Published in Pi Mu Epsilon Journal, Vol. 11, No. 6, pp 241-242, 2002

ON THE NUMBER 495
(A CONSTANT SIMILAR TO KAPREKAR'S CONSTANT)

PRATIK KUMAR RAY

Introduction. According to Gauss Mathematics is the queen of sciences and the theory of numbers is the queen of mathematics [1]. Number theory created interest among noted mathematicians as well as numerous amateurs from the time of Pythagoras.

Nannei discussed a problem by Barisien on one characteristic of 6-digit numbers [2]. Kaprekar dealt with 4-digit numbers and the number 6174 is known as Kaprekar's constant [3,4] (discussed later in this paper).This constant is obtained by a set of recurrent mathematical operations on any 4- digit number with at least two distinct digits. The present paper deals with a similar characteristic of 3-digit numbers.

An algorithm. The input is a 3-dgit natural number N_0 such that all its digits are not identical.

Set $i = 0$

while $N_i \neq 495$ do

begin

1. Arrange the digits of N_i in descending order to get the number A_i .
2. Arrange the digits of N_i in descending order to get the number B_i .
3. Set $N_{i+1} := A_i - B_i$.

End.

To illustrate the above, a numerical example is considered below.

EXAMPLE: Let $N_0 = 585$,

Output 1: $A_0 = 855$, $B_0 = 558$, and $N_1 = 297$.

Output 2: $A_0 = 972$, $B_0 = 279$, and $N_1 = 693$.

Output 3: $A_0 = 963$, $B_0 = 369$, and $N_1 = 594$.

Output 4: $A_0 = 954$, $B_0 = 459$, and $N_1 = 495$.

Observe that the while loop is executed four times before the algorithm terminates.

THEOREM 1. *The above algorithm terminates with the number 495 after at most six executions of the while loop.*

Dept. of Metallurgical Engineering, Regional Engineering College, Rourkela, India.

Proof. Let $a_1 \geq a_2 \geq a_3$ be the digits of N_0 . Since not all three digits are identical, we know that: $a_1 > a_3$. The digits of N_1 are $10 + a_3 - a_1$, 9, and $a_1 - a_3 - 1$. The middle digit must be 9 and the sum of the first and last digit is 9 as well. Clearly the first digit is at most 8 and the first and last digit can not be equal since this would imply $2a_3 - 2a_1 + 11 = 0$ which is impossible. So N_1 is a proper input. Let us denote the digits N_1 of by 9, b_1 and b_2 with $b_1 \geq b_2$. Computing N_2 we get the digits $10 + b_2 - 9 = b_2 + 1$, 9 and $9 - b_2 - 1 = 8 - b_2$. The middle digit is again 9, but the smaller digit of N_1 was increased by 1, while the larger digit was decreased by 1. Since after the first execution of the while loop the largest (non middle) digit is 9, we reach 495 after at most 6 executions of the loop.

Executing the while loop on the number 495 returns the number 495 again, so 495 is a fixed point of our algorithm. If we lift the restriction that not all the digits are the same, 000 is another fixed point.

If we change the input to our algorithm from 3 to 4-digit numbers, the fixed point 6174 is reached after at most 8 while-loop executions. 6174 is known as Kaprekar's constant [3,4]. For 5-digit numbers we do not obtain a fixed point, so the corresponding algorithm would not terminate; however, depending on the input, the output sequence becomes stationary repeating a set of two or four numbers. We observe that the sum of the digits of any output number is a multiple of 9. The interested readers are encouraged to prove that executing the while loop on an n-digit number yields a number whose sum is a multiple of 9.

Another spin-off is as follows. Instead of subtracting the two numbers after ordering the digits, we could add them. In [5] it is mentioned as an open problem (on page 82) whether this leads to a palindromic number after a finite number of steps. Note that the problem there is a bit different in that the digits of the number are merely reversed, but not ordered. We propose the following problem: Is it true that, given a 3-digit number, adding the two numbers obtained from it by ordering the digits in increasing and decreasing order and repeating this process, yields a palindromic number in a finite number of steps?

For example, performing this process on the number 196 yields 961+169=1130. Repeating we get 3110+0113=3223, which is a palindrome. However, without ordering the digits but merely reversing them, according to [5], might never yield a palindrome.

REFERENCES

[1] RICHARD COURANT and HERBERT ROBBINS, "What is Mathematics", Oxford University Press, 1947.

[2] LEONARD EUGENE DICKSON, "History of the Theory of Numbers- vol. I", Chelsa Publishing Co., N.Y., 1952.

[3] D. Salwi, Scientists of India, CBT Publications, 1997.

[4] D. SALWI, "Scientists of India", CBT Publications, India, 1997.

[5] HANS WALSER, "Symmetry", The Mathematical Association of America, 2000.

Pratik Kumar Ray is a freshman engineering student. He got interested in number theory during his preparation for the Regional Mathematics Olympiad.