Bearing capacity of shallow foundation under eccentrically inclined load

Capacité portante d'une fondation superficielle sous une charge inclinée excentrique

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ABSTRACT: Laboratory model tests were conducted in a dense sand to determine the bearing capacity of shallow strip foundation subjected to eccentrically inclined load. The embedment ratio (ratio of the depth of embedment D_f to the width of the foundation B) was varied from zero to one. Load eccentricity e was varied from zero to $0.15B$ and the load inclination with the vertical (α) was varied from zero to 20 degrees. Based on the results of the present study, an empirical nondimensional reduction factor has been developed. This reduction factor is the ratio of the bearing capacity of the foundation subjected to an eccentrically inclined load (average eccentrically inclined load per unit area) to the bearing capacity of the foundation subjected to a centric vertical load.

RÉSUMÉ: Des essais ont été réalisés sur des sables denses en utilisant des modèles au laboratoire afin de déterminer la capacité portante d'une fondation superficielle filante sous chargement inclinée excentrique. Le rapport d'enterrement de la fondation (rapport entre la profondeur d'enterrement D_f et la largeur de la semelle B) a été varié entre 0 et 1. L'excentricité de la charge e a été variée de 0 à $0.15B$ et l'inclinaison de 0 jusqu'à 20 degré. Sur la base des résultats de cette étude, un facteur empirique de réduction adimensionnel a été développé. Ce facteur de réduction est le rapport de la capacité portante d'une fondation soumise à une charge incliné excentrique (charge excentrique inclinée moyenne par unité de surface) par rapport à la capacité portante d'une fondation soumise à une charge verticale centrée.

KEYWORDS:Load eccentricity, load inclination, sand, shallow foundation, reduction factor, ultimate bearing capacity

1 INTRODUCTION

On some occasions shallow foundations are subjected to eccentrically inclined load as shown in Fig. 1 for the case of a strip foundation of width B supported by sand. In Fig. 1, Q_u is the ultimate load per unit length of the foundation applied with an eccentricity e and inclined at an angle α with respect to the vertical. Meyerhof (1963) proposed a relationship for the vertical component of the average ultimate load per unit area of the foundation based on the effective area concept. For granular soil it can be expressed as

$$q_{uv(e,\alpha)} = \frac{Q_u \cos\alpha}{B} = \frac{B'}{B}(qN_q d_q i_q + 0.5\gamma B' N_\gamma d_\gamma i_\gamma) \quad (1)$$

where $q_{uv(e,\alpha)}$ = average vertical component of the ultimate load per unit area with load eccentricity e and load inclination α, $q = \gamma D_f$, γ = unit weight of sand, D_f = depth of foundation, N_q, N_γ = bearing capacity factors, B' = effective width = $B - 2e$, d_q, d_γ = depth factors, and i_q, i_γ= inclination factors.

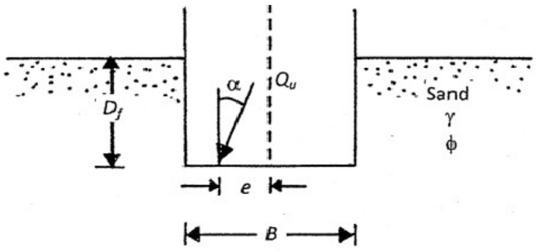

Figure 1. Shallow foundation on granular soil subjected to eccentrically inclined load.

The relationships for bearing capacity, depth and inclination factors are as follow,

$$N_q = e^{\pi \tan\phi}\left(\frac{1+\sin\phi}{1-\sin\phi}\right) \quad (2)$$

$$N_\gamma = (N_q - 1)\tan(1.4\phi) \quad (3)$$

$$d_q = d_\lambda = 1 + 0.1\left(\frac{D_f}{B}\right)\tan\left(45 + \frac{\phi}{2}\right) \quad (\text{for } \phi \geq 10°) \quad (4)$$

$$i_q = \left(1 - \frac{\alpha°}{90°}\right)^2 \quad (5)$$

and

$$i_\gamma = \left(1 - \frac{\alpha°}{\phi°}\right)^2 \quad (6)$$

where ϕ = soil friction angle.

Purakayastha and Char (1977) conducted stability analyses of eccentrically loaded strip foundations ($\alpha = 0$) supported by granular soil using the method of slices proposed by Janbu (1957). Based on their study it was proposed that, for a given D_f/B,

$$\frac{q_{uv(e,\alpha=0)}}{q_{uv(e=0,\alpha=0)}} = R = 1 - b\left(\frac{e}{B}\right)^c \quad (7)$$

where $q_{uv(e,\alpha=0)}$ = average ultimate vertical load per unit area of the foundation with load eccentricity e and load inclination α=0, $q_{uv(e=0,\alpha=0)}$ = average ultimate bearing capacity with centric vertical load, R = reduction factor, b and c = functions of D_f/B only and independent of soil friction angle ϕ. The variation of b and c with D_f/B [Eq. (7)] is given in Table 1.

Table 1. Variation of b and c with D_f/B—Analysis of Purkayastha and Char (1977)

D_f/B	b	c
0	1.862	0.73
0.25	1.811	0.785
0.5	1.754	0.80
1.0	1.820	0.888

For D_f/B between zero and 1, the average values of b and c are about 1.81 and 0.8 respectively. So Eq. (7) can be approximated as,

$$\frac{q_{uv(e,\alpha=0)}}{q_{uv(e=0,\alpha=0)}} \approx R = 1 - 1.8\left(\frac{e}{B}\right)^{0.8} \tag{8}$$

Saran and Agarwal (1991) performed a limit equilibrium analysis to evaluate the ultimate bearing capacity of strip foundation subjected to eccentrically inclined load. According to this analysis, for a foundation on granular soil,

$$q_{u(e/B,\alpha)} = qN_{q(e/B,\alpha)} + \frac{1}{2}\gamma B N_{\gamma(e/B,\alpha)} \tag{9}$$

where $q_{u(e/B,\alpha)}$ = average inclined load per unit area with load eccentricity ratio e/B and load inclination α, $N_{q(e/B,\alpha)}$ and $N_{\gamma(e/B,\alpha)}$ = bearing capacity factors expressed in terms of load eccentricity e and inclined at an angle α to the vertical. They are available in tabular and graphical form in the original paper of Saran and Agarwal (1991).

The purpose of the present study is to present several laboratory model test results for the average ultimate inclined load per unit area of a strip foundation, $q_{u(e,\alpha)}$, supported by dense sand [i.e. $q_{uv(e,\alpha)}/\cos\alpha$]. A reduction factor has been proposed to estimate $q_{u(e,\alpha)}$ at a given D_f/B from the ultimate bearing capacity with centric vertical loading $q_{uv(e=0,\alpha=0)}$ at similar D_f/B.

2 LABORATORY MODEL TESTS

Laboratory model tests were conducted using a poorly graded sand with effective size D_{10} = 0.325 mm, uniformity coefficient C_u = 1.45 and coefficient of gradation C_c = 1.15. The model tests were conducted in a tank measuring 1.0 m (length) × 0.504 m (width) × 0.655 m (height). The two length sides of the tank were made of 12mm thick high strength fiberglass. All four sides of the tank were braced to avoid bulging during testing. The model foundation measured 100 mm (width B) × 500 mm (length L) × 30mm (thickness t) and was made from a mild steel plate. The bottom of the footing was made rough by applying glue and then rolling the steel plate over sand. Since the width of the test tank and the length of the model foundation were approximately the same, a plane strain condition roughly existed during the tests.

Sand was poured into the test tank in layers of 25 mm from a fixed height by raining technique to achieve the desired average unit weight of compaction. The height of fall was fixed by making several trials in the test tank prior to the model test to achieve the desired unit weight of sand. The model foundation was placed at a desired D_f/B ratio at the middle of the box. Load to the model foundation was applied by a loading assembly which was capable of applying eccentrically inclined load. It consisted of three units: (a) the electrical control panel, (b) hydraulic power pack and (c) loading device. The loading device was a combination of a beam, four cylinders, four supporting columns and a base. The hydraulic cylinder was the device that converted fluid power into linear mechanical force and motion. It converted fluid energy to an output force in a linear direction for executing different jobs. The capacity of the hydraulic cylinder in universal static loading setup was 100 kN. The load could be applied to the model foundation in the range of 0 to 100 kN with an accuracy of 1 N. The inclination of the load could be changed by forward and backward movement of the cylinder. The inclination of the load remained intact throughout the testing period by the provision of the check valve. Settlement of the model foundation was measured by dial gauges placed on two edges along the width side of the model foundation.

The average values of the various parameters during the model tests are given in Table 2.

Table 2. Model Test Parameters

Parameters	Values
Unit weight of compaction of sand	14.36 kN/m^3
Relative density of compaction	69%
Soil friction angle	40.8°
D_f/B	0, 0.5, 1
e/B	0, 0.05, 0.1, 0.15
Load inclination α	0, 5°, 10°, 15°, 20°

3 MODEL TEST RESULTS

Based on the load-settlement curves, the average ultimate inclined loads per unit area of the foundation $q_{u(e,\alpha)}$ (= Q_u/B; see Fig. 1) obtained from the present tests are given in Table 3.

4 ANALYSIS OF MODEL TEST RESULTS

Based on Eqs. (5), 6) and (7), it was assumed that, for a given D_f/B,

$$RF = \text{reduction factor}$$
$$= \frac{q_{u(D_f/B,e,\alpha)}}{q_{u(D_f/B,e=0,\alpha=0)}} = \left[1 - a\left(\frac{e}{B}\right)^m\right]\left(1 - \frac{\alpha}{\phi}\right)^n \tag{10}$$

In order to determine the values of a, m and n, the following procedure was used:

Step 1: For vertical loading conditions (i.e. α =0), Eq. (10) takes the form

$$RF = \left[1 - a\left(\frac{e}{B}\right)^m\right] \tag{11}$$

With α = 0 and, for a given D_f/B, regression analyses were performed to obtain the magnitudes of a and m.

Step 2: Using the values of a and m obtained in Step 1 and Eq. (10), for a given D_f/B, a regression analysis was performed to obtain the value of n for α> 0°.

The values of a, m and n obtained from analyses described above are given below,

- D_f/B= 0 —a = 2.23, m = 0.81, n = 1.98
- D_f/B = 0.5 — a = 2.0, m = 0.88, n = 1.23
- D_f/B= 1.0 —a = 1.76, m = 0.92, n = 0.97

From the values of a, m and n, It can be seen that the variations of a and m with D_f/B are very minimal; however, the value of n decreases with the increase in embedment ratio. The average values of a and m are 1.97 and 0.87 respectively.

Table 3. Experimental Average Ultimate Loads Per Unit Area and Reduction Factors.

D_f/B (1)	α (deg) (2)	e/B (3)	Experimental $q_{u(D_f/B,e,\alpha)}$ (kN/m^2) (4)	Experimental *RF* [Eq. (10)] (5)	Calculated *RF* [Eqs. (10), (12), (13) and (14)] (6)	Deviation— $\frac{\text{Col.}6-\text{Col.}5}{\text{Col.}6}$ (%) (7)
0	0	0	166.77	1.0	1	0
0	0	0.05	133.42	0.8	0.9	11.11
0	0	0.1	109.87	0.659	0.8	17.65
0	0	0.15	86.33	0.518	0.7	26.05
0	5	0	128.51	0.771	0.77	-0.09
0	5	0.05	103.01	0.618	0.693	10.86
0	5	0.1	86.33	0.518	0.616	15.96
0	5	0.15	65.73	0.394	0.539	26.87
0	10	0	96.14	0.576	0.570	-1.16
0	10	0.05	76.52	0.459	0.513	10.54
0	10	0.1	62.78	0.376	0.456	17.42
0	10	0.15	51.99	0.312	0.399	21.85
0	15	0	66.71	0.4	0.4	-0.03
0	15	0.05	53.96	0.324	0.36	10.1
0	15	0.1	44.15	0.265	0.32	17.25
0	15	0.15	35.12	0.211	0.28	24.77
0	20	0	43.16	0.259	0.26	0.41
0	20	0.05	34.83	0.209	0.234	10.72
0	20	0.1	29.43	0.176	0.208	15.13
0	20	0.15	23.54	0.141	0.182	22.4
0.5	0	0	264.87	1.0	1.0	0
0.5	0	0.05	226.61	0.856	0.9	4.94
0.5	0	0.1	195.22	0.737	0.8	7.87
0.5	0	0.15	164.81	0.622	0.7	11.11
0.5	5	0	223.67	0.844	0.822	-2.74
0.5	5	0.05	193.26	0.73	0.74	1.37
0.5	5	0.1	165.79	0.626	0.658	4.81
0.5	5	0.15	140.28	0.530	0.575	7.95
0.5	10	0	186.39	0.704	0.656	-7.29
0.5	10	0.05	160.88	0.607	0.59	-2.9
0.5	10	0.1	137.34	0.519	0.525	1.18
0.5	10	0.15	116.74	0.441	0.459	4
0.5	15	0	151.07	0.57	0.503	-13.43
0.5	15	0.05	129.49	0.489	0.453	-8.03
0.5	15	0.1	111.83	0.422	0.402	-4.96
0.5	15	0.15	94.18	0.356	0.352	-1.01
0.5	20	0	115.76	0.437	0.364	-20.06
0.5	20	0.05	98.1	0.37	0.328	-13.05
0.5	20	0.1	85.35	0.322	0.291	-10.65
0.5	20	0.5	72.59	0.274	0.255	-7.56
1.0	0	0	353.16	1.0	1.0	0
1.0	0	0.05	313.92	0.889	0.9	1.23
1.0	0	0.1	278.6	0.789	0.8	1.39
1.0	0	0.15	245.25	0.694	0.7	0.79
1.0	5	0	313.92	0.889	0.877	-1.3
1.0	5	0.05	277.62	0.786	0.79	0.46
1.0	5	0.1	241.33	0.683	0.702	2.65
1.0	5	0.15	215.82	0.611	0.614	0.51
1.0	10	0	264.87	0.750	0.755	0.65
1.0	10	0.05	239.36	0.678	0.679	0.24
1.0	10	0.1	212.88	0.603	0.604	0.19
1.0	10	0.15	188.35	0.533	0.528	-0.93
1.0	15	0	225.63	0.639	0.632	-1.03
1.0	15	0.05	206.01	0.583	0.569	-2.5
1.0	15	0.1	179.52	0.508	0.506	-0.48
1.0	15	0.15	155.98	0.442	0.443	0.22
1.0	20	0	183.45	0.519	0.51	-1.89
1.0	20	0.05	166.77	0.472	0.459	-2.92
1.0	20	0.1	143.23	0.406	0.408	0.56
1.0	20	0.15	126.55	0.358	0.357	-0.41

Considering the uncertainties involved in any experimental evaluation of ultimate bearing capacity, we can assume without loss of much accuracy

$$a \approx 2 \tag{12}$$

$$m \approx 1 \tag{13}$$

$$n \approx 2 - \left(\frac{D_f}{B}\right) \tag{14}$$

The experimental values of *RF* defined by Eq. (10) are shown in Col. 5 of Table 1. For comparison purposes, the predicted values of the reduction factor *RF* obtained using Eqs. (10), (12), (13) and (14) are shown in Col. 6 of Table3. The deviations of the predicted values of *RF* from those obtained experimentally are shown in Col. 7 of Table 3. In most cases the deviations are ±15% or less; however, in some cases, the deviations were about 25%. Thus Eqs. (10), (12), (13) and (14) provide reasonable good and simple approximations to estimate the ultimate bearing capacity of strip foundations ($0 \le D_f/B \le 1$) subjected to inclined eccentric loading. Or, for a given D_f/B,

$$q_{u(D_f/B,e,\alpha)} = q_{u(D_f/B,e,\alpha=0)} \times \left[1 - 2\left(\frac{e}{B}\right)\right]\left(1 - \frac{\alpha}{\phi}\right)^{2-(D_f/B)} \tag{15}$$

5 CONCLUSIONS

The results of a number of laboratory model tests conducted to determine the ultimate bearing capacity of a strip foundation supported by sand and subjected to an eccentrically inclined load with an embedment ratio varying from zero to one have been reported. Tests were conducted on dense sand. The load eccentricity ratio e/B was varied from zero to 0.15, and the load inclination α was varied from zero to 20° (i.e. $\alpha/\phi \approx 0$ to 0.5). Based on the test results and within the range of parameters tested, an empirical relationship for a reduction factor *RF* has been proposed [Eq. (15)]. A comparison between the reduction factors obtained from the empirical relationships and those obtained experimentally shows, in general, a variation of ±15% or less. In a few cases, the deviation was about 25 to 30%.

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