TUNING OF PID CONTROLLER FOR HIGHER ORDER SYSTEM

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Abstract: Designing and tuning a proportional-integral-derivative (PID) controller appears to be conceptually intuitive, but can be hard in practice, if multiple (and often conflicting) objectives such as short transient and high stability are to be achieved. Usually, initial designs obtained by all means need to be adjusted. Good responses can be expected for processes with various dynamics, including those with low- and high-order, small and large dead time, and monotonic and oscillatory responses. The method is developed based on a second-order plus dead time modeling technique and a closed-loop pole allocation strategy through the use of root locus plot

Keywords: High performance, PID control, higher order reduction, frequency response

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1. Introduction:

PID controller has been applied in various kinds of industry control fields, as its tuning methods are developing. In the past few decades, Ziegler-Nicole method which is for first-order-plus-time-delay was proposed by Ziegler and Nichols[1], Chien-Hrones-Reswick method about generalized passive systems was proposed by Chien, Hrones and Reswick[2], and so many tuning methods were developed such as pole placement and zero-pole elimination method by Wittenmark and Astrom, internal model control (IMC) by Chien[3]. The gain and phase margin (GPM) method was proposed by Astrom and Hagglund[4], the tuning formulae were simplified by W K Ho[5].

They are thus more acceptable than advanced controllers in practical applications [6] unless response shows that it is not fulfill the requirement of the application. Owing to their popularity in the industrial world, many approaches have been developed to determine PID controller parameters for single input single output (SISO) systems [7]. In spite of the enormous amount of research work reported in the literature, many PID controllers are poorly tuned in practice [8], [9]. One of the reasons is that most of the tuning methods are derived for particular processes and situations, and therefore apply well only to their own areas. It is a common experience that it is not certain which tuning method should be chosen to provide good control to a given process. It would hence be desirable if there is a design method that works universally with high performance for general linear processes.

Let us take second order system for getting the good response after tuning. In this method first we reduce higher order system into second order system by using model reduction method. Complex variable is divided into two parts after applying angle condition. This model is able to generate peaks in its frequency response like those of oscillatory processes and these are not possible with the popular first-order plus dead time models on which most PID tuning formulae are based [10]. In spite of the low-order nature of this model, the fitting of its Nyquist plot to that of the real process is incredibly close over a frequency range important for control performance. After getting second order plus dead time system we need to design controllers which cancel out model pole. At last we get the integrator plus dead time system together with a constant gain that serves as a design parameter for determining the closed loop pole locations. Different closed loop poles are selected according to the damping ratio, delay to dead time ratio and dead time model. Satisfactory responses are obtained by using this relatively simple procedure. One more thing why it is convenient to use second order plus dead time for tuning of PID controller instead of first order plus dead time is that it generates peaks in the frequency response of oscillatory processes. Although first-order models are widely used for low-order modeling, they carry only real poles. Hence they are unable to generate peaks in the frequency response of oscillatory processes. So we are using second order plus dead time for PID tuning.

The paper is organized as follows: Section II includes higher order reduction method in which higher order system reduced into second order system and Section III include the PID tuning method in which damping ratio and speed of the response are used for tuning. Result and discussions are provided in Section IV. Section V concludes the paper.

2. **Higher Order Reduction Method:**

The transfer function $G(s)$ or the frequency response $G(jw)$ of a process is given. The single loop controller configuration as shown in Fig.1 is adopted. A PID controller in the form of

$$K(s) = K_P + \frac{K_I}{s} + K_d s \qquad (1)$$

is to be used to control the process. The tuning objective is, to find out K_p, K_I and K_d in such way that it improve the response of the system for a general class of linear processes with different dynamics.

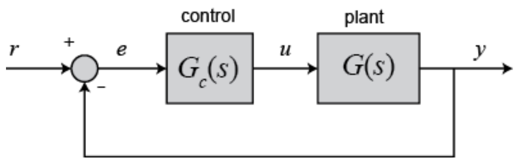

Fig.1: Single-loop Controller Feedback System

Consider the second-order plus dead time model with following structure:

$$G(s) = \frac{e^{-st_0}}{as^2 + bs + c} \qquad (2)$$

where a, b, c, and t_0 are unknowns to be determined. Depending on the values of a, b, and c , the model may have real or complex poles. Hence it is suitable for representing both monotonic and oscillatory processes and yet of sufficiently low order. In Eq. (2) we first put $s = jw$ then divide into two parts, real and imaginary part. We know that for finding out four unknowns we need four equations. So it can be constructed by fitting the process gain G(s) at two nonzero frequency points into Eq.(2). In this method, we pick the two points $s = jw_c$ and $s = jw_b$ where $\angle G(jw_c) = -\pi$

and $\angle G(jw_b) = -(\pi/2)$ such that $G(jw_c) = |G(jw_c)|$ and $G(jw_b) = |G(jw_b)|$

It follows that

$$G(jw_c) = |G(jw_c)| \angle G(jw_c)$$
$$G(jw_b) = |G(jw_b)| \angle G(jw_b)$$

After solving these equations we get the values of a ,b and c that are given below:

$$a = \frac{1}{w_c^{\,2} - w_b^{\,2}} \left[\frac{\sin(w_b t_0)}{|G(jw_b)|} + \frac{\cos(w_c t_0)}{|G(jw_c)|} \right] \quad (3)$$

$$b = \frac{\sin(w_c t_0)}{w_c |G(jw_c)|} \quad (4)$$

$$c = \frac{1}{w_c^{\,2} - w_b^{\,2}} \left[w_c^{\,2} \frac{\sin(w_b t_0)}{|G(jw_b)|} + w_b^{\,2} \frac{\cos(w_c t_0)}{|G(jw_c)|} \right] \quad (5)$$

For getting the value of t_0 it is required to made some assumptions

$$\frac{\sin(w_c t_0)}{\cos(w_b t_0)} - \frac{w_c |G(jw_c)|}{w_b |G(jw_b)|} = \Omega \quad (6)$$

To derive a good initial guess, we approximate sine and cosine function by the second order polynomial and then put that equation into equation (6), we get

$$-0.34(w_c^{\,2} - \Omega w_b^{\,2}) t_0^{\,2} + (1.7 w_c + \Omega(0.11) w_b) t_0 - \Omega = 0 \quad (7)$$

From here we get the value of t_0 that is delay function of the quadratic equation.

3. PID Tuning Method:

Now for tuning of the controller, first find the range, at which system is stable, by using Routh-Hurwitz criterion,

$$1 + G(s)H(s) = 0$$

It is solved to give

$$k \le \frac{b}{t_0}$$

From here, we get the range of k which gives sufficient gain that stabilizes the system. The equivalent time constant τ_0 of a process is inversely proportional to its speed of response. For monotonic processes, the speed of response is reflected by the locations of the dominant poles. For oscillatory ones, it is related to the real part of the complex poles which determines the system attenuation and hence serves as a measure of the process speed. According to equivalent time constant principle [11], we have

$$\frac{1}{\tau} = \begin{cases} \dfrac{c}{\sqrt{b^2 - 2ac}} & b^2 - 4ac < 0 \\ \dfrac{b}{2a} & b^2 - 4ac \ge 0 \end{cases} \quad (8)$$

Where a, b, c are model parameters that can be obtained from Eq. (3) to (5). Another variable is the damping ratio of the open loop plant which is defined as

$$\varepsilon_0 = \begin{cases} \dfrac{b}{2\sqrt{ac}} & b^2 - 4ac < 0 \\ 1 & b^2 - 4ac \ge 0 \end{cases} \quad (9)$$

The PID Controller which is written in Eq. (1) rewrite in new form as

$$K(s) = k \frac{(\alpha s^2 + \beta s + \delta)}{s} \quad (10)$$

Where $\alpha = (K_D/k)$, $\beta = (K_P/k)$ and $\delta = (K_I/k)$.We choose the controller zeros which cancel out model poles i.e. $\alpha = a$, $\beta = b$ and $\delta = c$. Then resultant open loop transfer function is

$$G(s)H(s) = \frac{ke^{-st_0}}{s} \quad (11)$$

Its closed loop pole can be selected from the root locus of the loop by assigning a proper value of k.

Real and complex poles can be obtained from the closed loop. So, two possibilities are there. First either both are real or complex. In this method model pole should be cancelled out by controller zero but exact cancellation is not possible so approximate the zero to the nearest of model poles. One more reason for not exact cancellation is process can be of any order while model is only of second order. For highly oscillatory processes, it is possible that the un-cancelled dynamics drive the system to heavy oscillation, and hence it is reasonable not to create additional oscillatory dynamics by having complex closed-loop poles. Real closed-loop poles are chosen for the system instead. On the other hand, for non-oscillatory or lightly oscillatory processes, the un-cancelled dynamics will not bring the system to severe oscillation, and hence it is good to introduce some overshoot by selecting complex closed-loop poles so as to speed up the response. Based on this theory it is separated closed loop selection into four parts.

Case1: ε_0>0.7071

Complex closed-loop poles on the root locus are chosen in this case. In order for a pair of the desired poles, $s = -\varepsilon_n w_n \pm w_n\sqrt{1-\varepsilon_n^2}$ where ε_n is the closed-loop damping ratio, to be on the root locus for the system, it follows

$$w_n = \frac{\cos^{-1}\varepsilon_n}{t_0\sqrt{1-\varepsilon_n^2}}$$

Now magnitude condition then assigns the value of k to

$$k = w_n e^{-w_n t_0 \varepsilon_n}$$

After applying phase condition on it and then put damping ratio = 0.707

$$k = \frac{0.5}{t_0}$$

Case II: $\varepsilon_0 \le 0.7071$ and $0.15 \le \frac{t_0}{\tau} \le 1$

In this case, we choose the closed-loop poles as real double poles on the root-locus

$$k = \frac{1}{\tau} e^{-(t_0/\tau)}$$

Case III: $\frac{t_0}{\tau} > 1$

In this case since delay to time constant is greater than 1 so value of k is slightly greater than in case 1, which is given as

$$k = \frac{0.6}{t_0}$$

Case IV: $0.05 < \frac{t_0}{\tau} < 0.15$

In this case complex close loop pole present so value of k is given as

$$k = \frac{0.4}{t_0}$$

now PID setting is

$$\begin{bmatrix} K_P \\ K_I \\ K_D \end{bmatrix} = k \begin{bmatrix} b \\ c \\ a \end{bmatrix}$$

4. Results and Discussions:

Ex.1: Considered a non-oscillatory higher order system

$$G(s) = \frac{e^{-0.5s}}{(s+1)^2(s+2)^2}$$

By using Model reduction second order system is

$$\hat{G}(s) = \frac{e^{-1.075s}}{8.216s^2 + 7.234s + 8.287}$$

The PID parameters are calculated as

$$K(s) = 2.0223 + \frac{2.2881}{s} + 2.2684s$$

Response of the system has been shown in Figure 2.

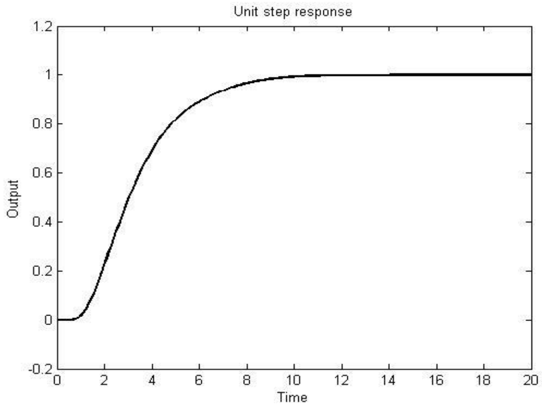

Fig.2: Step response of $G(s)=e^{-0.5s}/(s+1)^2(s+2)^2$

Analytical analysis has been shown in Table.

TABLE I

PARAMETERS	RESPONSE VALUES
% Overshoot	2%
Peak Time	9 sec
Rise Time	3.2sec
Peak Value	2
Settling Time	10 sec

Ex.2: Considered a high-order and moderately oscillatory process

$$G(s)=\frac{e^{-0.3s}}{(s^2+2s+3)(s+3)}$$

PID parameters are calculated as

$$K(s)=3.780+\frac{5.310}{s}+2.260s$$

Response of the system has been shown in Figure 3.

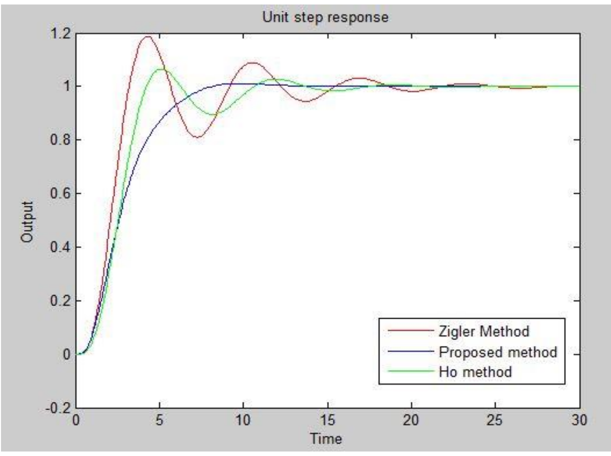

Fig.3: Step response of

$G(s)=e^{-0.3s}/(s^2+2s+3)(s+3)$

Analytical analysis has been shown in Table.

Table II:

PARAMETERS	RESPONSE VALUES		
	Z-N method	Ho Method	Proposed
% Overshoot	20%	11.7%	3.0%
Peak Time	4.8sec	4.8sec	8.7sec
Rise Time	2.3sec	2.6sec	2.7sec
Peak Value	1.20	1.17	1.03
Settling Time	18.1sec	10.2sec	7.2sec

In this high-order and moderately oscillatory process percent overshoot decreased by large extent and settling time is also reduced. So stability as well as speed of the response both improved simultaneously compares to the other method which is shown in Fig 3.

5. Conclusion:

PID controller tuning method that works for better performance in a self- regulating with different dynamics has been proposed. By this method it can be improved the performance of the system which is either monotonic or oscillatory. Higher order system reduced into second order plus dead time that is able to model process feature. With the help of pole zero cancellation in the model and controller, closed-loop poles can be easily assigned by the conventional root locus analysis method.

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