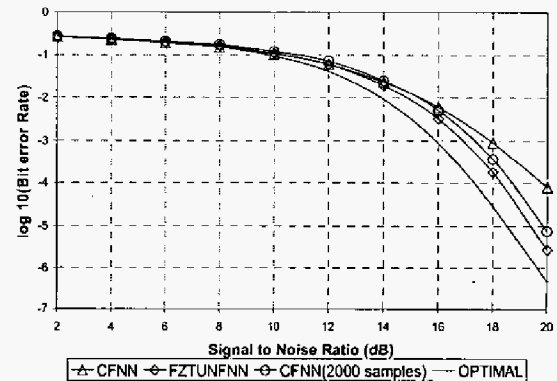


Fig.5 BER performance comparison of the proposed equaliser with conventional FNN for channel  $H_2(z)$

#### 4. CONCLUSION

This paper presents a novel Fuzzy tuned FNN (FZTUNFNN) equaliser designed on an FNN platform, where a fuzzy logic concept is employed to tune the slope ( $\phi$ ) of the sigmoid activation functions at all the nodes. The proposed equaliser structure on FNN framework is highly efficient in terms of BER performance and faster learning in comparison to a conventional MLP structure. Also the proposed equaliser with a reduced structure architecture provides a performance level close to optimal Bayesian equaliser. In addition to it, the fuzzy tuned equaliser in FNN domain comprising of reduced structural complexity is suitable for easy real-time implementation point of view.

## 5. REFERENCES

- [1] S. Chen, G.J. Gibson and C.F.N. Cowan, "Adaptive Channel Equalisation Using a Polynomial - Perceptron Structure", Proceedings-I of the IEE, Vol.137, pp.257-264, October 1990.
- [2] S. Siu, G.J. Gibson and C.F.N. Cowan, "Decision Feedback Equalisation Using Neural Network Structures and Performance Comparison with Standard Architecture", Proceedings-I of the IEE, Vol.137, No.4, pp. 221-225, August 1990.
- [3] A.F. Stronach, P. Vas and M. Neuroth, "Implementation of Intelligent Self-organising Controllers in DSP Controlled Electromechanical drives", IEE Proceedings, Control Theory Application, Vol.144, No.4, pp.324-330, July 1997.
- [4] J.K. Satapathy and C.J. Harris, "Application of Fuzzy-Tuned Adaptive Feedforward Neural Networks for Accelerating Convergence in Identification", 3<sup>rd</sup> International Conference on Industrial Automation, pp.6.1, June 1999.
- [5] S. K. Patra and B. Mulgrew, "Efficient Architecture for Bayesian Equalisation Using Fuzzy Filters", IEEE Transactions on Circuits and Systems II : Analog and Digital Signal Processing, Vol.45, No.7, pp. 812-820, July 1998.
- [6] S. Lambotharan and J.A. Chambers, "A New Blind Equalisation Structure for Deep-Null Communication Channels", IEEE Transactions on Circuits and Systems-II, Analog and Digital Signal Processing, Vol.45, No.1, pp.108-114, January 1998.