Effects of suction and injection on swirling flow near a rough disk

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Abstract

The flow due to uniform rotation of a viscous fluid at a larger distance from a rough stationary disk is investigated numerically. A uniform suction or injection is applied at the surface of the disk. The system of fully coupled, nonlinear similarity equations is integrated accurately for full range of flow parameters. Effects of wall roughness, suction and injection on the boundary layer are discussed with relevant physical interpretations.

Keywords: Swirling flow; Partial slip; Suction; Injection; Multiple shooting method.

1 Introduction

Steady swirling flow of a viscous, incompressible fluid near an infinite stationary disk (Bödewadt flow) is one of the few problems in fluid dynamics for which the Navier-Stokes equations admit an exact numerical solution, subject to the conventional no-slip boundary conditions. Swirling flows have many interesting features and immense industrial applications. Boundary layer problems in rotating flows are unique, as for many applications a strong interaction exists between boundary layer and the outer flow. Sharp gradients in the centrifugal force across the boundary layer can generate high velocities along the bounding surfaces and this results secondary flows. The interaction of rotating flow with a stationary rough surface has been the subject of recent study by Sahoo et al [2]. Unfortunately, in this work the effect of suction and injection on the momentum boundary layer was overlooked. It was pointed out by Schwiderski and Lugt [7] that the non-existence of a proper solution to the boundary value problems for rotating flows of von Kármán and Bödewadt is an indication that in reality the flow is separated from the disk surface. The simple 'Tea cup experiment' described in [2], displays very clearly a separation of the fluid from the bottom of the cup. Application of the suction is an effective device to reduce the chances of separation. With no-slip boundary conditions the momentum and the displacement thickness should decrease as the suction velocity increases. The objective of this paper is to study the combined effects of velocity slip and suction (or injection) on the momentum boundary layer. The effective direct multiple shooting method has been adopted to solve the resulting system of highly nonlinear similarity equations.

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2 Formulation of the problem

One considers a viscous fluid occupying the space $z > 0$ over an infinite stationary disk, which coincides with $z = 0$. The motion is due to the rotation of the fluid like a rigid body with constant angular velocity Ω at a larger distance from the disk. The flow is described in the cylindrical polar coordinates (r, ϕ, z). In view of the rotational symmetry, $\frac{\partial}{\partial\phi} \equiv 0$. Let $\mathbf{V} = (u, v, w)$ be the fluid velocity vector. Using the well known von Kármán [4] transformations $u = r\Omega F(\zeta), \quad v = r\Omega G(\zeta), \quad w = \sqrt{\Omega\nu}H(\zeta), \quad z = \sqrt{\frac{\nu}{\Omega}}\zeta, \quad p - p_\infty = -\rho\nu\Omega P$ and following [2], the equations of continuity and motion take the form:

$$\frac{dH}{d\zeta} + 2F = 0, \tag{2.1}$$

$$\frac{d^2F}{d\zeta^2} - H\frac{dF}{d\zeta} - F^2 + G^2 = 1, \tag{2.2}$$

$$\frac{d^2G}{d\zeta^2} - H\frac{dG}{d\zeta} - 2FG = 0, \tag{2.3}$$

$$\frac{dP}{d\zeta} - H\frac{dH}{d\zeta} + \frac{d^2H}{d\zeta^2} = 0 \tag{2.4}$$

which has to be solved subject to the following partial slip boundary conditions [2, 3, 5]:

$$F(0) = \lambda F'(0), \quad G(0) = \eta G'(0), \quad H(0) = \frac{W_0}{\sqrt{\Omega\nu}} = W,$$
$$F(\infty) \to 0, \quad G(\infty) \to 1. \tag{2.5}$$

where λ, η are non-dimensional slip parameters and W_0 is the is the uniform suction ($W_0 < 0$) or injection ($W_0 > 0$) velocity.

3 An exact numerical solution

There is no doubt that the present problem can be solved by a shooting method [3, 5]. However, there are some drawbacks in the single shooting method. For a highly nonlinear system of equations, the initial guesses must be chosen very carefully. Initial values that are chosen slightly off the true solution may lead to singularities or breakdown of the method. Here, one has adopted the direct multiple shooting method to solve the system of nonlinear differential equations (2.1)-(2.3) subject to slip boundary conditions (2.5). Subsequently, Eq. (2.4) can be integrated directly to get the non-dimensional pressure P. A finite value large enough has been substituted for ζ_∞, the numerical infinity. The whole domain of integration $[0, \zeta_\infty)$ has been divided into subintervals by introducing additional grid points $\zeta_0 = 0 < \zeta_1 < \zeta_2 < \ldots < \zeta_N = \zeta_\infty$. The aforementioned system of equations can be written as a system of five first order liner equations. Let:

$$y_1 = F, \quad y_2 = G, \quad y_3 = H, \quad y_4 = F', \quad y_5 = G' \tag{3.1}$$

Eqn. (2.1)-(2.3) can be rewritten as:

$$
\begin{aligned}
\frac{dy_1}{d\zeta} &= y_4; \quad y_1(0) = \lambda s_1 \\
\frac{dy_2}{d\zeta} &= y_5; \quad y_2(0) = \eta s_2 \\
\frac{dy_3}{d\zeta} &= -2y_1; \quad y_3(0) = W \\
\frac{dy_4}{d\zeta} &= y_3 y_4 + y_1^2 - y_2^2 + 1; \quad y_4(0) = s_1 \\
\frac{dy_5}{d\zeta} &= y_3 y_5 + 2y_1 y_2; \quad y_5(0) = s_2
\end{aligned}
\tag{3.2}
$$

The initial guesses for y_i at each nodal points are borrowed from [2]. The solutions obtained in each interval are pieced together to form continuous trajectories of the velocity profiles. Utmost care has been taken while refining values of the missing initial guesses s_1 and s_2 by the Broyden's method [8].

4 Results

The velocity profiles for the Bödewadt flow exhibit oscillations unlike the von Kármán one. The oscillations occurring in the boundary layer when the fluid rotates near a stationary disk can be explained in the following manner. The radial inflow, induced by the delay of the tangential velocity in the vicinity of the stationary disk, tends to conserve the angular momentum of the flow and thus to increase the tangential velocity with decreasing radius. For an overshoot, radial convection of the angular momentum near the disk must be strong enough to more than balance the dissipation of angular momentum caused by the wall shear. This inward radial convection of surplus angular momentum is possible as long as the distribution of circulation in the outer flow increases with increasing radius. A local overshoot in the tangential velocity increases the centrifugal force locally which then tends to induce a radial outflow. This radial outflow convects an angular momentum defect to force an undershoot in the tangential velocity, and the above process is repeated to yield oscillatory approach to infinity.

Figs. 1-3 display the effects of injection and suction on the radial velocity component F in presence of slip. It is evident that injection enhances the oscillation in the velocity profiles, which is significant in absence of slip ($\lambda(=\eta)=0$). On the other hand, suction dominates oscillations in the velocity profiles. For the no-slip case, Bödewadt's solution shows that the boundary layer effects extend out to about $\zeta = 8$. But it is evident from all figures that injection increases the boundary layer thickness and suction has an opposite effect. Variations of transverse velocity profiles with slip and in presence of suction/injection are shown in Figs. 4-6. It is clear that in case of no-slip, G assumes its asymptotic value 1 far away from the disk surface with small injection velocity. However, even in presence of injection, the boundary layer thickness decreases with an increase in λ. Axial velocity profiles are plotted in Figs 7-9. These figures also depict that in presence of slip, suction and injection have substantial effects. Finally, the variation of the moment coefficient, C_m with W is plotted in Fig. 10. $C_m = \frac{-\pi G'(0)}{\sqrt{\mathbb{R}}}$ is the measure of torque required to maintain the disk at rest. It is interesting to find that C_m decreases in magnitude when W changes its sign from negative to positive and attains the value zero after a critical value of W. This critical value depends on the slip parameter $\lambda(=\eta)$.

5 Concluding remarks

In this note, one has adopted an effective multiple shooting method to obtained an exact numerical solution to the resulting system of fully coupled and nonlinear differential equations, arising due to the swirling flow of a viscous fluid near a rough infinite stationary disk. The conventional no-slip conditions are replaced by partial slip boundary conditions. A uniform suction/injection (W) velocity is applied at the surface of the disk. It is observed that even a small value of injection enhances the oscillation in the velocity profiles and increases the boundary layer thickness. Whereas, both suction and slip dominates the oscillations and decrease the boundary layer thickness. The moment coefficient C_m becomes zero after a critical value of W.

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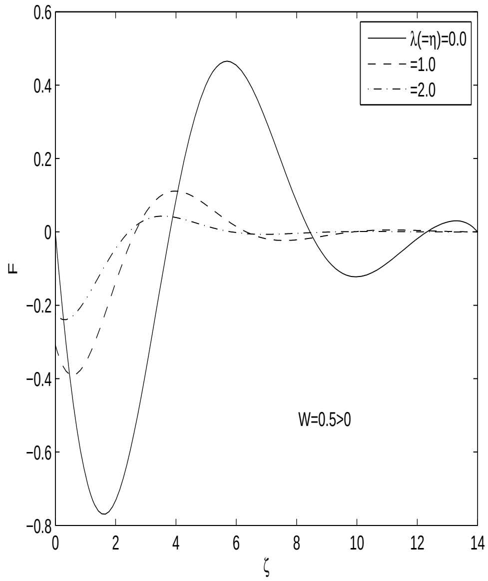

Figure 1: Variation of F with $\lambda(=\eta)$ when $W = 0.5$.

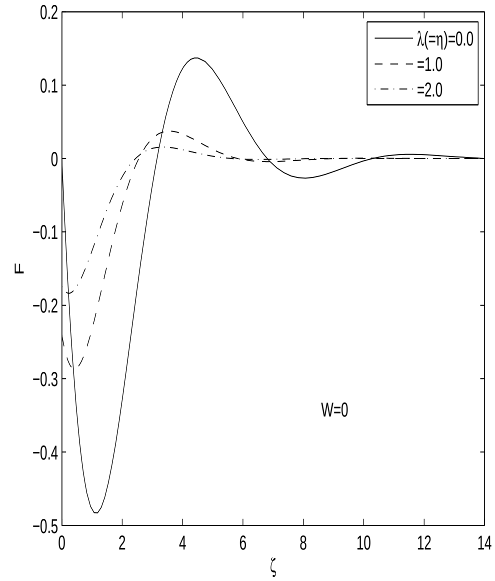

Figure 2: Variation of F with $\lambda(=\eta)$ when $W = 0$.

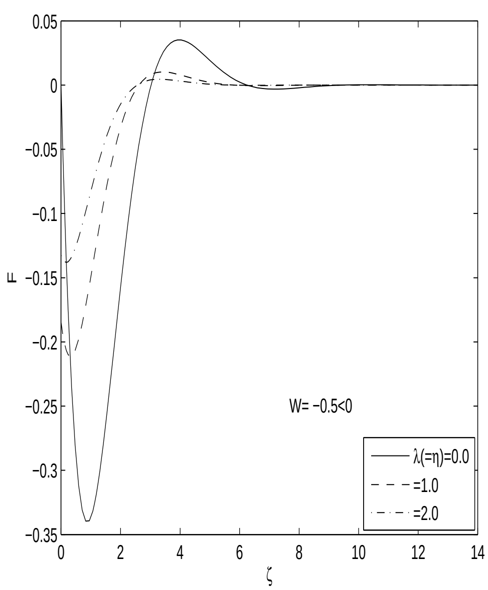

Figure 3: Variation of F with $\lambda(=\eta)$ when $W = -0.5$.

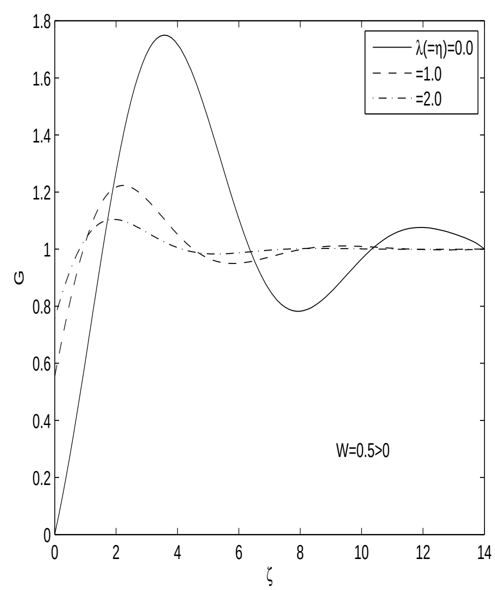

Figure 4: Variation of G with $\lambda(=\eta)$ when $W = 0.5$.

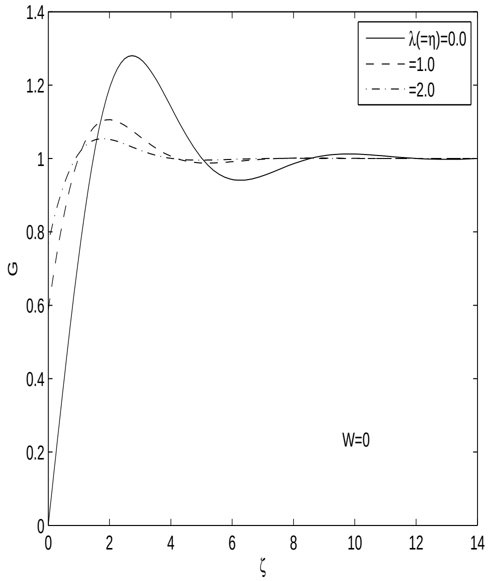

Figure 5: Variation of G with $\lambda(=\eta)$ when $W=0$.

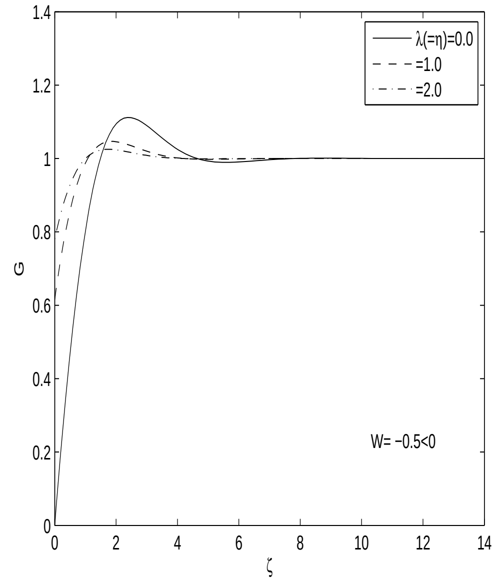

Figure 6: Variation of G with $\lambda(=\eta)$ when $W=-0.5$.

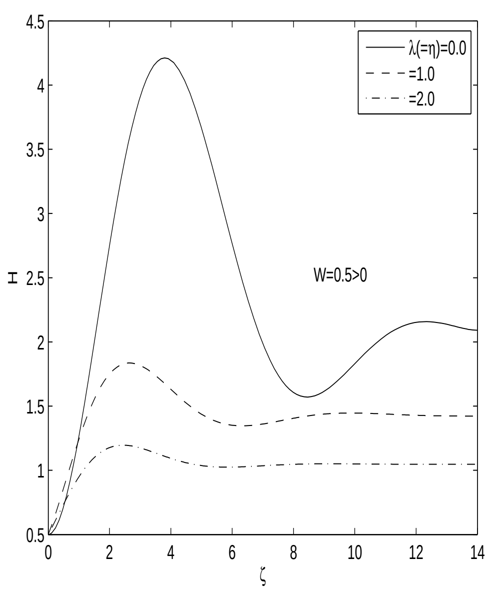

Figure 7: Variation of H with $\lambda(=\eta)$ when $W=0.5$.

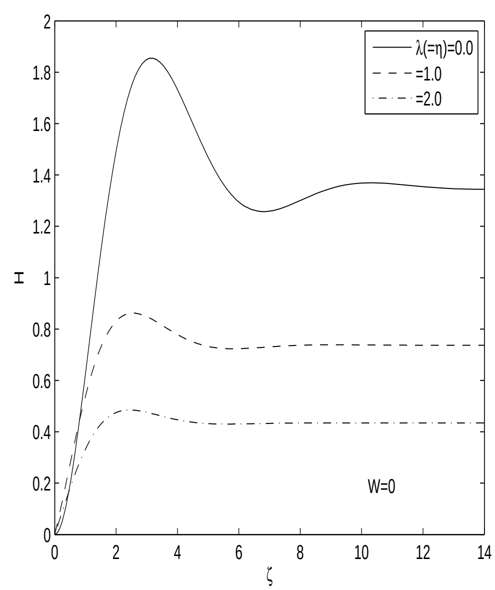

Figure 8: Variation of H with $\lambda(=\eta)$ when $W=0$.

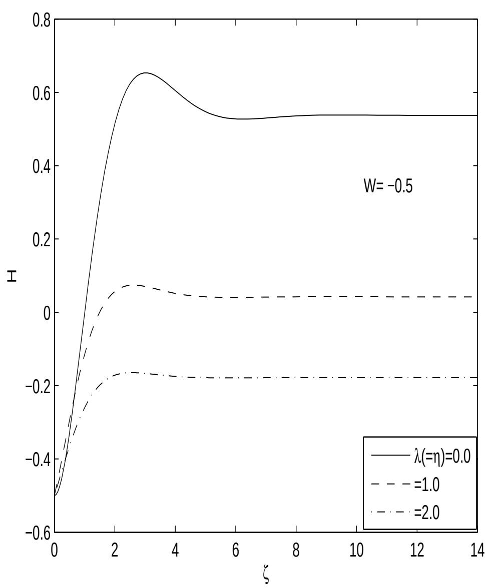

Figure 9: Variation of H with $\lambda(=\eta)$ when $W = 0.5$.

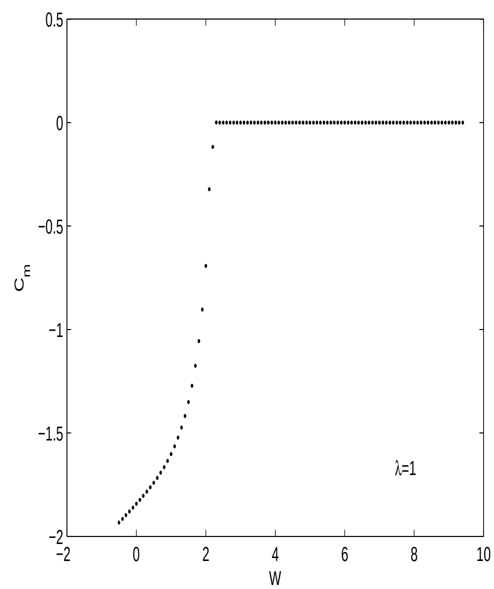

Figure 10: Variation of C_m with W at $\lambda(=\eta) = 1$.