

# Performance Comparison of Various ISI cancellation SFBC-OFDM Decoders in Highly Frequency Selective Channel

Jyoti P. Patra  
Dept. of Electronics and Communication  
National Institute Of Technology  
Rourkela-769008, India  
E mail<sup>1</sup>: jyotiprasannapatra@gmail.com

Poonam Singh  
Dept. of Electronics and Communication  
National Institute Of Technology  
Rourkela-769008, India  
E mail: psingh@nitrrkl.ac.in

**Abstract**— The Alamouti coded Space Frequency Block Code (SFBC) based Orthogonal Frequency Division Multiplexing OFDM scheme is upon analyzed by assuming that the Channel Frequency Response (CFR) remains constant over Alamouti code period (two adjacent OFDM subcarriers). But when the channel is highly frequency selective, this assumption does not hold good and causes inter-symbol-interferences (ISI). Thus, due to the ISI effect, the simple Alamouti decoder is not sufficient to recover the original transmitted signal from the mixed transmitted signals at the receiver side. In this paper, we have investigated several ISI cancellation SFBC OFDM decoders which include SIC, ZF and DF. Finally, the performances of the above decoders are compared on the basis of symbol error rate (SER) in highly frequency selective channel environments.

**Index Terms** — SFBC, OFDM, ISI, ZF, SIC, DF

## 1. INTRODUCTION

In recent years, transmit diversity have attracted much more attention due to link reliability and spectral efficiency in modern wireless communication systems [1]-[2]. The first transmit diversity scheme popularly known as space time block code (STBC) was first proposed by Alamouti for flat fading channel [3]. But in practical scenario the channel is frequency selective rather than flat fading. Hence, the STBC scheme is applied to OFDM technique as OFDM converts the frequency selective fading channel into many narrow parallel flat fading channels [4]. The use of OFDM also offers the other possibility of coding in the frequency dimension in a form of space frequency block code OFDM (SFBC-OFDM) [5]. The SFBC-OFDM scheme outperforms the STBC-OFDM scheme in fast fading channel. However, in frequency selective channel SFBC OFDM suffer from ISI and leads to performance degradation [5].

In literature, various detection methods have been addressed for cancelling the effect of ISI SFBC OFDM scheme in frequency selective channel. In [6], a Diagonalized Maximum Likelihood Detector (DMLD) is proposed to improve the system performance for SFBC OFDM and STBC-OFDM in both the multipath fading environments and the fast fading environments respectively. But the DMLD method is more computational complexity. Hence, a similar performance

with low computational complexity ZF is proposed in [7] for STBC OFDM in fast fading channel. This scheme can be easily extended to SFBC OFDM in frequency selective channel. Therefore, we use the ZF scheme to improve the SFBC OFDM system performance by removing the ISI effect. A successive interference cancellation (SIC) method is proposed for STBC-OFDM system in [8]. As the properties of STBC OFDM scheme in fast fading channel is similar to SFBC OFDM in frequency selecting channel, the SIC scheme is also applied to SFBC OFDM. The decision feedback (DF) method is originally proposed in [9]. Then it is extended for STBC/SFBC OFDM system in [10]. The performance of DF method is better than the Alamouti, SIC and ZF methods but with a little bit higher computational cost. In this paper, we have studied the above ISI cancellation decoders for SFBC-OFDM system in highly frequency channel.

The rest of the paper is organized as follows. In Section 2, the system model for the SFBC based OFDM system along with the channel model is discussed. The different ISI cancellation decoders are presented in Section 3. In Section 4, the performances of these decoders are compared on the basis of SER. Finally, Section 5 concludes the paper.

## 2. SYSTEM MODEL

In this section, an SFBC based OFDM system with two transmit antenna and one receive antenna is described along with the exponential decaying power delay profile model is discussed as multipath Rayleigh fading channel.

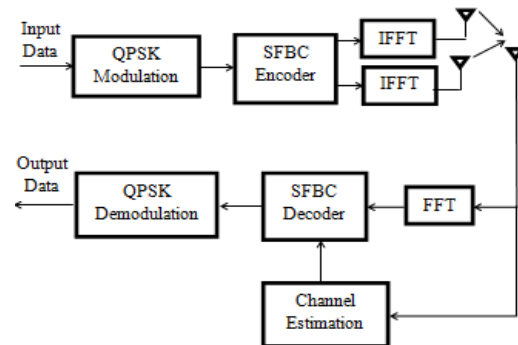


Fig.1 Block diagram of an SFBC-OFDM system model

### A. SFBC-OFDM System Model

The SFBC OFDM is formed over space and frequency domain. The schematic diagram of SFBC-OFDM for two transmit antenna and one receive antenna is shown in the Fig 1. The data sequence is generated, modulated and passed into the SFBC encoder. The SFBC encoder converts the modulated output data sequence  $X(k)$  into two data vector  $X_1(k)$  and  $X_2(k)$  and are given by

$$\begin{aligned} X(k) &= [s_0, s_1, s_2, s_3, \dots, s_{N-2}, s_{N-1}] \\ X_1(k) &= [s_0, -s_1^*, s_2, -s_3^*, \dots, s_{N-2}, -s_{N-1}^*] \\ X_2(k) &= [s_1, s_0^*, s_3, s_2^*, \dots, s_{N-1}, s_{N-2}^*] \end{aligned} \quad (1)$$

For simplicity of operation, the SFBC-OFDM can be described in terms of even and odd components. Let  $X_{1e}(k)$ ,  $X_{1o}(k)$ ,  $X_{2e}(k)$ ,  $X_{2o}(k)$  be two length  $N/2$  vectors denoting the even and odd component vectors of  $X_1(k)$  and  $X_2(k)$  and are given by

$$\begin{aligned} X_{1e} &= X_e & X_{1o} &= -X_o^* \\ X_{2e} &= X_o & X_{2o} &= X_e^* \end{aligned} \quad (2)$$

Through the signal manipulations such as the Inverse Discrete Fourier Transform (IDFT), addition and removal of the cyclic prefix and the DFT operation, we can obtain the received symbol vector  $Y$  as

$$Y(k) = \begin{bmatrix} Y_e(k) \\ Y_o^*(k) \end{bmatrix} = \begin{bmatrix} H_{1e}(k) & H_{2e}(k) \\ H_{2o}^*(k) & -H_{1o}^*(k) \end{bmatrix} \begin{bmatrix} X_e(k) \\ X_o(k) \end{bmatrix} + \begin{bmatrix} Z_e(k) \\ Z_o^*(k) \end{bmatrix} = HX + Z \quad (3)$$

Assuming that channel frequency response remain constant over two adjacent OFDM subcarriers, that is  $H_{1e} = H_{1o}$  and  $H_{2e} = H_{2o}$ . The SFBC decoding operation is performed by multiplying  $H^H$  on both the side of the equation (2), the original transmitted signal can be recovered after taking the hard decision of the decoded signal and can be written as

$$\hat{X}(k) = Q \left( \frac{\tilde{X}(k)}{\Phi(k)} \right) \quad (4)$$

where  $\tilde{X} = H^H(k)Y(k)$  and

$$\Phi(k) = |H_{1o}(k)|^2 + |H_{1e}(k)|^2 = |H_{2o}(k)|^2 + |H_{2e}(k)|^2$$

Hence, both the even  $X_e(k)$  and odd  $X_o(k)$  estimated signals are decoupled from each other at the receiver side. However, in the highly frequency selective channel, the channels are not constant for two adjacent OFDM subcarriers. Hence,  $H^H H$  is no longer a orthogonal matrix and is given by

$$G = H^H H = \begin{bmatrix} \alpha_1(k) & \beta(k) \\ \beta^*(k) & \alpha_2(k) \end{bmatrix} \quad (5)$$

where

$$G(k) = H^H(k, k) * H(k, k) = \begin{bmatrix} \alpha_1(k) & \beta(k) \\ \beta^*(k) & \alpha_2(k) \end{bmatrix}$$

$$\text{and } \alpha_1(k) = |H_{1,e}(k)|^2 + |H_{2,o}(k)|^2$$

$$\alpha_2(k) = |H_{1,o}(k)|^2 + |H_{2,e}(k)|^2$$

$$\beta(k) = H_{1,e}^*(k)H_{2,e}(k) - H_{1,o}^*(k)H_{2,o}(k)$$

$\alpha_1(k)$ ,  $\alpha_2(k)$  are the desired diversity gain terms and  $\beta(k)$ ,  $\beta^*(k)$  are the ISI terms. By multiplying  $G(k)$ , the detected output signal vector can be written as

$$\tilde{X} = \begin{bmatrix} \tilde{X}_e(k) \\ \tilde{X}_o(k) \end{bmatrix} = \begin{bmatrix} \alpha_1(k)X_e(k) + \beta(k)X_o(k) + Z'_e(k) \\ \beta^*(k)X_e(k) + \alpha_2(k)X_o(k) + Z'_o(k) \end{bmatrix} \quad (6)$$

The original transmitted signal can be recovered after taking the hard decision of the decoded signal.  $\alpha_1(k)X_e(k)$  and  $\alpha_2(k)X_o(k)$  are the desired signal.  $\beta(k)X_o(k)$  and  $\beta^*(k)X_e(k)$  are the ISI signal which are coupled with the desired signal at the receiver side. Hence, in order to accurately recover the original transmitted signal, these two ISI signals are to be cancelled.

### B. Channel Model

The wireless channel model is assumed to be highly frequency selective channel. The frequency selective is employed as exponential decaying power delay profile (PDP) [11]-[12]. The channel is modeled as finite impulse response (FIR) with total  $L+1$  non-zero path with zero mean and average power  $\sigma_1^2$ . The Rayleigh channel can be expressed as

$$h_1 = N(0, \sigma_1/2) + jN(0, \sigma_1/2) \quad (7)$$

where  $N(0, \sigma_1/2)$  is the zero mean with variance  $\sigma_1^2$ .

The power of multipath component decreases exponentially. The first path of the model is chosen to be

$$\sigma_0^2 = \frac{1 - e^{-\frac{1}{d}}}{1 - e^{-\frac{L+1}{d}}}, \quad d = \frac{\tau_{\text{rms}}}{T_s} \quad (8)$$

where  $d$  is the normalized delay spread and  $\tau_{\text{rms}}$  is the root mean squared delay of the channel.  $T_s = 1/W$ , where  $W$  is the channel (OFDM signal) bandwidth.

The energy of the  $l$ -th path can be written as

$$\sigma_l^2 = \sigma_0^2 \lambda^l, \quad l = 0, 1, 2, \dots, L. \quad (9)$$

The total number of path is given by

$$L = \tau_{\max}/T_s, \tau_{\max} = -\tau_{\text{rms}} \ln A \quad (10)$$

where  $\tau_{\text{rms}}$  is the maximum excess delay and A is ratio of non-negligible path power to first path power. For high frequency selective channel, we have taken A= -20dB and d=4 or 8. The total number of fading path for d=4 and 8 is calculated to be 12 and 24 respectively.

### 3. ISI CANCELLATION SFBC-OFDM DECODERS

In this section, we describe different decoders namely the SIC decoder, the ZF decoder and the DF decoder in order to cancel the effect of ISI in the SFBC based OFDM system for highly frequency selective channel.

#### A. SIC Decoder

The successive interference cancellation method is proposed in [8]. Due to frequency selective nature of channel, the two diversity gain terms are not same but the gains of the two ISI terms are equal as given in the Equation (5) in SFBC OFDM system. Hence the SIC decoder is based on this gain difference property to recover the original transmitted signal and is illustrated below.

$$1. \tilde{X}_e(k) = \alpha_1(k)X_e(k) + \beta(k)X_o(k) + Z'_e(k) \quad (11)$$

$$2. \tilde{X}_o(k) = \beta^*(k)X_e(k) + \alpha_2(k)X_o(k) + Z'_o(k) \quad (12)$$

$$3. \text{ if } \alpha_1(k) > \alpha_2(k)$$

$$\hat{X}_e(k) = Q(\tilde{X}_e(k)/\alpha_1(k)) \quad (13)$$

$$\hat{X}_o(k) = Q((\tilde{X}_o - \beta^*(k)\hat{X}_e(k))/\alpha_2(k))$$

$$4. \text{ else}$$

$$\hat{X}_o(k) = Q(\tilde{X}_o(k)/\alpha_2(k)) \quad (14)$$

$$\hat{X}_e(k) = Q((\tilde{X}_e - \beta(k)\hat{X}_o(k))/\alpha_1(k))$$

The SIC decoder gives better results than Alamouti decoder with a low computational cost, however it is not accurate enough to recover the original transmitted signal due to its error propagation problem.

#### B. ZF Decoder

We adopt the ZF method to cancel the effect of ISI. In the frequency selective channel, multiplying  $H^H$  with H does not give an orthogonal matrix as explained in the Equation (5). In order to make (5), an orthogonal matrix, a  $\Omega$  matrix is multiplied with the H matrix [6] and is given by

$$\Omega H = \text{diag}(\varphi_1, \varphi_2) \quad (15)$$

where  $\varphi_1$  and  $\varphi_2$  are the complex number.

After simplification of the Equation (15), we get

$$\Omega(k) = \begin{bmatrix} H_{1,o}^*(k) & H_{2,e}(k) \\ H_{2,o}^*(k) & -H_{1,e}(k) \end{bmatrix} \quad (16)$$

The  $\varphi_1$  and  $\varphi_2$  have the same value and can be written as

$$\begin{aligned} \varphi_1 = \varphi_2 = \varphi \\ = H_{1,o}^*(k)H_{1,e}(k) + H_{2,e}(k)H_{2,o}^*(k) \end{aligned} \quad (17)$$

Substituting  $\Omega$  in the place of  $H^H$ , the Equation (6) becomes

$$\tilde{X} = \Omega Y = \text{diag}(\varphi, \varphi)X + \Omega Z \quad (18)$$

The estimated original transmitted signal can be obtained by dividing the value of  $\varphi$  on both the side of equation (18) and then taking the hard decision.

$$\hat{X}_e(k) = Q\left(\frac{\tilde{X}_e(k)}{\varphi(k)}\right) = X_e + Z \quad (19)$$

$$\hat{X}_o(k) = Q\left(\frac{\tilde{X}_o(k)}{\varphi(k)}\right) = X_o + Z \quad (20)$$

The ZF decoder is simple and gives better result than SIC decoder as it simply forces the two ISI terms to zero.

#### C. DF Decoder

The DF decoder was proposed for SFBC-OFDM system in [7]. The algorithm of DF detection method is given below.

1. Estimate the first signal  $\hat{X}_e(k)$  using ZF detection method as given in the equation (19).

2. Cancel the contribution of  $\hat{X}_e(k)$  by subtracting it from the mixed detected output signal in equation (12). Then take the hard decision to obtain the other signal  $\hat{X}_o(k)$  and is given by

$$\hat{X}_o(k) = Q((\tilde{X}_o - \beta^*(k)\hat{X}_e(k))/\alpha_2(k)) \quad (21)$$

## 4. SIMULATION RESULTS AND DISCUSSION

The performance of various ISI cancellations SFBC OFDM decoders which include Alamouti, SIC, ZF and DF are compared on the basis of SER. The total simulation parameters used for simulation is listed on the Table1.

**Table1 Simulation Parameter**

Parameter	Value
FFT Size	128
Number of Subcarrier	128
Number of CP	32
Data Modulation	16QAM
Channel Model	Exponential decaying PDP
RMS Delay spread (d)	4/8
Number of Multipath	12/24

The performance comparison of various decoders for 16QAM modulated SFBC OFDM with different receive antennas over exponential PDP with normalized delay spread of d=4 and d=8 are shown in the Fig 2 and Fig 3 respectively.

The simulation result shows that Alamouti method suffers severe performance degradation due to ISI effect. The performance of SIC method is better than Alamouti detection method but is not enough accurate due to error propagation problem. The ZF decoder simply forces the ISI signal to zero and gives better results than SIC method. The DF decoder outperforms all of the above decoders including Alamouti, SIC and ZF methods. From the Fig.2 and Fig.3, it is seen that, when, the normalized RMS delay spread increases, the SER increases drastically. The performances of different decoders in the descending order are given as follows: DF, ZF, SIC and Alamouti.

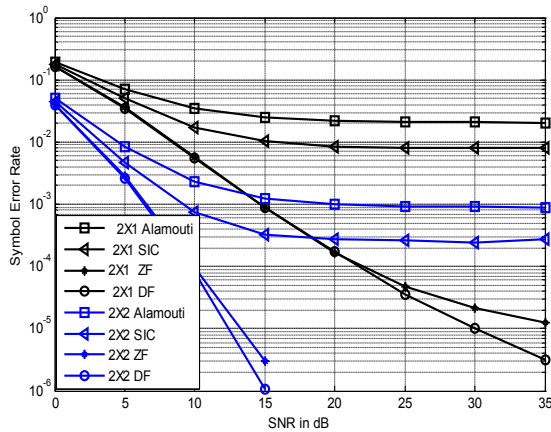


Fig.2 SER performance of various detection methods for normalized delay spread  $d=4$

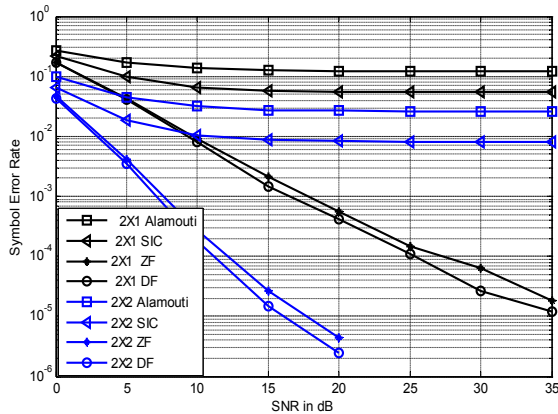


Fig.3 SER performance of various detection methods for normalized delay spread  $d=8$

#### 4. CONCLUSION

In this paper, we have investigated various ISI cancellation SFBC OFDM decoders including Alamouti, SIC, ZF and DF in highly frequency selective channel. The Alamouti method performs the worst due to the severe ISI effect. The SIC method gives better results than Alamouti method but suffer from error propagation problem. The ZF decoder simply

forces the ISI signal to zero and hence gives better results than SIC decoder as no error propagation is occurred. The DF decoder outperforms all of the described decoders because it uses the first estimated signal correctly by using ZF method and then cancels its contribution from the mixed transmitted signal at the receiver side to estimate the other data signal. The simulation results show that as the normalized delay spread increases the performance of various detection methods decreases drastically. Furthermore, we also see that as the number of receive antenna increases the SER decreases for different SFBC OFDM decoders.

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