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Computation of Velocity, Pressure and Temperature Profiles in a Cryogenic Turboexpander

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Abstract

An indigenous programme on design and development of a cryogenic turboexpander has been taken up at NIT, Rourkela. This paper presents the detailed computational procedure for determining the velocity, pressure and temperature profiles in the turbine wheel, nozzle and diffuser of the turboexpander. The procedure allows any arbitrary combination of fluid species, inlet conditions and expansion ratio, the fluid properties being properly taken care of in the relevant equations. The computational process is illustrated with an example.
Keywords: turboexpander; pressure; temperature; velocity.

Nomenclature

b	channel width
C	absolute velocity
D_{tr}	turbine wheel diameter
D_{tip}	eye tip diameter
D_{hub}	eye hub diameter
d_s	specific diameter
h	enthalpy (J/kg)
$\Delta h_{in\text{-}3s}$	adiabatic enthalpy drop across turbine wheel (J / kg)
K_e	free parameter
K_h	free parameter
$\dot{m}$	mass flow rate
M	mach number
N	number of revolution
n_s	specific speed
P	power produced
p	pressure
Q	volumetric flow rate (m^3 / s)
R_m	radius of curvature of meridional streamline
r	radius
s	direction and arclength of a meridional streamline
s	entropy (J/kg-K)
t_b	blade thickness
T	temperature
U	circumferential velocity
W	relative velocity
Z	number of blades
r, θ, z	cylindrical coordinate fixed to rotor

Greek Symbols

η	efficiency
ω	rotational speed (rad/s)
ε	ratio of tip diameter to turbine wheel diameter
λ	ratio of hub diameter to tip diameter
β	relative velocity angle
τ	time coordinate
δ	angle between meridional velocity component and axial coordinate
ρ	density of fluid

Subscripts

in	inlet to nozzle
1	exit to nozzle
2	inlet to turbine wheel
3	inlet to diffuser (exit to turbine wheel)
ex	exit from diffuser
hub	hub of turbine wheel at exit
tip	tip of turbine wheel at exit
m	meridional
u	circumferential
tr	turbine
s	isentropic condition
0	stagnation condition

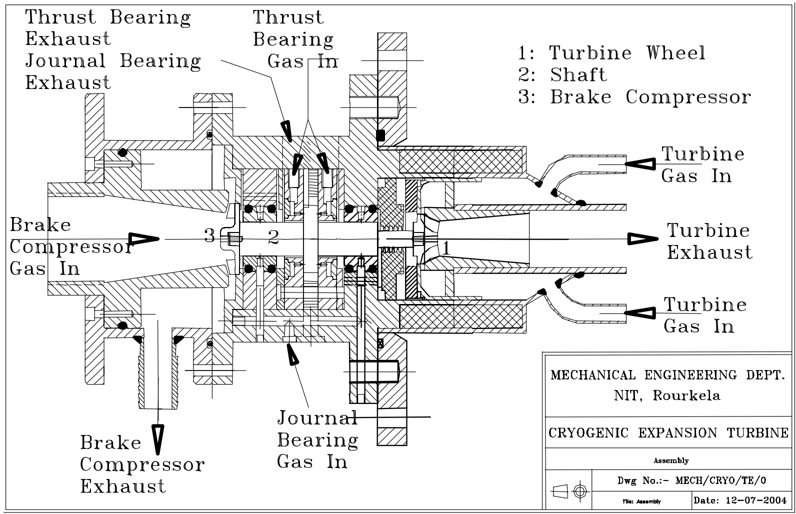


Fig.1: Longitudinal section of the expansion turbine displaying the layout of the components.

1. Introduction

The expansion turbine constitutes the most critical component of a large number of cryogenic process plants like air separation units, helium liquefiers and low temperature refrigerators. The primary function of this unit is to produce cooling by expanding a pre-cooled high pressure gas stream where power is extracted from the fluid and enthalpy of the gas decreases. Compared with the other expansion devices the use of expansion turbine offers greater economy, safety and flexibility. We can eliminate problems like high maintenance cost, large size, difficult valve operation and improper sealing by using a turboexpander. In our laboratory a turboexpander has been designed as shown in Fig.1 having the following specifications.

Working fluid	:Nitrogen
Turbine inlet temperature ($T_{0,in}$)	:130K
Turbine inlet pressure ($p_{0,in}$)	:6 bar
Exit pressure (p_{ex})	:1.5 bar
Throughput $(\dot{m})$	:180 nm^3/hr
Expected efficiency ($\eta_{T\text{-}st}$)	: 75%

This turbine is comparable in characteristics to that developed earlier at IIT Kharagpur [1,2] and draws heavily from that experience.

A turboexpander assembly consists of the following basic units:

- the turbine wheel, nozzles and diffuser,
- the shaft,
- the brake compressor,
- a pair of journal bearings and a pair of thrust bearings,
- appropriate housing.

2. Computational Procedure

The design of turbine wheel has been done following the method outlined by Balje [3] and by Kun and Sentz [4], which are based on the well known "similarity principles". The similarity laws state that for given Reynolds number, Mach number and Specific heat ratio of the working fluid, to achieve optimized geometry, two dimensionless parameters: specific speed and specific diameter uniquely determine the major dimensions of the wheel and its inlet and exit velocity triangles. Specific speed (n_s) and specific diameter (d_s) are defined as:

Specific speed $$n_s = \frac{\omega \times \sqrt{Q_3}}{\left(\Delta h_{in-3s}\right)^{3/4}} \qquad (1)$$

Specific diameter $$d_s = \frac{D_{tr} \times \left(\Delta h_{in-3s}\right)^{1/4}}{\sqrt{Q_3}} \qquad (2)$$

The true values of Q_3 and h_{3s}, which define n_s and d_s are not known a priori. Kun and Sentz [4], however suggest two empirical factors k_1 and k_2 for finding out these parameters.

$$Q_3 = k_1 Q_{ex} = k_1 \frac{\dot{m}}{\rho_{ex}} \qquad (3)$$

$$\rho_3 = \rho_{ex} / k_1 \qquad (4)$$

$$\Delta h_{in-3s} = k_2 \left(h_{0in} - h_{exs}\right) \qquad (5)$$

The factors k_1 and k_2 account for the difference between the states '3' and 'ex' caused by pressure recovery and consequent rise in temperature and density in the diffuser. Following the suggestion of Kun and

Sentz [4], we have taken k_2 = 1.03. The factor k_1 represents the ratio Q_3 / Q_{ex}, which is also equal to ρ_{ex} / ρ_3. The value of Q_{ex} is known at this stage, but that of Q_3 is not known. A guess value of k_1 is necessary to start the calculations. By taking a guess value of k_1 we find out the initial value of volume flow rate (Q_3)and density (ρ_3) at the exit of the turbine wheel by using equation (3) and (4). This Q_3 is used for determining the initial values of D_{tr} and ω from equations (1) and (2). After solving equations (11) and (12), we get the absolute velocity C_3 at the exit of turbine.

Using the standard thermodynamic relations,

$h_{03} = h_{0ex}$

$s_3 = s_{ex}$ and

$$h_3 = h_{03} - \frac{C_3}{2}$$

and using the property tables we calculate the value of ρ_3, and from that the value of the parameter k_1. The calculated ρ_3 and initial ρ_3 should be same.

We started from a guess value of 1.02 for k_1. But that led to a final value of k_1 much different from what we assumed. After examining the guessed and calculated values, we found that the assumed and calculated values agree closely if we take 1.07 for k_1.

From Balje [3] the peak efficiency of a radial inflow turbine corresponds to the values of:

$$n_s = 0.54 \quad \text{and } d_s = 3.4 \tag{6}$$

Rohlik [5] prescribes that the ratio of inlet diameter to exit tip diameter should be limited to a minimum value of 1.42 to avoid excessive shroud curvature. Corresponding to the peak efficiency point [3]:

$$\varepsilon = D_{tip} / D_{tr} = 1.45, \tag{7}$$

According to Reference [5], the exit hub to tip diameter ratio should maintained above a value of 0.4 to avoid excessive hub blade blockage and energy loss. Kun and Sentz [4] have taken a hub ratio of 0.35 citing mechanical considerations.

$$\lambda = D_{hub} / D_{tip} = 0.35 \tag{8}$$

From continuity equation, the ratio of blade height at entrance to that of the wheel is computed as:

$$b_{2tr} = \frac{\dot{m}}{(\pi D_{tr} - Z_{tr} t_{tr}) \rho_{2tr} C_{m2tr}} \tag{9}$$

For small turbines, the hub circumference at exit and diameter of milling cutters available determine the number of blades. In summary, the major dimensions for our prototype turbine have been computed as follows:

Rotational speed:	N	= 138,800 r/min
		= 14537.29 rad/s
Wheel diameter:	D_{tr}	= 26.23 mm
Eye tip diameter:	D_{tip}	= 18.09 mm
Eye hub diameter:	D_{hub}	= 6.33 mm
Number of blades:	Z_{tr}	= 7
Thickness of blades	t_{tr}	= 1 mm
Blade height at entrance	b_{2tr}	= 1.10 mm.

$$U_{3m} = \frac{\omega(D_{tip} + D_{hub})}{4} \tag{10}$$

$$\tan \beta_{3m} = \frac{C_{3m}}{U_{3m}} \tag{11}$$

$$Q_3 = A_{3m} C_{3m} = C_{3m} \left[\frac{\pi}{4} \left(D_{tip}^{\ 2} - D_{hub}^{\ 2} \right) - \frac{Zt(D_{tip} - D_{hub})}{2 \sin \beta_{3m}} \right] \tag{12}$$

where, A_3 is area at the exit of the turbine wheel.

$$W_3 = \sqrt{C_3^2 + U_3^2} \tag{13}$$

Equations (11) and (12) are now solved simultaneously for exhaust velocity C_3 and mean relative velocity angle β_{3m}, giving:

U_{3m} = 88.75 m/s

C_{3m} = 75.84 m/s

β_{3m} = 40.5°

State Points:

In	Inlet to nozzle (overall inlet)
1	Exit of nozzle
2	Inlet to wheel (exit from nozzle)
3	Inlet to diffuser (exit from wheel)
4	Exit from diffuser (overall exit)

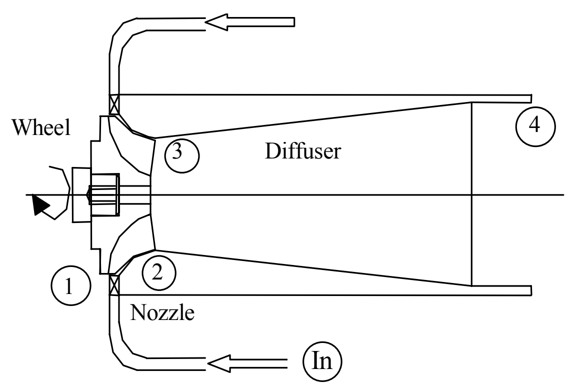


Fig. 2: The fluid flow path in a turbine and definition of the state points.

Table 1: Thermodynamic properties at different states of turboexpander

	Inlet State (stagnation condition)	State 1	State 2	State 3	State 4
Pressure (bar)	6.00	3.37	3.02	1.36	1.50
Temperature (K)	130.00	111.26	107.8	93.47	96.17
Density (kg/m^3)	16.46	10.74	9.92	5.06	5.43
Absolute Velocity (m/s)	0	188.20	205.18	75.84	20.00

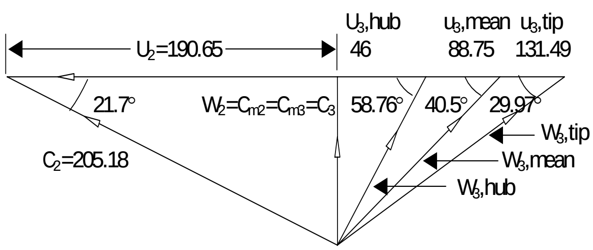


Fig.3: Composite velocity diagram for expansion turbine. (All velocities are in units of m/s)

The blade profiles have been worked out using the technique of Hasselgruber [6] maintaining the assumptions made there in such as: i) constant acceleration of the relative velocity, ii) equal meridional velocity at the wheel inlet and at exit, iii) relative flow angle at the wheel inlet = 90°. Hasselgruber's formulation leads to three characteristic functions defined as follows.

$$f_1\left(\frac{s}{s_2}\right) = \sqrt{(cosec(\beta_{3m}))^2 + \left\{(cosec(\beta_2))^2 - (cosec(\beta_{3m}))^2\right\} \times A} \tag{14}$$

where,

$$A = \left[\frac{\frac{s}{s_2} \times (k_h + 1) \times cosec(\beta_2) + (cosec(\beta_{3m}) - cosec(\beta_2)) \times \left\{1 - \left(1 - \frac{s}{s_2}\right)^{k_h+1}\right\}}{k_h \times cosec(\beta_2) + cosec(\beta_{3m})}\right]^{k_e} \tag{15}$$

$$f_2\left(\frac{s}{s_2}\right) = \frac{1}{cosec(\beta_2) + \{cosec(\beta_{3m}) - cosec(\beta_2)\} \times \left(1 - \frac{s}{s_2}\right)^{k_h}} \tag{16}$$

$$f_3\left(\frac{s}{s_2}\right) = f_1\left(\frac{s}{s_2}\right) \times \sqrt{1 - f_2^2\left(\frac{s}{s_2}\right)} \tag{17}$$

The function f_1 depicts the variation of the relative acceleration of the fluid from the wheel inlet to exit. The function f_2 gives the relative flow angle along the flow path while function f_3 is a combination of f_1 & f_2.

The radius of curvature of meridional streamline path is expressed in terms of the three characteristic functions as:

$$R_m = \left[\frac{f_1\left(\frac{s}{s_2}\right) \times f_2\left(\frac{s}{s_2}\right)}{\frac{r}{r_{3m} \times tan(\beta_{3m})} - f_3\left(\frac{s}{s_2}\right)}\right]^2 \times \frac{r}{cos(\delta)} \tag{18}$$

The angle between meridional velocity component and axial coordinate is derived to be:

$$\delta = \int_0^s \left(\frac{1}{R_m}\right) ds \tag{19}$$

Other relations on the path of determining the velocity, density, pressure and temperature profiles are summarized below.

$$r = \int_0^s (sin\,\delta) ds \tag{20}$$

$$\theta = \int_0^s \left(\frac{\sqrt{1 - f_2^2\left(\frac{s}{s_2}\right)}}{r \times f_2\left(\frac{s}{s_2}\right)}\right) ds \tag{21}$$

$$z = \int_0^s (cos\,\delta) ds \tag{22}$$

$$\beta = A\,sin\, f_2\left(\frac{s}{s_2}\right) \tag{23}$$

$$C_m = C_3 \times f_1\left(\frac{s}{s_2}\right) \times f_2\left(\frac{s}{s_2}\right) \tag{24}$$

$$b_w = (2\pi r\, sin\,\beta - z_{tr} \times t_b) / z_{tr} \tag{25}$$

$$U = \frac{C_3 \times r}{r_3 \times tan\beta_3} \quad (26)$$

$$W = C_3 \times f_1\left(\frac{s}{s_2}\right) \quad (27)$$

$$C_u = U - W\,cos\beta \quad (28)$$

$$C = \sqrt{(C_m{}^2 + C_u{}^2)} \quad (29)$$

$$M = \frac{C_3}{V_s} \quad (30)$$

Where

r = radial coordinate of mean stream line
θ = tangential coordinate of mean streamline
z = axial coordinate of mean streamline
β = relative velocity angle
C_m = meridian component of absolute velocity
b_w = width of the flow channel between two blades
U = circumferential velocity
W = relative velocity
C_u = circumferential component of absolute velocity
C = absolute velocity
V_s = sound velocity
M = Mach number of the absolute velocity

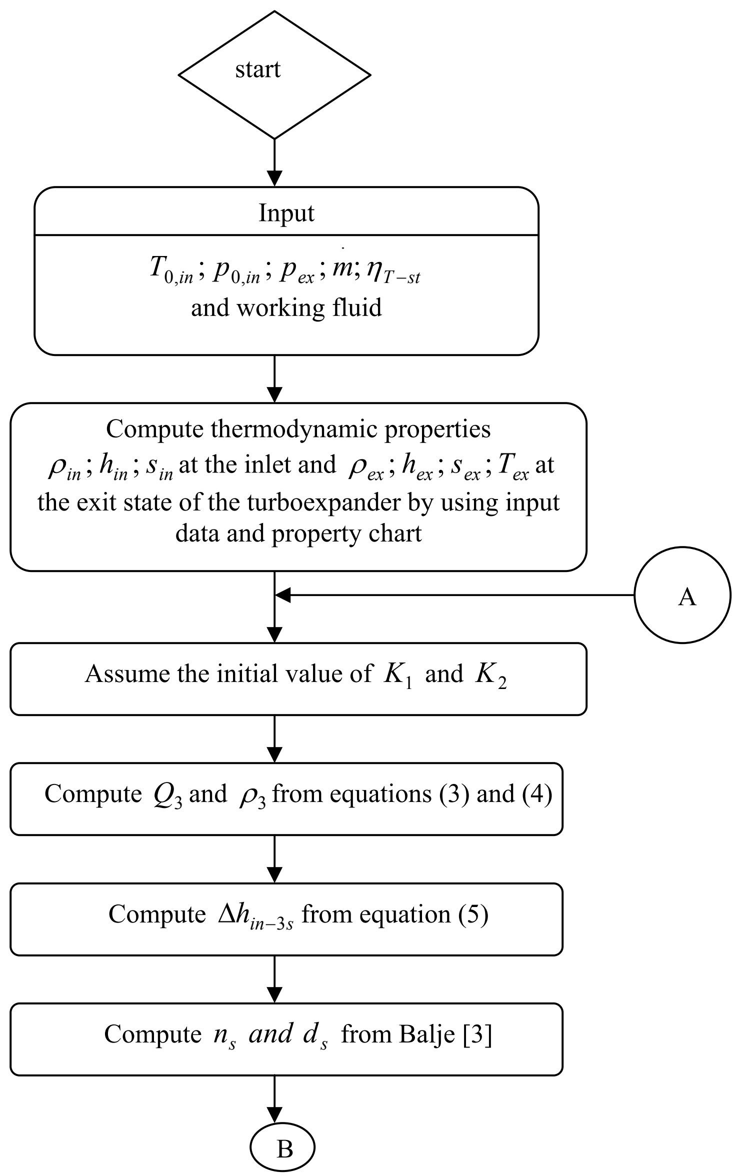
The thermodynamic quantities are computed using the following relations:

$$\rho = \rho_3 \times \left(1 + M^2 \times \frac{(m-1)\gamma}{2m} \times \frac{C_3{}^2 + U^2 - W^2}{C_3{}^2}\right)^{\frac{1}{m-1}} \quad (31)$$

$$P = P_3 \times \left(\frac{\rho}{\rho_3}\right)^m \quad (32)$$

$$T = T_3 \times \left(\frac{\rho}{\rho_3}\right)^{m-1} \quad (33)$$

Figure 4 presents a flow chart of the computational process.



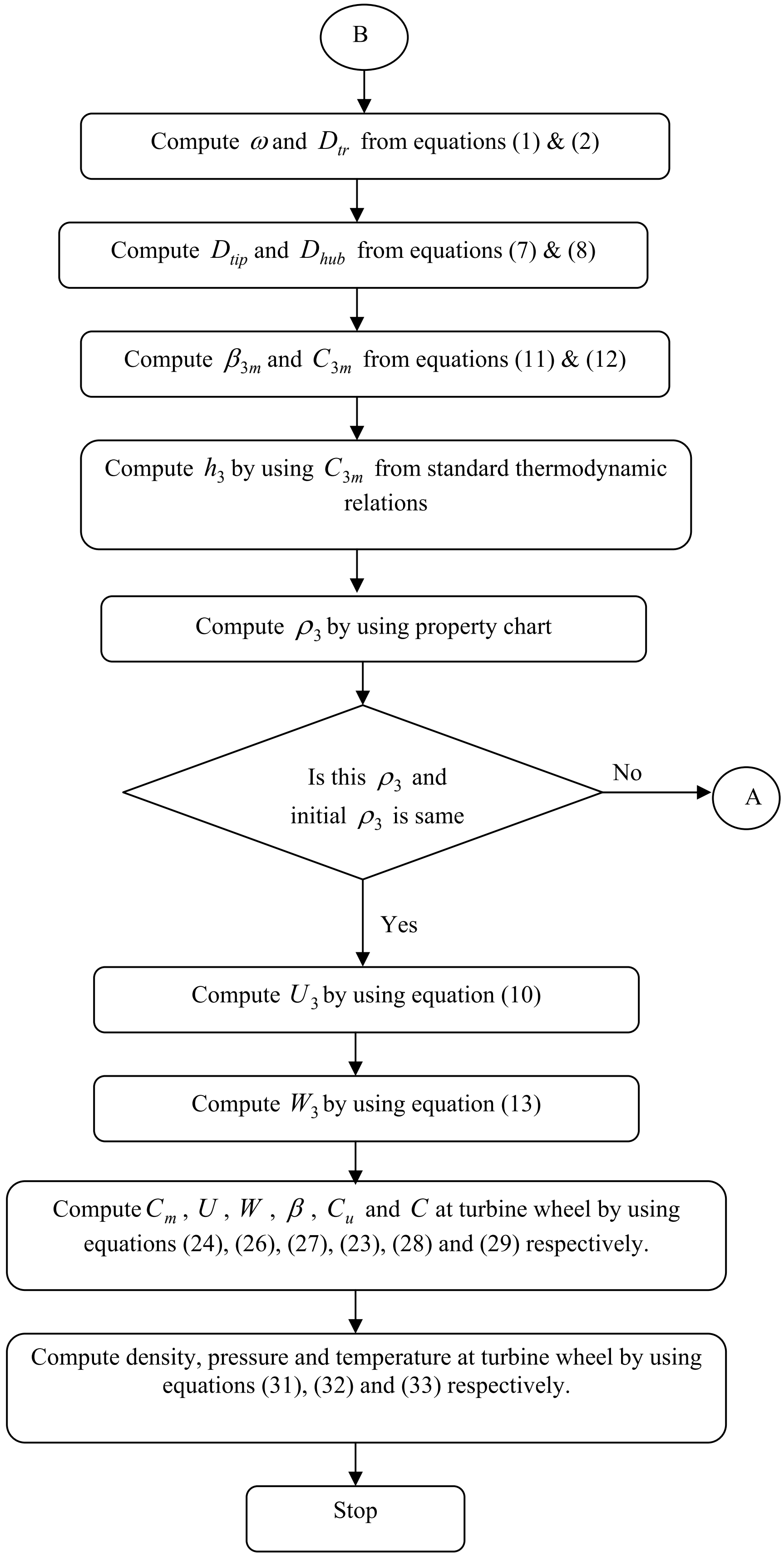


Fig. 4: Flow chart of the computer program for calculation of velocity, pressure, temperature profiles in cryogenic turboexpander

3. Results and Discussion

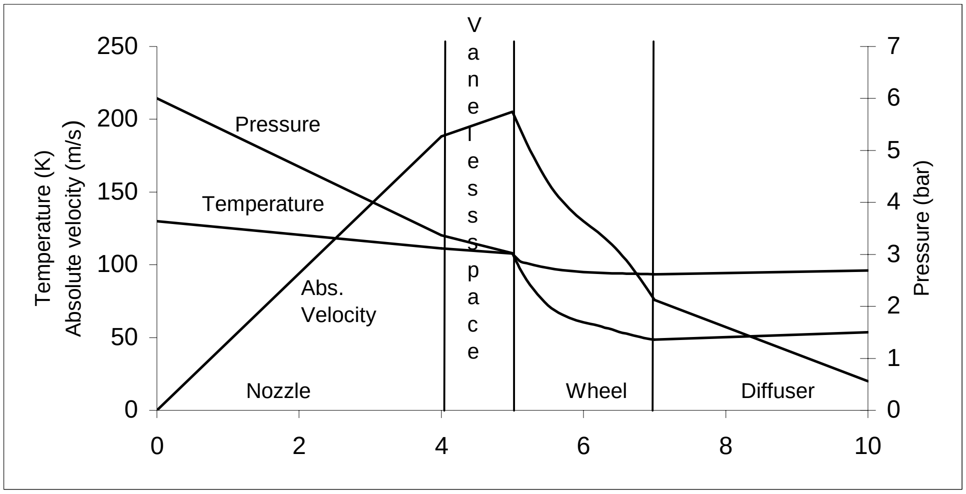


Fig. 5: Variation of pressure, temperature, and absolute velocity in turboexpander (length not to scale)

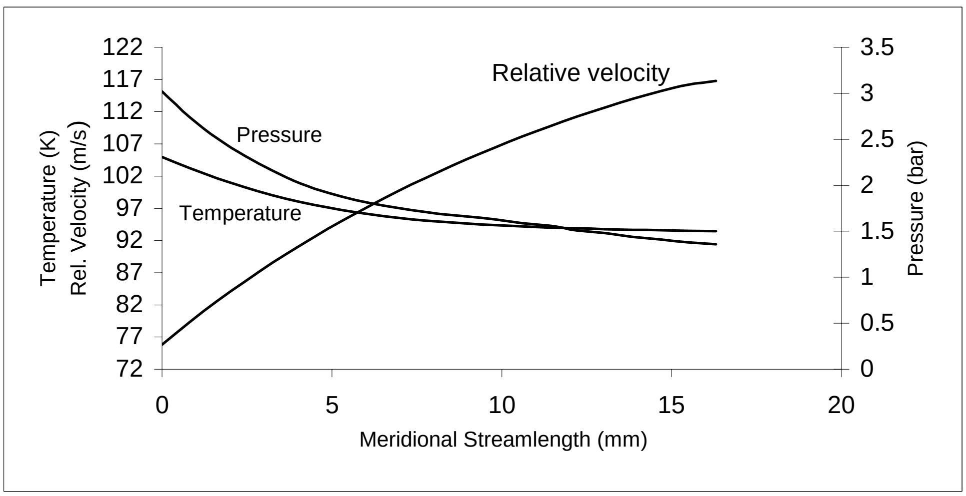


Fig. 6: Pressure, temperature, relative velocity distribution along the meridional streamline of the turbine wheel.

Figure 5 presents qualitatively the velocity, temperature and pressure profiles from inlet to exit, while Figure 6 shows the same profiles along the middle streamline in the wheel. The results may be used to compute the net axial thrust by integration of the pressure over the projected area of the turbine wheel.

$$F_{a\uparrow} = \int_{r_3}^{r_2} 2\pi rp\,dr + \pi r_3^2 p_3 \quad (34)$$

$$F_{a\downarrow} = \pi\left(r_2^2 - r_s^2\right) \times p_2 \quad (35)$$

where,

F_a = axial thrust force (Newton)

r_s = radius of the shaft

Net axial force = 5.83 N

4. Conclusions

The turboexpander is an important mechanical device in the cryogenic plant, which has very wide applicability. The turboexpander is in the process of fabrication. This paper gives detailed computational procedure with an example. Suitability of the computational process would be confirmed by conducting experiments.

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