PAPR Reduction in OFDM systems

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***Abstract*—In a communication system, operation of the transmitter power amplifier is limited to linear range. Input signal with an amplitude more than the transmitter power amplifier linear range results in signal distortion. Hence, the input signal to the transmitter should be with low peak to average power ratio. This paper presents a new method of reducing the peak to average power ratio in OFDM system. The proposed method is based on DCT aided successive addition and subtraction of OFDM symbols inside the single OFDM frame. Performance of the proposed method is evaluated and found to be superior to PTS, SLM techniques.**

***Index Terms*—OFDM, PAPR, DCT, PTS, SLM.**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is an orthogonal multicarrier communication system. Bandwidth efficiency, high data rate and immune to fading makes the OFDM systems preferred choice for modern communication system and are being implemented in many modern communication systems like Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB), Wireless Local Area Network (WLAN) and Long Term Evolution (LTE). Although OFDM gives many advantages, it suffers from many technical difficulties. Few of these difficulties include tight frequency synchronisation, time offset, peak to average power ratio (PAPR) and channel estimation.

In a multicarrier communication system, like OFDM, independent phases of subcarriers may have constructive or destructive effect. When all subcarriers have same phase then the constructive effect gives high peak amplitude and produces high peak to average power ratio (PAPR). If amplitude of the OFDM signal is greater than linear range of transmitter amplifier, the amplifier may operate in saturation region which leads to nonlinear distortion. To mitigate this high PAPR many methods have been used [1–5].

The simplest technique to mitigate nonlinearity around the peak is clipping technique. Since OFDM signal is bounded in frequency, not in time, clipping technique suffers from in-band and out-band radiations thereby, destroying the orthogonality among subcarriers. Clipping technique includes clipping and filtering, peak windowing and peak canceling as mentioned in [3, 6–8].

Another method to overcome high PAPR is coding technique where, specific code words are used to minimise the PAPR. Though, coding technique has no distortion effect and no out-of-bound radiation, still this technique has an adverse effect on bandwidth efficiency as the code rate is reduced. Another disadvantage of this technique is the complexity associated with finding best codes. If large number of subcarriers are used, a large memory size is required to store the look up table for coding and decoding. Some of coding technique includes Reed Muller Code, Hadamard Code and M sequence codes [2, 9–16].

Scrambling is another technique to reduce PAPR. This method is a probabilistic approach. In this method, the input data block of the OFDM symbols are scrambled (i.e multiplied with different phase) and the resulting OFDM symbols with minimum PAPR is chosen to transmit. It does not suffer from out of bound radiation but the spectral efficiency decreases. Again, computational complexity increases with increase in the number of subcarrier. Few of technique based on this includes partial transmit sequence (PTS) [1, 4, 17, 18], selective mapping (SLM) technique [19–21], tone injection (TI) technique and tone reservation (TR) technique [22, 23].

Discrete Fourier Transform (DFT) spreading technique is also a method of minimising the PAPR. This technique is specifically useful for mobile terminals and they are used in uplink transmission. In this method, the input signal is spread with DFT that can be subsequently taken by IFFT. This technique reduces the PAPR of the OFDM signal up to single carrier transmission [5, 24–26].

Present research in PAPR reduction are based on modified SLM technique, PTS technique and Discrete Cosine Transform (DCT) based PAPR reduction methods proposed in [8, 19, 20, 22, 27, 28]. Above study shows DCT based methods significantly reduces the PAPR. In this paper we proposed a new method that is based on iterative addition and subtraction of OFDM symbols followed by DCT. This technique gives better performance than only DCT based PAPR reduction.

In this section brief back ground study about PAPR reduction was presented. The rest of the paper is organised as follows. Section II describes the PAPR system model of OFDM system. Section III reveals the proposed method. Simulation study and result is discussed in Section IV followed by conclusion in Section V.

II. System Model

A OFDM system block diagram is presented in Fig. 1. First binary data are grouped and mapped into multi-amplitude-multi-phase signals. After pilot insertion, the modulated data

$X(k)$ are sent to an IFFT. The transformed data are multiplexed into $x(n)$ as

$$x(n) = IFFT\{X(k)\} = \sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N} \quad for \;\; n = 0, 1, ..., N-1 \tag{1}$$

where N is the number of subcarriers. The guard interval N_g is inserted to prevent inter-symbol interference (ISI). The resultant samples $x_g(n)$ are

$$x_g(n) = \begin{cases} x(N+n) & n = N_g, N_g-1, \ldots, -1 \\ x(n) & n = 0, 1, \ldots, N-1 \end{cases} \tag{2}$$

where N_g is the number of samples in the guard interval. The transmitted signal is then sent to a frequency selective multipath fading channel. The received signal can be represented by

$$y_g(n) = x_g(n) \otimes h(n) \; + \; w(n) \tag{3}$$

Where $h(n)$ is the channel impulse response (CIR) and $w(n)$ is the Additive White Gaussian Noise (AWGN) and $\otimes$ is the circular convolution.

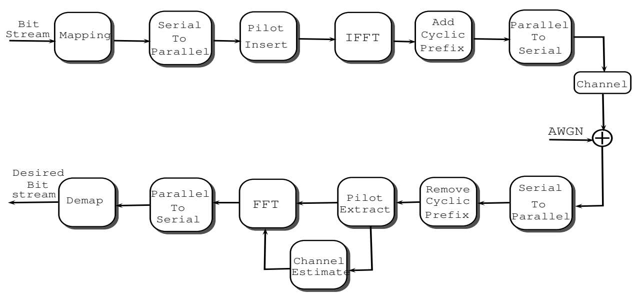

Fig. 1. Block Diagram of the OFDM systems

PAPR is associated with the time domain signal. Here our time domain signal is $x(n)$. In communication system baseband signal is transmitted with a transmitted pulse. So the multi-amplitude-multi-phase sequence of $x(n)$ is given by

$$s(t) = \sum_{k}^{N} x(n) g(t - kT_s) \tag{4}$$

Where $g(t)$ is the transmit pulse and T_s symbol duration. PAPR is defined as ratio of maximum power to the average power of complex signal. Mathematically it is given by

$$PAPR\{s(t)\} = \frac{max|s(t)|^2}{E\{|s(t)|^2\}}$$

In OFDM system maximum power appears when all subcarrier components happens to be in same phase and results in $PAPR = N$. The PAPR is usually calculated through Cumulative Distribution Function (CDF) which is a probabilistic approach. A complex OFDM symbols is composed of real parts and imaginary parts. These real and imaginary parts are asymptotically Gaussian distribution for a large number of subcarriers. Hence, the amplitude of OFDM symbols follows Rayleigh distribution. Assuming average power of $s(t)$ is equal to one, $E\{|s(t)|^2\} = 1$; then z_n are i.i.d. Rayleigh random variables normalised with its own average power will have Probability density Function (PDF) [29]

$$f_{Z_n}(Z) = \frac{z}{\sigma^2} e^{-\frac{z^2}{2\sigma^2}} = 2ze^{-z^2}$$

where Z_n is the magnitude of complex samples $\{s(nT_s/N)\}_{n=0}^{N-1}$ and $E\{Z_n^2\} = 2\sigma^2 = 1$.

Now CDF Z_{max} is given as

$$\begin{aligned} F_{Z_{max}}(z) &= P(Z_{max}) < z \\ &= P(Z_0 < z).P(Z_1 < z) \ldots P(Z_{N-1} < z) \\ &= (1 - e^{-z^2})^N \end{aligned} \tag{5}$$

Where

$$Z_{max} = \max_{n=0,1,2\ldots,N-1} Z_n$$

is called the crest factor (CF).

To find the probability that CF exceeds z we consider Complementary CDF (CCDF). CCDF can be given as

$$\begin{aligned} \tilde{F}_{Z_{max}}(z) &= P(Z_{max}) > z \\ &= 1 - P(Z_{max} \leq z) \\ &= 1 - F_{max}(z) \\ &= 1 - (1 - e^{-z^2})^N \end{aligned} \tag{6}$$

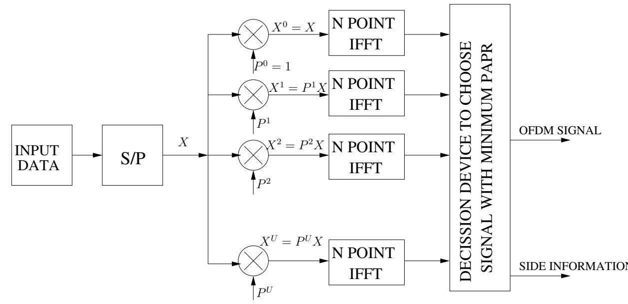

Fig. 2. Selective Mapping Technique for PAPR reduction

III. PAPR REDUCTION METHODS

Different type of PAPR reduction methods have been proposed as described in Section. I. This section, however, describes SLM, PTS and the proposed method. SLM and PTS schemes are seen to minimise PAPR effectively. So, performance of proposed method will be compared against SLM and PTS technique.

A. Selective mapping technique (SLM)

In SLM technique the input data sequence is multiplied by U number of phases independently. The block diagram for selective mapping technique is shown in Figure. 2. The input data block $X = [X(0) \; X(1) \; \ldots \; X(N-1)]$ is multiplied with different independent phases $P^U = [P_0^U \; P_1^U \; \ldots \; P_{N-1}^U]$. Where $P_v^u = e^{j\phi_v^u}$ and $\phi_v^u \in [1, 2\pi)$ for $v = 0, 1, \ldots \; N-1$

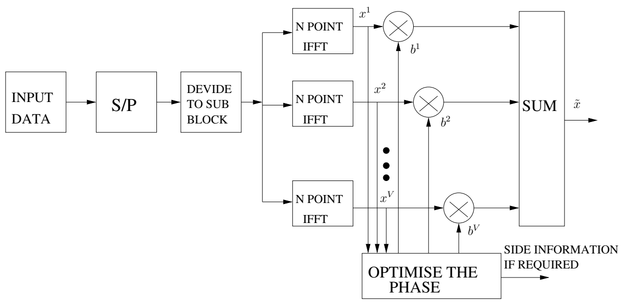

Fig. 3. Partial Transmit Sequence Scheme for PAPR reduction

and $u = 1, 2, \ldots U$. The modified data is then represented by $X^U = [X^U(0)\ X^U(1)\ X^U(2)\ \ldots\ X^U(N-1)]^T$. IFFT of U number of independent sequences, X^U, are computed. After IFFT operation the time domain signal is represented as $x^U = [x^U(0)\ x^U(1)\ x^U(2)\ \ldots\ x^U(N-1)]^T$. The decision device calculates PAPR of each of independent $x^u$and chooses the signal with minimum PAPR that is to be transmitted. The corresponding phase sequence for which PAPR is minimum will be sent as side information. The phase for which PAPR is minimum is given by

$$\tilde{U} = \underset{u=1,2,\ \ldots\ U}{\arg\min} \left(\max_{n=1,2,\ \ldots\ N-1} |x^u(n)| \right) \tag{7}$$

B. Partial Transmit Sequence (PTS) technique

In PTS scheme the input data block of length N is divided into V disjoint sub blocks. V number of sub blocks can be given as

$$X = [X^1\, X^2\, \ldots\, X^V]$$

Where X^i, for $i = 1, 2, \ldots V$, are sub blocks that are located consecutively and are of equal length. Note that incase of SLM technique, scrambling is applied to each subcarrier where as in case of PTS scheme scrambling is applied to a set of subcarrier which constitutes sub block. The PTS scheme is shown in Figure. 3. Each of the sub block is multiplied by a phase factor $b^v = e^{j\phi_v}$ for $v = 1, 2, \ldots V$. Subsequently, the IFFT of each sub block is computed.

Hence, the time domain signal after IFFT operation is given by

$$\begin{aligned} x &= IFFT\left\{ \sum_{v=1}^{V} b^v\ X^v \right\} \\ &= \sum_{v=1}^{V} b^v\ IFFT(X^v) \\ &= \sum_{v=1}^{V} b^v\ x^v \end{aligned}$$

x^v is called the partial transmit sequence. The phase vector for minimum PAPR is given by

$$\tilde{b}^1\ \tilde{b}^2\ \ldots \tilde{b}^V = \underset{[b^1\ b^2\ \ldots b^V]}{\arg\min} \left(\max_{n=0,1,\ \ldots N-1} \left| \sum_{v=1}^{V} b^v\ x^v(n) \right| \right) \tag{8}$$

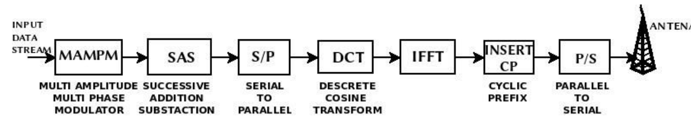

Fig. 4. Transmitter using the proposed method of PAPR reduction

The corresponding time domain signal with minimum PAPR is given by

$$\tilde{x} = \sum_{v=1}^{V} b^v\ x^v$$

C. Proposed method

Basically, the principle behind this method is to change the phase of each successive symbols inside the OFDM frame. For N number of subcarriers we have N number of OFDM symbols. Each odd OFDM symbol is multiplied by +1 and each even OFDM symbol is multiplied by -1. First OFDM symbol to be transmitted is maintained as it is. The second OFDM symbol to be transmitted is addition of first two OFDM symbols (after multiplication by either +1 or -1). Similarly the process goes on for N number of symbols. After getting transformed N OFDM symbols we take the DCT of the transformed signal. Then it is passed through IFFT block. Mathematically the transformed $X(K)$ is given by

$$X_t(k) = \begin{cases} X(k) & \text{for n=1} \\ \sum_{i=odd}^{k} X(i) - \sum_{i=even}^{k} X(i) & \text{k=1,2,...,N} \end{cases}$$

Where $X_t(k)$ is the transformed signal.So the time domain signal after IFFT is

$$x(n) = IFFT\{DCT\{X_t\}\}$$

The required transmitter using the proposed method of PAPR reduction is shown in Fig. 4

IV. Simulation Study and Result

CCDF of the proposed method is carried out by simulation. In this simulation QPSK and 16-QAM modulation were used. 512 number of subcarriers were incorporated for data transmission. Fig. 5 and Fig. 6 are the PAPR performance comparison of the proposed method using QPSK and 16-QAM modulation. From Fig. 5 it is evident that the proposed method gains a PAPR margin of 3dB lower than the theoretical PAPR. Moreover, the performance of proposed method is asymptotically same as of DCT method after a certain threshold of PAPR. At the lower PAPR the proposed method outperforms the DCT method too. In Fig. 6 it is seen that performance of proposed method is better when higher constellation modulation is used. Fig. 7 shows PAPR performance of different methods like PTS,SLM, DCT and proposed method. In this paper, PTS scheme is performed by deviding the input data block into 4 sub blocks and only two phases (+1 and -1) are used to get minimum PAPR. From Fig. 7 it is clear that the proposed method performs better than the SLM and DCT technique. However , at higher PAPR PTS technique outperforms the proposed method.

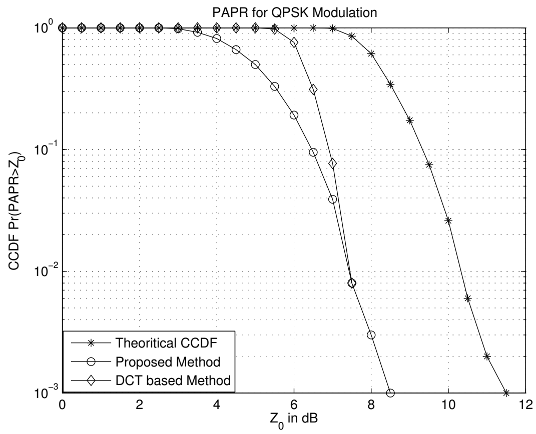

Fig. 5. PAPR performance comparison of proposed method using QPSK modulation

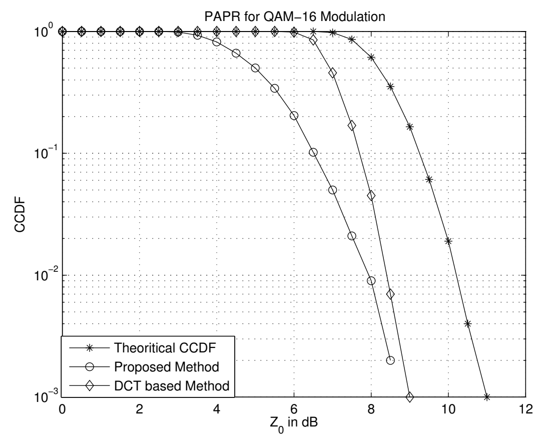

Fig. 6. PAPR performance comparison of proposed method using QAM-16 modulation

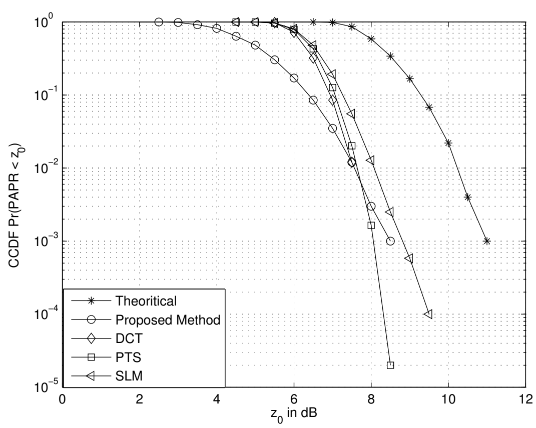

Fig. 7. PAPR comparison of different methods

V. Conclusion

A new method based on successive addition subtraction aided DCT for PAPR is presented. The performance in term of CCDF was evaluated. The proposed algorithm performance was found to be better than other algorithms. Using this method, in the receiver side, one can easily decode the transmitted symbols by taking simply the differentiation after IDCT. One more advantage of the proposed method is that side information is not required to receive the signal at the receiver side. The use of side information reduces spectral efficiency of the OFDM system.

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