Simulation Analysis of Zeta Converter with Continuous and Discontinuous Conduction Modes

K Mahesh Dash, Samresh Satapathy and B Chitti Babu

Abstract- **This paper presents a comparative analysis of dynamic computer simulation model for the pulse-width modulation (PWM) ZETA converter working in both discontinuous conduction mode (DCM) and continuous conduction mode (CCM). The modelling is performed considering only resistive load. Initially a brief analysis of the converter is presented and then the characteristics that make possible the dynamic analysis in both CCM and DCM are mentioned. The detailed analysis of CCM technique and a generalized switch averaging technique is discussed with the small-signal analysis of the ZETA converter in DCM is presented so that it can be used for DC applications such as battery charging by integrating the converter with a PV array and a MPPT controller. The systems are modeled and simulated in a user friendly MATLAB/Simulink environment. The paper then concludes which strategy is useful for operation with photovoltaic array.**

Index Terms--**Averaged modelling, averaged switch model, discontinuous conduction mode, continuous conduction mode and PWM converters.**

1. Introduction

Various AC-DC and DC-DC power converters are used in a gargantuan manner in recent years due to the type and varied applications that are arising for mankind. The converters have successfully achieved the required power conversion and at the same time have offered fewer harmonic and a better power factor. The ZETA DC-DC is a fourth order PWM DC-DC converter which consists of two inductors and a series capacitor, which is sometimes called a flying capacitor. The ZETA converter has its main switch in series with the input power supply; because of this reason this converter is subjected to a greater level of stress in the input current than other basic fourth-order converters like Cuk or SEPIC converters. Therefore the concerned converter received less attention as compared to the other two, but the converter due to some peculiar advantageous features came into attention recently as described by Lopez (2009) and Peres (1994).

This paper discusses the two operating modes in the converter and presents a dynamic computer simulation model for the ZETA converter working in DCM mode and then the CCM mode. This paper also cites which one is suitable and shows better dynamic response by simulation results done in a user friendly MATLAB/Simulink environment. This paper is organized with the operation of the ZETA converter working in DCM in Section 2 where the technique employed to obtain the computational model is also discussed, Section 3 follows with the analysis of the model in CCM mode, Section 4 then summarises the circuit specification to be used and also compares both the dynamic response for the two modes, and section 5 ends up with a general conclusion.

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2. ZETA Converter in DCM

The ZETA converter is a fourth-order structure, similar to the SEPIC converter. The converter topology is shown in Figure 1.

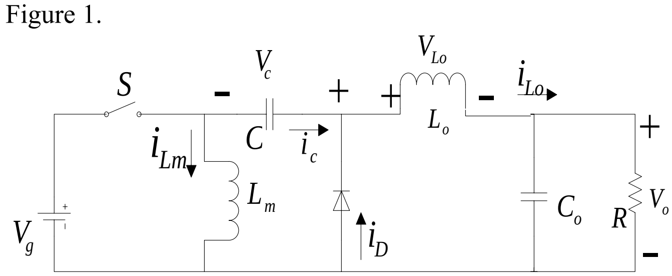

Fig 1. Circuit diagram of ZETA Converter

A. Stages of Operation:-

Assuming that the switch used is ideal, then stages of operation of the ZETA converter connected to the load can be analysed in a proper way. The load assumed is a resistive load. In DCM this converter presents three stages of operation, which are represented schematically in Figure 2.

1) First stage ($0 < t < t_c$): At the time the switch S is closed and the voltage v_g is applied to inductors L_m and L_o, the currents i_{Lm} and i_{Lo} will increase linearly with a slope of v_g /L_m and v_g/L_o, respectively. The diode will work in the reverse bias condition due to the reverse polarity of the source, so no current will pass through it. Thus, the current in the switch S is the sum of the current passing through both the inductors, and it will increase at the rate of v_g /L_{eq}, where L_{eq} is the equivalent parallel inductance of the magnetising inductance and the output inductance. V_g is the input voltage (V_{IN}).

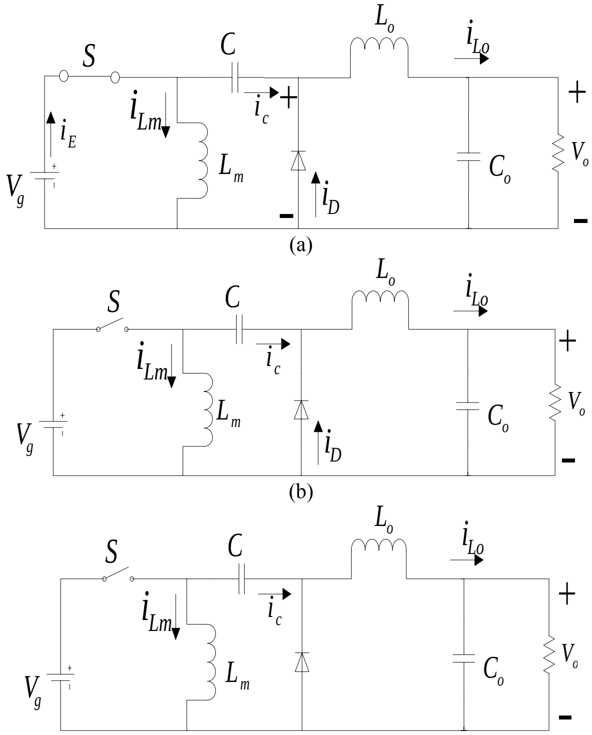

Fig. 2. (a) stage 1 (b) stage 2 (c) stage 3 operation of a ZETA converter in DCM

2) Second stage ($t_c < t < t_c + t_d$): When the switch S is open the diode turns its stage into the conduction region. Now the stored output voltage will act as input to the inductors. L_m transfers the stored energy to the coupling capacitor C, in a similar way L_o works as a voltage source for the load. Now the currents i_{Lm} and i_{Lo} decrease linearly with slope $-v_o/L_m$ and $-v_o/L_o$, respectively. The diode forward- bias current i_D is the sum of the currents i_{Lm} and i_{Lo}, and decreases linearly with slope $-v_o/L_{eq}$. The current will decrease until the diode current reaches to zero.

3) Third stage ($t_c + t_d < t < T$): Due to the extinction of diode current the diode circuit will behave as an open circuit and till that time also the switch S remains open. The coupling capacitor current i_C is constant and equal to the output inductor current L_o (i_{Lo}), which has the opposite direction of the magnetizing inductor current L_m (i_{Lm}), causing the voltage at the inductors to be equal zero.

B. Static Transfer Characteristic:-

The instantaneous static gain *g(t)* of the ZETA converter in DCM can be expressed as the function of instantaneous duty cycle *d(t)* and D_1 is a parameter that depends on the diode conduction time and the switching frequency as mentioned by Martins and Barbi (2008).

$$g(t) = \frac{V_O}{V_{IN}} = \frac{D}{D_1} \tag{1}$$

Where $$D_1 = \sqrt{\frac{2L_{eq}f}{R}} \tag{2}$$

This characteristic implies a linear relationship between the input and output voltage. Here f is the switching frequency and R is the load resistance, L_{eq} is the equivalent parallel inductance value of the magnetizing inductance and the output inductance. As mentioned earlier this converter can be modeled by generalized switch averaging technique where well known models derived from CCM can be used for DCM analysis by including an additional switching network through a feedback which is shown below. The state space averaging technique (SSA) could be used as it is used for other converters but in the DCM analysis it is avoided because it is less accurate, the reason being that the converter transfer function is dependent on the type of load which is a great deal of problem.

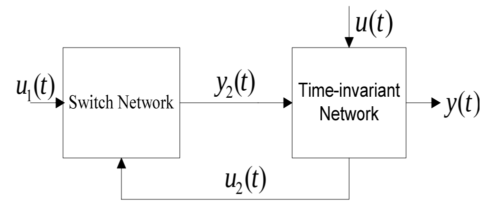

Fig. 3. Generalized Switching Averaging technique

In the above figure $u(t)$ refers to input to the converter which is a function of input voltage $V_g(t), y(t)$ refers to output, $y_2(t)$ is the output of the switching network, u_1 is the control input which is a function of instantaneous duty cycle $d(t)$, $u_2(t)$ is the function of state variable and the independent input $u(t)$ of the converter. The switching network method is based on the average switching modeling approach by Vorperian (1989) and in the loss- free resistor model, proposed by Singer (1990). According to this, the switching model can be expressed as combination of loss free resistor model in order to correlate between the input and the output parameter by the switching conversion ratio. In state space the converter of figure 1 can be represented as

$$\begin{cases} \dot{x}(t) = A_a x(t) + B_a(t) \\ y(t) = C_a x(t) + E_a(t) \end{cases} \tag{3}$$

where

$$\begin{aligned} A_a &= A_1\mu(t) + A_2(1-\mu(t)) \\ B_a &= B_1\mu(t) + B_2(1-\mu(t)) \\ C_a &= C_1\mu(t) + C_2(1-\mu(t)) \\ E_a &= E_1\mu(t) + E_2(1-\mu(t)) \end{aligned} \tag{4}$$

The above matrix represent the averaged matrix over a given duty cycle. For calculating the small signal AC model, (3) can be represented as (5) where by perturbation and linearization each term is a combination of a steady state value and a small signal variation.

$$\begin{aligned} x(t) &= X + \hat{x}(t) \\ \mu(t) &= \mu_0 + \hat{\mu}(t) \\ u(t) &= U + \hat{u}(t) \end{aligned} \tag{5a}$$

$$y(t) = Y + \hat{y}(t)$$

$$u_2(t) = U_2 + \hat{u}_2(t) \tag{5b}$$

$$u_1(t) = U_1 + \hat{u}_1(t)$$

Thus the steady state solution of the system is given by (6) and (7) is the small signal AC model that contains the dynamic behavior.

$$\begin{cases} X = -AB^{-1}U \\ Y = (-CA^{-1}B + E)U \end{cases} \tag{6}$$

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + B\hat{u}(t) + B_D\hat{\mu}(t) \\ \hat{y}(t) = C\hat{x}(t) + E\hat{u}(t) + E_D\hat{\mu}(t) \end{cases} \tag{7}$$

where
$$\begin{aligned} A &= A_1\mu_0 + A_2(1-\mu_0) \\ B &= B_1\mu_0 + B_2(1-\mu_0) \\ C &= C_1\mu_0 + C_2(1-\mu_0) \\ E &= E_1\mu_0 + E_2(1-\mu_0) \\ B_D &= (A_1 - A_2)X + (B_1 - B_2)U \\ E_D &= (C_1 - C_2)X + (D_1 - D_2)U \end{aligned}$$

(8)

μ_0 Can be obtained from μ (switching ratio) which is a function of converter voltage and instantaneous duty-cycle at the quiescent operating point and the linearized switching conversion ratio $\hat{\mu}(t)$ that can be obtained from $\hat{\mu}_1(t)$ and $\hat{\mu}_2(t)$ with a gain parameter k_1 and k_2 respectively where k_1 and k_2 are the function of duty-cycle and inputs at the quiescent operating point. The relation of μ is given as $\mu=\mu_1*k_1+\mu_2*k_2$ (where '^' has been removed for convenience). The values of k_1 and k_2 can be obtained as (9). (μ_o=D/D+D_1 at quiescent point of operation which is chosen for the small signal analysis.) For our small signal analysis the D is chosen to be 0.5 .

$$k_1 = \frac{2D_1}{(D+D_1)^2}$$
$$k_2 = \frac{1}{V_g}\left[-\frac{D_1^3}{D(D+D_1)^2} \quad \frac{L_o D_1^2}{(D+D_1)^2} \quad \frac{CD_1^2}{D(D+D_1)^2} \quad \frac{D_1^2}{(D+D_1)^2} \right] \tag{9}$$

In the above expression D is the linearized steady state duty cycle value and D_1*T is the diode conduction time where T is the switching time period. Taking r_{Lm}, r_{Lo} and r_c as the equivalent series resistance of both the inductors and capacitor respectively and applying Kirchhoff's law for the first and second stage of operation that is figure 2(a) and 2(b) respectively the indexed matrix are obtained. The matrixes obtained are presented in equation 12. The equations for the circuits in the two stages are cited in equation 11.

$$\begin{aligned} \frac{di_{Lm}}{dt} &= \frac{1}{L_m}(-r_{Lm}i_{Lm} + v_g) \\ \frac{di_{Lo}}{dt} &= \frac{1}{L_o}(-(r_C + r_{Lo})i_{Lo} - v_C - v_{Co} - v_g \\ \frac{dv_C}{dt} &= \frac{1}{C}(i_{Lo}) \\ \frac{dv_{Co}}{dt} &= \frac{1}{C_O}(i_{Lo} - \frac{1}{R}v_{Co}) \\ y &= v_{Co} \end{aligned} \tag{10}$$

$$\begin{aligned} \frac{di_{Lm}}{dt} &= \frac{1}{L_m}(-(r_c + r_{Lm})i_{Lm} + v_C) \\ \frac{di_{Lo}}{dt} &= \frac{1}{L_o}(r_{Lo}i_{Lo} - v_{Co}) \\ \frac{dv_C}{dt} &= \frac{1}{C}(-i_{Lm}) \\ \frac{dv_{Co}}{dt} &= \frac{1}{C_O}(i_{Lo} - \frac{1}{R}v_{Co}) \\ y &= v_{Co} \end{aligned} \tag{11}$$

$$A_1 = \begin{bmatrix} -\frac{r_{Lm}}{L_m} & 0 & 0 & 0 \\ 0 & -\frac{r_{Lo}+r_c}{L_o} & -\frac{1}{L_o} & -\frac{1}{L_o} \\ 0 & \frac{1}{C} & 0 & 0 \\ 0 & \frac{1}{C_0} & 0 & -\frac{1}{RC_0} \end{bmatrix}$$
$$A_2 = \begin{bmatrix} -\frac{r_{Lm}+r_c}{L_o} & 0 & \frac{1}{L_m} & 0 \\ 0 & -\frac{r_{Lo}}{L_o} & 0 & -\frac{1}{L_o} \\ -\frac{1}{C} & 0 & 0 & 0 \\ 0 & \frac{1}{C_0} & 0 & -\frac{1}{RC_0} \end{bmatrix} \tag{12}$$
$$B_1 = \begin{bmatrix} \frac{1}{L_m} & \frac{1}{L_o} & 0 & 0 \end{bmatrix}^T \qquad B_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$$
$$C_1 = C_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \qquad E_1 = E_2 = \begin{bmatrix} 0 \end{bmatrix}$$

Upon modeling the system in Simulink with specific circuit elements the zeta converter can be made to operate in DCM. On simulating the above model the graph obtained is shown in figure 5.

3. ZETA Converter in CCM

The operation of a Zeta converter in CCM is analysed with the help of a switching network Figs. 4a, 4b. In this mode the current through the inductor never becomes zero and the circuit operates continuously. The first stage begins when the switch is turned on at t = 0, the equivalent circuit of which is indicated in Fig. 4a. The currents in both the inductors rise linearly with voltage across as mentioned before. The second stage begins when the switch is turned off. The equivalent circuit is shown in Fig. 4b. In this mode, as the capacitors charge through the inductors, so inductor currents start decreasing in a linearly which is the basic concept of CCM. The DCM occurs in converters when the inductor current or capacitor voltage ripple is large enough to cause the switch current or voltage to reverse polarity. This phenomenon is avoided in a CCM.

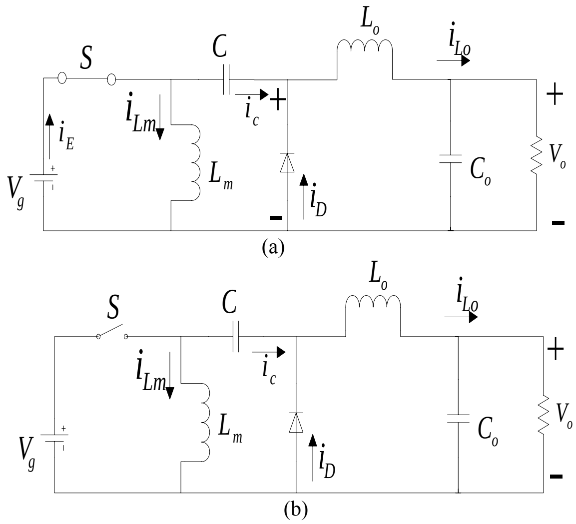

Fig. 4. (a) Stage 1 (b) Stage 2 operation modes in CCM

A.Modelling in the CCM mode

In the CCM mode the duty cycle is related to the output and input voltage as given below

$$D = \frac{V_{OUT}}{V_{IN} + V_{OUT}} \tag{13}$$

This can be written as

$$\frac{D}{1-D} = \frac{I_{IN}}{I_{OUT}} = \frac{V_{OUT}}{V_{IN}} \tag{14}$$

For designing any PWM switching regulator first thing is to decide the inductor ripple current, $\Delta I_{L(PP)}$, and this ripple current is a convolution of input current with some gain parameter. The ripple current is decided according to tolerable current fluctuation in the load side. The magnetizing inductance is an important factor in deciding the mode of operation. The mode of operation depends upon the equivalent inductance L_{eq}, which is the parallel combination of the magnetizing inductance and an output inductance. For maximum load the equivalent inductance must satisfy the following condition. If this relation is not satisfied then the circuit would enter discontinuous mode.

$$L_{eq} < \frac{R_L (1-D)^2}{2f} \tag{15}$$

Now the output inductor, L_o which has a dependency over the switching frequency, ripple current, input voltage and duty cycle can be found out by

$$L_O = \frac{V_{IN} D}{f \Delta I_{L(PP)}} \tag{16}$$

$$L_m = \frac{L_{eq} L_O}{L_O - L_{eq}} \tag{17}$$

In the above equation V_{IN} is the input voltage, D is the duty cycle, f is the duty cycle while the Δ symbol is for the allowable ripple which for our case we have assumed to be 2%. Solving the above two equations the value of the inductors can be found out. The output inductor has a significant effect on output voltage ripple. So the output capacitor is selected on the basis of this peak-to-peak ripple in output voltage (ΔV) as

$$C_O = \frac{V_O}{\omega R_L \Delta V} \tag{18}$$

A large value of input capacitor if used can distort the input current waveform as the high value capacitor will store large amount of reactive energy and which cannot be fed to the input supply due to the presence of a unidirectional diode in the second stage of operation. Thus it is required to choose a small value of an input capacitor if used.

After all the parameters are obtained the model can be modelled using Simulink. Now since no discontinuous state is being involved in this mode the previous generalized switching state space analysis is not needed. The dynamic response obtained upon modelling is shown in figure 6.

4. Simulation Results and Discussions

To analyse the relative pros and cons of the two models the dynamic response of the two models were compared since it indicates which strategy is better for load fluctuations. Step input was given to both the models where the voltage was 0V till 0.3ms and then the input voltage of 34V was applied. The graphs of load voltage obtained are shown below.

The circuit parameters taken are tabulated in figure 7.

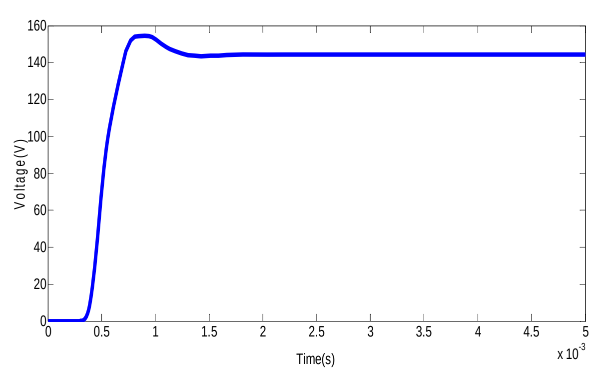

Fig. 5. Dynamic response of ZETA converter in DCM mode for d(t)=.75

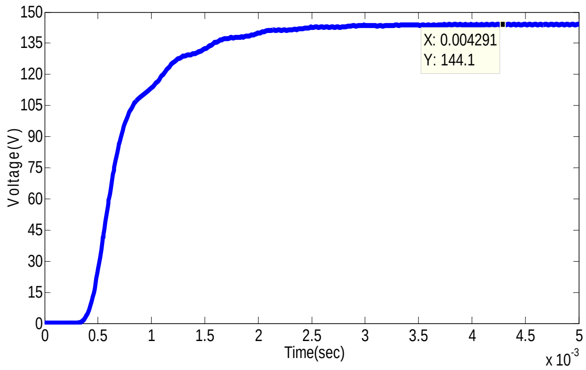

Fig. 6. Dynamic response of ZETA converter in CCM mode for D=.82

It is quite clear that the dynamic response in the DCM mode is quite good.In the DCM mode the output quickly stabilises on sudden changes in contrast CCM which takes more time to stabilise. This is because CCM in its transfer function has a right half plane zero which enhances instability due to extra phase. Apart from the graph the better response of DCM is further strengthened by the fact that the loop gain has two poles in a CCM which makes it potentially unstable if feedback is applied while a DCM has only one pole which makes it only unconditionally unstable.

5. CONCLUSIONS

Despite the better dynamic response of the converter in DCM the CCM mode enjoys the advantage of yielding better efficiency. Further the I_{peak} and V_{peak} is low on the switch stress in the CCM which means the switches used can be less expensive. Hence depending upon the application to be used a ZETA converter can be used in either of the mode as required. For example where load fluctuation is more like a photovoltaic panel tied to a grid it is economical to use the converter in DCM mode.

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APPENDIX: TABLE-1 PARAMETERS FOR SIMULATION

Component	***Definition***	***Value in CCM***	***Value in DCM***
V_g	Input Voltage	34 V	34 V
V_{OUT}	Output Voltage	145 V	145 V
f	Switching Frequency	20 kHz	20 kHz
R	Load Resistance	125 ohm	125 ohm
r_{lm}	ESR of Magnetizing Inductance	0.0 ohm	0.0 ohm
r_C	ESR of Output Inductance	0.0 ohm	0.0 ohm
L_m	Magnetizing Inductance	229µH	85 µH
Lo	Output Inductance	69mH	22 mH
C	Coupling Capacitance	680nF	680 nF
Co	Output Capacitance	462 nF	820 nF