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Universal constructions for fuzzy topological spaces

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Abstract

The concepts of fuzzy products and their duals, namely, fuzzy coproducts are already known for fuzzy topological spaces. In this paper the concepts of fuzzy equalizers and fuzzy pullbacks and their duals, namely, fuzzy coequalizers and fuzzy pushouts are introduced for fuzzy topological spaces and various results concerning fuzzy products, fuzzy equalizers and fuzzy pullbacks and their duals are explored.

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1. Introduction

Ever since the introduction of the revolutionary concept of fuzzy set by Zadeh [12] and fuzzy topological space by Chang [1], numerous topological concepts in general topology have been generalized successfully to fuzzy settings. There are many interesting universal objects, such as, products, equalizers, pullbacks and their duals, namely, coproducts, coequalizers, pushouts [6] in general topology, out of which only two, namely, products and coproducts have been studied in fuzzy settings by Hutton [5], Mashhour et al. [7], Pu and Liu [8,9] and Wong [10,11]. In this paper we introduce the concepts of fuzzy equalizers, fuzzy pullbacks and their duals, namely, fuzzy coequalizers, fuzzy pushouts for fuzzy topological spaces and explore the results concerning all such universal objects.

2. Preliminaries

Most of the concepts, notations and definitions which we have used in this paper are standard by now. But, for the sake of completeness, we recall the terminologies used in the sequel. The other unexplained notations and definitions can be referred from [1,8,9,12].

Let X be a nonempty set. A fuzzy set (f-set) A in X is a function $A : X \to [0,1]$. 1_X and 0_X are the constant fuzzy sets taking values 1 and 0, respectively, on X. The collection of all fuzzy sets in X is denoted by I^X. We denote fuzzy sets in X by capital letters $A, B, C, \ldots$. A fuzzy topological space (fts) is denoted by (X, T) or simply by X unless explicitly mentioned otherwise. Also in this paper, we define fuzzy functions via fuzzy points x_t as has been done in [10,11], even though by the usual concept of Zadeh [12], for any function $f: X \to Y$ and any fuzzy set A in X with $f(A)(y) = \sup\{A(x): f(x) = y\}$ we always get $f(x_t) = f(x)_t$.

3. Fuzzy product

In fact, the concepts of fuzzy products have already appeared in the research work of Hutton [5], Mashhour et al. [7], Pu and Liu [8] and Wong [10,11]. In order to emphasize the universal property of the products we formally put this concept in the form of a theorem.

Theorem 3.1. *For any arbitrary family $\{A_\alpha: \alpha \in L\}$ of fuzzy topological spaces the following hold*:

(1) *There exists a pair (P, p_α) where P is an fts and $p_\alpha: P \to A_\alpha$ is a fuzzy continuous (f-continuous) map for each $\alpha \in L$.*

(2) (*Universal property*) *For any fts X and a family $\{\varphi_\alpha: X \to A_\alpha, \alpha \in L\}$ of f-continuous maps, there exists a unique f-continuous map $\theta: X \to P$ such that $p_\alpha \theta = \varphi_\alpha$ for each $\alpha \in L$.*

Proof. (1) We take P to be the Cartesian product $\prod A_\alpha$ [10] together with the product fuzzy topology [10]. Recall that the product fuzzy topology on the Cartesian product $\prod A_\alpha$ is the weakest fuzzy topology that makes the projections $p_\alpha: \prod A_\alpha \to A_\alpha$ f-continuous [10]; moreover, the family of fuzzy open sets (f-open sets) of the form $p_\alpha^{-1}(U_\alpha)$, where U_α is f-open in A_α, forms a subbase for the product fuzzy topology on $\prod A_\alpha$ [10].

(2) Define $\theta: X \to P$ by $\theta(x_t) = (\varphi_\alpha(x_t))_{\alpha \in L}$ for all fuzzy points (f-points) [8] $x_t \in X$. Clearly $p_\alpha \theta = \varphi_\alpha$ for each $\alpha \in L$. θ is f-continuous since for any f-open set $V = p_\alpha^{-1}(U_\alpha)$ (belonging to the fuzzy subbase) in P, $\theta^{-1}(V) = \theta^{-1} p_\alpha^{-1}(U_\alpha) = \varphi_\alpha^{-1}(U_\alpha)$ is fuzzy open in X (since φ_α is f-continuous) for each $\alpha \in L$. $\square$

Definition 3.2. The pair (P, p_α) in Theorem 3.1 is called the *fuzzy product* of the family $\{A_\alpha: \alpha \in L\}$ of fuzzy topological spaces.

The following proposition is an immediate consequence of Theorem 3.1; it may be referred to as *mass cancellation law* for fuzzy products.

Proposition 3.3. *If (P, p_α) is a fuzzy product of the family $\{A_\alpha: \alpha \in L\}$ of fuzzy topological spaces and $\varphi, \psi: X \to P$, X being an fts, are f-continuous maps such that $p_\alpha \varphi = p_\alpha \psi$ for each $\alpha \in L$, then $\varphi = \psi$.*

Proof. Consider the family $\{p_\alpha \varphi: X \to A_\alpha, \alpha \in L\}$ of f-continuous maps. By Theorem 3.1(2) $p_\alpha \varphi = p_\alpha \psi$ factors through p_α (φ and ψ being the factorizing f-continuous maps). From the uniqueness of factorization in the fuzzy products, it follows that $\varphi = \psi$. $\square$

Proposition 3.4. *The fuzzy product of a family $\{A_\alpha: \alpha \in L\}$ of fuzzy topological spaces is unique upto fuzzy homeomorphism.*

Proof. Let (P, p_α) and (Q, q_α) be two fuzzy products of the family $\{A_\alpha: \alpha \in L\}$ of fuzzy topological spaces. By the universal property of (P, p_α), there exists a unique f-continuous map $\varphi: Q \to P$ such that $p_\alpha \varphi = q_\alpha$, for each $\alpha \in L$. Similarly by the universal property of (Q, q_α) there exists a unique f-continuous map $\psi: P \to Q$ such that $q_\alpha \psi = p_\alpha$, for each $\alpha \in L$. Thus $p_\alpha = q_\alpha \psi = p_\alpha \varphi \psi$ for each $\alpha \in L$. Also $p_\alpha = p_\alpha 1_P$ for each $\alpha \in L$. From the uniqueness condition of the factorization of the fuzzy products, it follows that $\varphi \psi = 1_P$; similarly, one proves that $\psi \varphi = 1_Q$. Thus P and Q are fuzzy homeomorphic [1]. $\square$

4. Fuzzy coproduct

The dual notion of fuzzy product is fuzzy coproduct. The concept of fuzzy coproduct has already appeared in the research work of Wong [10,11] and Pu and Liu [9]. In order to emphasize the couniversal property we put this concept in the form of a theorem.

Theorem 4.1. *For any arbitrary family $\{A_\alpha: \alpha \in L\}$ of fuzzy topological spaces the following hold*:

(1) *There exists a pair (S, i_α), where S is an fts and $i_\alpha: A_\alpha \to S$ is an f-continuous map for each $\alpha \in L$.*

(2) (*Couniversal property*) *For any fts X and a family $\{\varphi_\alpha: A_\alpha \to X, \alpha \in L\}$ of f-continuous maps, there exists a unique f-continuous map $\theta: S \to X$ such that $\theta i_\alpha = \varphi_\alpha$ for each $\alpha \in L$.*

Proof. (1) We take S to be the fuzzy topological sum $\coprod A_\alpha$ [10] of the family $\{A_\alpha: \alpha \in L\}$ of fuzzy topological spaces, i.e., S is the union of the disjoint fuzzy topological spaces $A_\alpha \times \{\alpha\}$; therefore,

$$S = \coprod_{\alpha \in L} A_\alpha = \bigvee_{\alpha \in L} (A_\alpha \times \{\alpha\}).$$

An f-set U is f-open in S if and only if $U \wedge (A_\alpha \times \{\alpha\})$ is f-open in $A_\alpha \times \{\alpha\}$, for each $\alpha \in L$. We observe that $U \wedge (A_\alpha \times \{\alpha\})$ is the restriction $U|_{(A_\alpha \times \{\alpha\})}$ of the fuzzy set U to the (ordinary) set $A_\alpha \times \{\alpha\}$ [10]. We have fuzzy inclusion maps $i_\alpha : A_\alpha \to S$, $\alpha \in L$, defined by $i_\alpha(z_t) = (z_t, \alpha)$, $z_t \in A_\alpha$ and i_α is f-continuous [10].

(2) Define $\theta : S \to X$ by $\theta(z_t, \alpha) = \varphi_\alpha(z_t)$ for $z_t \in A_\alpha, \alpha \in L$. Clearly, θ is the only fuzzy map with the property that $\theta i_\alpha = \varphi_\alpha$. For proving the f-continuity of θ, consider any f-open set V in X. Then

$$\theta^{-1}(V) = \bigvee_{\alpha \in L} (\theta^{-1}(V) \wedge (A_\alpha \times \{\alpha\}))$$

$$= \bigvee_{\alpha \in L} \varphi_\alpha^{-1}(V)$$

which is f-open in S since φ_α is f-continuous for each $\alpha \in L$. Thus θ is f-continuous. $\square$

Definition 4.2. The pair (S, i_α) in Theorem 4.1 is called the *fuzzy coproduct* of the family $\{A_\alpha : \alpha \in L\}$ of fuzzy topological spaces.

The following proposition is an immediate consequence of Theorem 4.1; it may be referred to as *mass cancellation law* for fuzzy coproducts.

Proposition 4.3. *If (S, i_α) is a fuzzy coproduct of the family $\{A_\alpha : \alpha \in L\}$ of fuzzy topological spaces and $\varphi, \psi : S \to Y$, Y being an fts, are f-continuous maps such that $\varphi_{i_\alpha} = \psi i_\alpha$ for each $\alpha \in L$; then $\varphi = \psi$.*

Proof. The arguments of the proof are dual to the arguments of the proof of Proposition 3.3. $\square$

Proposition 4.4. *The fuzzy coproduct of a family $\{A_\alpha : \alpha \in L\}$ of fuzzy topological spaces is unique upto fuzzy homeomorphism.*

Proof. The arguments of the proof are dual to the arguments of the proof of Proposition 3.4. $\square$

5. Fuzzy equalizer

We now deal with another universal construction, namely the fuzzy equalizer.

Theorem 5.1. *For any pair of f-continuous maps $f, g : A \to B$ the following hold:*

(1) *There exist an fts E and an f-continuous map $e : E \to A$ such that $fe = ge$.*

(2) (*Universal property*) *For any fts X and f-continuous map $\varphi : X \to A$ satisfying $f\varphi = g\varphi$, there exists a unique f-continuous map $\theta : X \to E$ such that $\varphi = e\theta$.*

Proof. (1) We define E by $E = \{x_t \in A : f(x_t) = g(x_t)\}$ and impose the fuzzy subspace topology [4] on E. The fuzzy inclusion map $e : E \to X$ is defined by $e(x_t) = x_t$, $x_t \in E$; clearly e is f-continuous [10] and $fe = ge$.

(2) Define an f-map $\theta : X \to E$ by $\theta(x_t) = \varphi(x_t)$, $x_t \in X$. Since $f(\varphi(x_t)) = f\varphi(x_t) = g\varphi(x_t) = g(\varphi(x_t))$, we see that $\varphi(x_t) \in E$. Since $e\theta(x_t) = e(\theta(x_t)) = \theta(x_t) = \varphi(x_t)$, $x_t \in X$ we have $e\theta = \varphi$. θ is f-continuous since for any f-open set U in E, $\theta^{-1}(U) = \theta^{-1}(e^{-1}(U)) = \theta^{-1}e^{-1}(U) = \varphi^{-1}(U)$ is f-open in X (since φ is f-continuous). Clearly, θ is unique. $\square$

Definition 5.2. The pair (E, e) in Theorem 5.1 is called the *fuzzy equalizer* of the f-continuous maps $f, g : A \to B$.

6. Fuzzy coequalizer

The dual notion of fuzzy equalizer is fuzzy coequalizer.

Theorem 6.1. *For any pair of f-continuous maps $f, g : A \to B$ the following hold:*

(1) *There exist an fts Q and an f-continuous map $q : B \to Q$ such that $qf = qg$.*

(2) (*Couniversal property*) *For any fts X and f-continuous map $\varphi : B \to X$ satisfying $\varphi f = \varphi g$, there exists a unique f-continuous map $\theta : Q \to X$ such that $\varphi = \theta q$.*

Proof. (1) Define $\tilde{Q}$ by $\tilde{Q} = \{(f(x_t), g(x_t)) : x_t \in A\} \subset B \times B$. Note that $\tilde{Q}$ need not be a fuzzy equivalence relation on B [10]. Let R be the smallest fuzzy subset of $B\times B$ containing $\tilde{Q}$ that defines a fuzzy equivalence relation $\sim$ on B; for $y_s\in B$, let $[y_s]$ denote the fuzzy equivalence class of y_s. Let $Q=B/{\sim}$; Q has the quotient fuzzy topology [10]. Define $q:B\to Q$ by $q(y_s)=[y_s]$, $y_s\in B$; q is f-continuous [10]. Since for any $x_t\in A$, $f(x_t)\sim g(x_t)$, it follows that $q(f(x_t))=[f(x_t)]=[g(x_t)]=q(g(x_t))$, i.e., $qf=qg$.

(2) Define an f-map $\theta:Q\to X$ by $\theta([y_s])=\varphi(y_s)$. To show that θ is well defined, let $[y_s]=[z_u]\in Q$, i.e., $q(y_s)=q(z_u)$ such that $(y_s,z_u)\in R$. Now if we define $R_q=\{(y_s,z_u)\in B\times B\colon\ \varphi(y_s)=\varphi(z_u)\}$ then it is easy to see that R_q is a fuzzy equivalence relation. Moreover, since $\varphi(f(x_t))=\varphi(g(x_t))$ for every $x_t\in A$, it follows that $(f(x_t),g(x_t))\in R_q$; hence $\tilde{Q}\subset R_q$. Therefore $R_q\supset R$ (as R is the smallest fuzzy equivalence relation on B containing $\tilde{Q}$). Thus, $(y_s,z_u)\in R\subset R_q$ and we have $\varphi(y_s)=\varphi(z_u)$. θ is therefore well defined. θ is unique, f-continuous [10] and $\varphi=\theta q$. $\square$

Definition 6.2. The pair (Q,q) in Theorem 6.1 is called the *fuzzy coequalizer* of the f-continuous maps $f,g:A\to B$.

7. Fuzzy pullback

We shall now deal with another universal construction, namely, the fuzzy pullback.

Theorem 7.1. *For any pair of f-continuous maps $A\xrightarrow{f}C\xleftarrow{g}B$ the following hold*:

(1) *There exist an fts P and f-continuous maps $\mu:P\to A$, $\delta:P\to B$ such that $f\mu=g\delta$.*

(2) (*Universal property*) *For any fts X and f-continuous maps $\varphi:X\to A$, $\psi:X\to B$ satisfying $f\varphi=g\psi$, there exists a unique f-continuous map $\theta:X\to P$ such that $\varphi=\mu\theta$, $\psi=\delta\theta$.*

Proof. (1) Let $P=\{(x_t,y_s)\in A\times B\colon\ f(x_t)=g(y_s)\}\subset A\times B$ where $A\times B$ has product fuzzy topology [10] and P has fuzzy subspace topology [4]. Let $\mu:P\to A$, $\delta:P\to B$ be defined by $\mu(x_t,y_s)=x_t$, $\delta(x_t,y_s)=y_s$ for all $(x_t,y_s)\in P$, the usual fuzzy projection maps [10]; μ and δ are f-continuous [10]. Clearly, for any $(x_t,y_s)\in P$, $f(\mu(x_t,y_s))=f(x_t)=g(y_s)=g(\delta(x_t,y_s))$, so that $f\mu=g\delta$ and hence, (1) holds.

(2) Define an f-map $\theta:X\to P$ by $\theta(z_u)=(\varphi(z_u),\psi(z_u))$. Since $f\varphi(z_u)=g\psi(z_u)$, it follows that $(\varphi(z_u),\psi(z_u))=\theta(z_u)\in P$. Clearly, $\varphi=\mu\theta$ and $\psi=\delta\theta$. To show that θ is f-continuous, recall that the f-open sets of P are of the form $U\times V=\mu^{-1}(U)\wedge\delta^{-1}(V)$ where U and V are f-open in A and B, respectively; hence $\theta^{-1}(U\times V)=\theta^{-1}(\mu^{-1}(U)\wedge\delta^{-1}(V))=\theta^{-1}\mu^{-1}(U)\wedge\theta^{-1}\delta^{-1}(V)=\varphi^{-1}(U)\wedge\psi^{-1}(V)$ is f-open in X since φ and ψ are f-continuous [1]. For showing the uniqueness of θ, suppose that there exists another $\theta':X\to P$ such that $\varphi=\mu\theta'$, $\psi=\delta\theta'$. Thus $\mu\theta=\mu\theta'=\varphi$ and $\delta\theta=\delta\theta'=\psi$. By the uniqueness property of the fuzzy projection maps (Theorem 3.1) we have $\theta=\theta'$. $\square$

Definition 7.2 The triplet (P,μ,δ) as stated in Theorem 7.1 is called the *fuzzy pullback* of the f-continuous maps

$$A\xrightarrow{f}C\xleftarrow{g}B.$$

8. Fuzzy pushout

The dual notion of fuzzy pullback is fuzzy pushout.

Theorem 8.1. *For any pair of f-continuous maps*

$$B\xleftarrow{f}A\xrightarrow{g}C$$

the following hold:

(1) *There exists an fts Q and f-continuous maps $\mu:B\to Q$, $\delta:C\to Q$ such that $\mu f=\mu g$.*

(2) (*Couniversal property*) *For any fts X and f-continuous maps $\varphi:B\to X$, $\psi:C\to X$ satisfying $\varphi f=\psi g$, there exists a unique f-continuous map $\theta:Q\to X$ such that $\varphi=\theta\mu$, $\psi=\theta\delta$.*

Proof. (1) Let $B\vee C$ be the disjoint union of B and C, i.e., $B\vee C=(B\times\{1\})\vee(C\times\{2\})$. On $B\vee C$ define a fuzzy equivalence relation $\sim$ so that $(f(x_t),1)\sim(g(x_t),2)$, $x_t\in A$. Let $Q=(B\vee C)/{\sim}$; Q has the quotient fuzzy topology [10]. Let $\mu:B\to Q$ and $\delta:C\to Q$ be defined by $\mu(y_s)=[(y_s,1)]$, $y_s\in B$, $\delta(z_u)=[(z_u,2)]$, $z_u\in C$. Clearly, $\mu(f(x_t))=[(f(x_t),1)]=[(g(x_t),2)]=\delta(g(x_t))$, $x_t\in A$, so that $\mu f=\delta g$. μ and δ are f-continuous [10].

(2) Define $\theta : Q \to X$ by $\theta([(y_s, 1)]) = \varphi(y_s)$, $\theta([(z_u, 2)]) = \psi(z_u)$. Clearly $\varphi = \theta\mu$ and $\psi = \theta\delta$. To show that θ is f-continuous, consider an f-open set U in X. Since φ is f-continuous $\varphi^{-1}(U) = \mu^{-1}(\theta^{-1}(U))$ is f-open in B. Since μ is an f-identification [1], we conclude that $\theta^{-1}(U)$ is f-open in Q. For showing the uniqueness of θ, suppose that there exists another $\theta' : Q \to X$ such that $\varphi = \theta'\mu$ and $\psi = \theta'\delta$. Thus $\theta\mu = \theta'\mu = \varphi$ and $\theta\delta = \theta'\delta = \psi$. By the uniqueness property of the fuzzy quotient map (Theorem 4.1) we have $\theta = \theta'$. $\square$

Definition 8.2. The triplet (Q, μ, δ) as stated in Theorem 8.1 is called the *fuzzy pushout* of the f-continuous maps

$$B \xleftarrow{f} A \xrightarrow{g} C.$$

9. Some results

The proof of the following theorem shows that fuzzy pullbacks and fuzzy equalizers can be related via fuzzy products.

Theorem 9.1. *The following are equivalent*:

(a) *The fuzzy equalizer of any two f-continuous maps $f, g : A \to B$ exists.*

(b) *The fuzzy pullback of any two f-continuous maps $A \xrightarrow{f} C \xleftarrow{g} B$ exists.*

Proof. (a) $\Rightarrow$ (b): Consider the arbitrary f-continuous maps $A \xrightarrow{f} C \xleftarrow{g} B$. Let $A \times B$ be the fuzzy product of the fuzzy topological spaces A and B with fuzzy projection maps $p_A : A \times B \to A$ and $p_B : A \times B \to B$. Let (E, e) be the fuzzy equalizer of the f-continuous maps $fp_A, gp_B : A \times B \to C$; so $e : E \to A \times B$ is f-continuous and $fp_Ae = gp_Be$ (Theorem 5.1). Let $p_Ae = q_A$ and $p_Be = q_B$; so $q_A : E \to A$, $q_B : E \to B$. We claim that the triplet (E, q_A, q_B) is the fuzzy pullback of the f-continuous maps $A \xrightarrow{f} C \xleftarrow{g} B$. Let the fts X and f-continuous maps $\varphi : X \to A$ and $\psi : X \to B$ be arbitrary with $f\varphi = g\psi$. Since $A \times B$ is the fuzzy product of the fuzzy topological spaces A and B, by Theorem 3.1, there exists a unique f-continuous map $\mu : X \to A \times B$ such that $p_A\mu = \varphi$, $p_B\mu = \psi$. Since $(fp_A)\mu = f(p_A\mu) = f\varphi = g\psi = g(p_B\mu)(gp_B)\mu$ and (E, e) is the fuzzy equalizer of the f-continuous maps fp_A and gp_B, by Theorem 3.1, there exists a unique f-continuous map $\theta : X \to E$ such that $e\theta = \mu$. Hence $q_A\theta = p_Ae\theta = p_A\mu = \varphi$, $q_B\theta = p_Be\theta = p_B\mu = \psi$. Thus the required fuzzy pullback is obtained.

(b) $\Rightarrow$ (a): Consider an arbitrary pair of f-continuous maps $f, g : A \to B$. Consider the f-continuous maps

$$A \xrightarrow{\mu} B \times B \xleftarrow{\Delta_B} B$$

defined by $\mu(x_t) = (f(x_t), g(x_t))$, $x_t \in A$ and $\Delta_B(y_s) = (y_s, y_s)$, respectively. Let (E, e, e') be the fuzzy pullback of μ and Δ_B (Theorem 7.1) so that $e : E \to A$ and $e' : A \to B$ are f-continuous maps with $\mu e = \Delta_B e'$. We claim that (E, e) is the fuzzy equalizer of the f-continuous maps $f, g : A \to B$. We note that $p_1\mu = f$, $p_2\mu = g$ and $p_1\Delta_B = 1_B$, $p_2\Delta_B = 1_B$ where $p_i : B \times B \to B$ are the fuzzy projection maps, $i = 1, 2$. Therefore $fe = p_1\mu e = p_1\Delta_B e' = 1_B e' = e'$, $ge = p_2\mu e = p_2\Delta_B e' = 1_B e' = e'$ and hence $fe = ge$. Let X be an arbitrary fts and $\varphi : X \to A$ be an arbitrary f-continuous map with $f\varphi = g\varphi$. Then $p_1\mu\varphi = f\varphi = g\varphi = 1_B g\varphi = p_1\Delta_B g\varphi$, $p_2\mu\varphi = g\varphi = 1_B g\varphi = p_2\Delta_B g\varphi$. By Theorem 3.1, $\mu\varphi = \Delta_B g\varphi$. Since (E, e, e') is the fuzzy pullback of the f-continuous maps $A \xrightarrow{\mu} B \times B \xleftarrow{\Delta_B} B$, by Theorem 7.1, there exists a unique f-continuous map $\theta : X \to E$ such that $\varphi = e\theta$ (and $e'\theta = g\varphi$) and this proves the point. $\square$

Theorem 9.1 can be dualized as follows:

Theorem 9.2. *The following are equivalent*:

(a) *The fuzzy coequalizer of any two f-continuous maps $f, g : A \to B$ exists.*

(b) *The fuzzy pushout of any two f-continuous maps $B \xleftarrow{f} A \xrightarrow{g} C$ exists.*

Proof. The proof can be obtained by dualizing the arguments of the proof of Theorem 9.1. $\square$

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