Experimental and Numerical Study on Vibration of Industry Driven Woven Fiber Carbon/Epoxy Laminated Composite Plates

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ABSTRACT

The present research is an experimental and numerical investigation on parametric study of vibration characteristics of industry driven woven fiber carbon composite panels. The effects of different geometry, boundary conditions, lamination parameters and fibre on the frequencies of vibration of carbon fiber reinforced polymer (CFRP) panels are studied in this investigation. The vibration study is carried out using B &K FFT analyzer, accelerometer, impact hammer excitation. The PULSE software is used to convert the responses from time domain to frequency domain. The Frequency Response Function (FRF) spectrum are studied with the coherences to obtain a clear understanding of the vibration characteristics of the CFRP plates. The experimental results are compared with the numerical predictions using the FEM program as well as software package ANSYS 13.0. A very good agreement was observed between the results. Different mode shapes were plotted to interpret the different modes of vibration using ANSYS. Benchmark results are presented showing the effects of different parameters on the natural frequencies of CFRP plates.

KEYWORDS
Vibration, CFRP, modal testing, FRF spectrum, FFT, FEM

1. Introduction

Composite materials have extensive applications in various fields including fuselage panels of aeroplane, turbine blades, automobile body panels, cryogenic fuel tanks etc. The recent Boeing 787 uses nearly 50% of composites of which the major components including fuselage and wings consists of carbon composites. Thus, the vibration characteristics of the woven fibre laminated composite panels are of tremendous practical importance in prediction of the dynamic behaviour of carbon composite panels.

Most of the previous investigations were focused either on numerical analysis of unidirectional composite plates. The related literature was critically reviewed so as to provide the background information on the problems to be considered in the research work and to emphasize the relevance of the present study. Most of the previous studies are limited to theoretical results by adopting various methods including analytical and numerical approach like Ritz and finite element method but with unidirectional fibres. The experimental results on vibration measurement or modal analysis of composite plates are less in open literature. Cawley and Adams [1] investigated the natural modes of square aluminium plates and square composite plates with different ply orientations for free-free boundary conditions, both theoretically as well as experimentally. Cawley and Adams [2] also used dynamic analysis to detect, locate and roughly quantify damage to components fabricated from fibre reinforced plastic. Crawley [3] experimentally determined the mode shapes and natural frequencies of composite plates, cylindrical shell sections and Aluminium hybrid plates for various laminates and aspect ratio using electro-magnetic shaker and compared the results to that obtained from finite element analysis. The natural frequency and the specific damping capacity of CFRP and GFRP were predicted by Lin *et al*. [4] using zoom-FFT based on transient testing technique and computer based programme implementing finite element method. Chai [5] presented an approximate method based on Rayleigh-Ritz approach to determine the free vibration frequencies of generally laminated composites for different ply orientation and different boundary conditions. Maiti and Sinha [6] used the first order shear deformation theory (FSDT) and higher order shear deformation theories (HSDT) to develop FEM methods to study the bending, free vibration and impact response of thick laminated composite plates. The effects of delamination on the free vibration of composite plates were analysed by Ju *et al.* [7]. Chen and Chou [8] developed 1D elasto-dynamic analysis method for vibration analysis orthogonal woven fabric composites. The free vibration frequencies of cross ply laminated square plates for twelve different boundary conditions were determined using Ritz method by Aydogdu and Timarci [9]. Ferreira *et al.* [10] conducted analytical studies using FSDT in radial basis functions procedure for moderately thick symmetrically laminated composite plates. Xiang *et al.* [11] carried out ttheoritical studies of laminated composite plates using Guassian radial basis functions and first order shear deformation theory. Xiang and Wang [12] studied the free vibration analysis of symmetric laminated composite plates using trigonometric theory and inverse multiquadriatic radial basis function. Maheri [13] used theoretical predictions of modal response of square layered FRP panel to study the variation of modal damping under various boundary conditions.

Woven fabric composites are a class of composite materials with a fully integrated, continuous spatial fibre network that provide excellent integrity and conformability for advanced structural composite applications. These materials have gained tremendous popularity for possessing excellent durability, corrosion resistance and high strength to weight ratio. Ease of installation, versatility, anti-seismic behaviour, electromagnetic neutrality, excellent fatigue behaviour and fire resistance make it a better alternative to steel and other alloys. The studies on woven fiber composites are limited to static/ impact studies, damage initiation or failure mode of woven or braided composite plates. The computation of natural frequencies is important to predict the behaviour of structures under dynamic loads. The modal analysis can be used a non-destructive technique of assessment of stiffness of structures. Measurement of changes in vibrational characteristics can be used to detect, locate and roughly quantify damage in CRPF panels. This study is also necessary in order to avoid resonance of large structures under dynamic loading.

However vibration of industry driven woven fiber composite plates are scarce in literature. Linear analysis on CFRP faced sandwich plates with an orthotropic aluminium honeycomb core has done using principle of minimum total potential and double Fourier series by Kanematsu *et al*. [14]. Chai *et al.* [15] used TV holography technique to obtain the vibrational response of the unidirectional laminated carbon fibre-epoxy plates and carried out finite element studies simultaneously. Chakraborty *et al.* [16] determined the frequency response of GFRP plates experimentally and validated the results using commercial finite element package (NISA). The analytical values were compared with the experimental values obtained with fully clamped boundary condition. Holographic technique was used to study the modes and deflection. Hwang and Chang [17] used impulse technique for vibration testing of composite plates for determination elastic constants of materials and modelled undamped free vibration using ANSYS 5.3. Lei *et al.* [18] studied the effect of different woven structures of the glass fibre on the dynamic properties of composite laminates.

The present study deals with modal testing of CRFP plates and compared with the numerical modelling using finite element in MATLAB environment and also by ANSYS. Various mode shapes are plotted using ANSYS and discussed. The effects of different geometry, boundary conditions and lamination parameters on the frequencies of vibration of carbon fiber reinforced polymer (CFRP) panels are studied in this investigation.

2. Mathematical formulation

The basic configuration of the problem considered here is a woven fiber carbon fiber composite laminated plate of sides ‘a’ and ‘b’ as shown in the Figure 1. The lamination sequence is also shown in Figure 2.

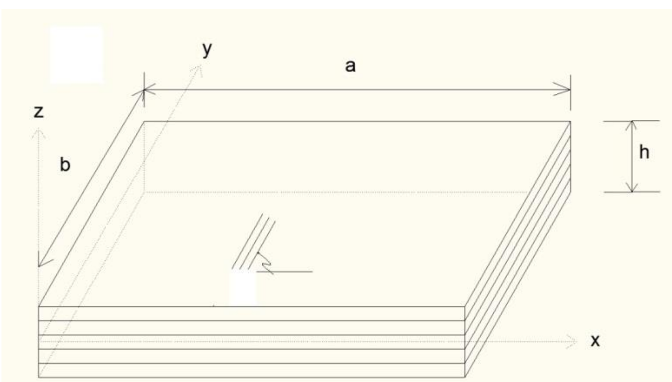


Figure 1- Laminated Composite Plate under in-plane harmonic Loading

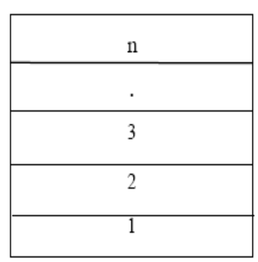


Figure 2- Lamination sequence

The governing equations for the structural behavior of the laminated plates are derived on the basis of first order shear deformation theory. The element elastic stiffness, geometric stiffness and mass matrices are derived on the basis of principle of minimum potential energy and Lagrange’s equation. The assumptions made in this analysis are summarized as follows.

2.1. Governing Differential Equation

The equation of motion is obtained by taking a differential element of plate. The governing differential equations for vibration of general laminated composite plates derived on the basis of first order shear deformation theory (FSDT) are:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = P_1 \frac{\partial^2 u}{\partial t^2} + P_2 \frac{\partial^2 \theta_x}{\partial t^2} \tag{1}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = P_1 \frac{\partial^2 v}{\partial t^2} + P_2 \frac{\partial^2 \theta_y}{\partial t^2} \tag{2}$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = P_1 \frac{\partial^2 w}{\partial t^2} \tag{3}$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = P_3 \frac{\partial^2 \theta_x}{\partial t^2} + P_2 \frac{\partial^2 u}{\partial t^2} \tag{4}$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = P_3 \frac{\partial^2 \theta_y}{\partial t^2} + P_2 \frac{\partial^2 v}{\partial t^2} \tag{5}$$

Where N_x, N_y and N_{xy} are the in-plane stress resultants, M_x, M_y and M_{xy} are moment resultants and Q_x, Q_y= transverse shear stress resultants.

$$(P_1, P_2, P_3) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} (\rho)_k (1, z, z^2) dz \tag{6}$$

Where n= number of layers of laminated composite plates, $(\rho)_k$= mass density.

The equation of motion for vibration of a laminated composite panel, subjected to generalized in-may be expressed in the matrix form as:

$$[[\mathrm{K}] - \omega^2 [\mathrm{M}]]\{\mathrm{q}\} = 0 \tag{7}$$

2.2 Finite Element Formulation

For problems involving complex geometry, material and boundary conditions, analytical methods are not easily adaptable and numerical methods like finite element methods (FEM) are preferred. The finite element formulation is developed hereby for the structural analysis of woven fiber composite plates based on first order shear deformation theory. An eight nodded isoparametric element is employed in the present analysis with five degrees of freedom u, v, w, θ_x and θ_y per node. A Composite plate of length 'a' and width 'b' consisting of 'n' number of thin homogeneous arbitrarily oriented orthotropic layers having a total thickness 'h' is considered as shown in figure 3. The x-y axes refer to the reference axes and the principal material axes are indicated by the axes 1-2. The angle 'θ' measured in the anti-clockwise direction of x-axis represents the fiber orientation. The displacement field assumes that mid-plane normal remains straight before and after deformation, but not normal even after deformation so that:

$$\left.\begin{aligned} u(x,y,z) &= u^0(x,y) + z\theta_x(x,y) \\ v(x,y,z) &= v^0(x,y) + z\theta_y(x,y) \\ w(x,y,z) &= w^0(x,y) \end{aligned}\right\} \tag{8}$$

Where u, v, w are displacements in the x, y, z directions respectively for any point, u^0,v^0, w^0 are those at the middle plane of the plate. θ_x, θ_y are the rotations of the cross section normal to the y and x axis respectively.

2.3 Strain Displacement Relations

The linear part of the strain is used to derive the elastic stiffness matrix. The linear generalized shear deformable strain displacement relations are [6]

$$\begin{aligned} \varepsilon_{xl} &= \frac{\partial u}{\partial x} + zk_x \\ \varepsilon_{yl} &= \frac{\partial v}{\partial y} + zk_y \\ \gamma_{xyl} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + zk_{xy} \\ \gamma_{xzl} &= \frac{\partial w}{\partial x} + \theta_x \\ \gamma_{yzl} &= \frac{\partial w}{\partial y} + \theta_y \end{aligned} \tag{9}$$

The bending strains k_j are expressed as,

$$k_x = \frac{\partial \theta_x}{\partial x}, \quad k_y = \frac{\partial \theta_y}{\partial y}$$

$$k_{xy} = \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} \tag{10}$$

The linear strain $\{\varepsilon\}$ can be expressed in terms of displacement as:

$$\{\varepsilon\} = [B]\{\delta_e\} \tag{11}$$

Where $\{\delta_e\} = \{u_1, v_1, w, \theta_{x1}, \theta_{y1} ...., u_8, v_8, w_8 \theta_{x8}, \theta_{y8}\}^T$, (12)

And [B] = [[B1], [B2]……………………………… [B8]] (13)

$$[B] = \sum_{i=1}^{8} \begin{bmatrix} N_{i,x} & 0 & 0 & 0 & 0 \\ 0 & N_{i,y} & 0 & 0 & 0 \\ N_{i,y} & N_{i,x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & N_{i,x} \\ 0 & 0 & 0 & N_{i,y} & 0 \\ 0 & 0 & 0 & N_{i,x} & N_{i,y} \\ 0 & 0 & N_{i,x} & 0 & N_i \\ 0 & 0 & N_{i,y} & N_i & 0 \end{bmatrix} \tag{14}$$

[B] is called the strain displacement matrix

2.4 Constitutive Relations

The elastic behavior of each lamina is essentially two dimensional and orthotropic in nature. The elastic constants for the composite lamina are given as [6].

The stress strain relation for the kth lamina is,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{15}$$

Where

$$\begin{aligned} & Q_{11} = \frac{E_{11}}{(1-\nu_{12}\nu_{21})}, Q_{12} = \frac{E_{11}\nu_{21}}{(1-\nu_{12}\nu_{21})}, Q_{21} = \frac{E_{22}\nu_{12}}{(1-\nu_{12}\nu_{21})}, Q_{22} = \frac{E_{22}}{(1-\nu_{12}\nu_{21})} \\ & Q_{66} = G_{12} \\ & Q_{44} = kG_{13} \\ & Q_{55} = kG_{23} \end{aligned} \tag{16}$$

Where

E_{11} = Modulus of Elasticity of Lamina along 1-direction

E_{22} = Modulus of Elasticity of Lamina along 2-direction

G_{12} = Shear Modulus

ν_{12}= Major Poisson's ratio

ν_{21} = Minor Poisson's ratio

The on-axis elastic constant matrix $[Q_{ij}]_k$ for the material axes 1-2 for k^{th} layer is given by

$$\left.\begin{aligned} \left[Q_{ij}\right]_k &= \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \text{For} \quad i, j = 1, 2, 6 \\ \left[Q_{ij}\right]_k &= \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \quad \text{For} \quad i, j = 4, 5 \end{aligned}\right\} \tag{17}$$

For obtaining the off-axis elastic constant matrix, $[Q_{ij}]_k$ corresponding to any arbitrarily oriented reference x-y axes for the k^{th} layer ,appropriate transformation is required. Hence the off-axis elastic constant matrix is obtained from the on axis elastic constant matrix by the relation:

$$\left.\begin{aligned} \left[\overline{Q}_{ij}\right]_k &= \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \text{for i ,j =1,2,6} \\ \left[\, \overline{Q}_{ij} \,\right]_k &= \begin{bmatrix} \overline{Q}_{44} & \overline{Q}_{45} \\ Q_{45} & Q_{55} \end{bmatrix}_k \qquad \text{for i ,j =4,5} \end{aligned}\right\} \tag{18}$$

$$[\overline{Q_{ij}}]_k = \left[T\right]^{-1} \left[Q_{ij}\right]_k \left[T\right] \tag{19}$$

$$\text{Where [T] = Transformation matrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix}_k$$

The off-axis stiffness values are:

$$\begin{aligned}
\overline{Q}_{11} &= Q_{11}m^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}n^4 \\
\overline{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})m^2n^2 + Q_{12}(m^4 + n^4) \\
\overline{Q}_{22} &= Q_{11}n^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}m^4 \\
\overline{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})m^3n + (Q_{12} - Q_{22} + 2Q_{66})n^3m \\
\overline{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})mn^3 + (Q_{12} - Q_{22} + 2Q_{66})m^3n \\
\overline{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})m^2n^2 + Q_{66}(m^4 + n^4)
\end{aligned} \tag{20}$$

The stiffness corresponding to transverse deformations are:

$$\begin{aligned}
\overline{Q}_{44} &= G_{13}m^2 + G_{23}n^2 \\
\overline{Q}_{45} &= (G_{13} - G_{23})mn \\
\overline{Q}_{55} &= G_{13}n^2 + G_{23}m^2
\end{aligned} \tag{21}$$

Where m=cosθ and n=sinθ; and θ=angle between the arbitrary principal axis with the material axis in a layer.

The force and moment resultants are obtained by integrating the stresses and their moments through the laminate thickness as given by

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \sigma_x z \\ \sigma_y z \\ \tau_{xy} z \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz \tag{22}$$

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{44} & S_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{45} & S_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ k_x \\ k_y \\ k_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{23}$$

This can also be stated as

$$\begin{Bmatrix} N_i \\ M_i \\ Q_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} & 0 \\ B_{ij} & D_{ij} & 0 \\ 0 & 0 & S_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_j \\ k_j \\ \gamma_m \end{Bmatrix} \tag{24}$$

Or

$$\{F\} = [D]\{\varepsilon\} \tag{25}$$

Where A_{ij}, B_{ij} and S_{ij} are the extensional, bending- stretching coupling, bending and transverse shear stiffnesses. They may be defined as

$$A_{ij} = \sum_{k=1}^{n} \left(\overline{Q_{ij}}\right)_k (z_k - z_{k-1}) \tag{26}$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} \left(\overline{Q_{ij}}\right)_k (z_k^2 - z_{k-1}^2) \tag{27}$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} \left(\overline{Q_{ij}}\right)_k (z_k^3 - z_{k-1}^3) \qquad \text{For } i, j = 1, 2, 6 \tag{28}$$

(23)

$$S_{ij} = \kappa \sum_{k=1}^{n} \left(\overline{Q_{ij}}\right)_k (z_k - z_{k-1}) \qquad \text{For } i, j = 4, 5 \tag{29}$$

κ = shear correction factor =5/6 in-line with previous studies [Whitney and Pagano [1970] and Reddy [1979]]

z_k, z_{k-1}= top and bottom distance of lamina from mid-plane.

2.5 Elastic stiffness matrix

The element matrices in natural coordinate system are derived as

$$\left[\mathrm{K}\right]_{\mathrm{e}}=\int_{-1}^{+1}\int_{-1}^{+1}\left[\mathrm{B}\right]^{\mathrm{T}}\left[\mathrm{D}\right]\left[\mathrm{B}\right]\left|\mathrm{J}\right|\mathrm{d}\xi\mathrm{d}\eta \tag{30}$$

Where [B] is called the strain displacement matrix

2.6 Element mass matrix

$$\left[\mathrm{M}\right]_{\mathrm{e}}=\int_{-1}^{+1}\int_{-1}^{+1}\left[\mathrm{N}\right]^{\mathrm{T}}\left[\mathrm{P}\right]\left[\mathrm{N}\right]\left|\mathrm{J}\right|\mathrm{d}\xi\mathrm{d}\eta \tag{31}$$

Where the shape function matrix

$$\left[N\right]=\sum_{i=1}^{8}\begin{bmatrix} N_i & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix} \tag{32}$$

$$\left[P_1\right]=\begin{bmatrix} P_1 & 0 & 0 & 0 & 0 \\ 0 & P_1 & 0 & 0 & 0 \\ 0 & 0 & P_1 & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix} \tag{33}$$

In which, $P_1=\sum_{k=1}^{n}\int_{ek-1}^{ek}\rho dz$ And $I=\sum_{K=1}^{n}\int_{ek-1}^{ek}z^2\rho dz$ (34)

The element load vector due to external transverse static load ‘p’ per unit area is given by

$$\left[\mathrm{P}\right]_{\mathrm{e}}=\int_{-1}^{+1}\int_{-1}^{+1}\left[\mathrm{N_i}\right]\begin{bmatrix}\mathrm{p}\\ 0\\ 0\end{bmatrix}\left|\mathrm{J}\right|\mathrm{d}\xi\mathrm{d}\eta\ . \tag{25}$$

2.7. Computer Program

A computer program is developed by using MATLAB environment to perform all the necessary computations. The element stiffness and mass matrices are derived using the formulation. Numerical integration technique by Gaussian quadrature is adopted for the element matrices. The overall matrices $[K]$ and $[M]$ are obtained by assembling the corresponding element matrices. The boundary conditions are imposed restraining the generalized displacements in different nodes of the discretized structure.

2.8 Modeling using ANSYS 13.0

The CFRP plate was modeled using a commercially available finite element package, ANSYS 13.0. [20] The natural frequencies and mode shapes are obtained by modal analysis. The element type used is SHELL281 which is an 8 noded structural shell, suitable for analyzing thin to moderately thick shell structures. The element has 8 nodes with 6 degrees of freedom at each node. The accuracy in modeling composite shells is governed by the first order shear deformation theory. The whole domain is divided into 8 x 8 mesh for all the cases. The boundary conditions of CCCC, CSCS, SSSS and CFFF were introduced by limiting the degrees of freedom at each node. FFFF condition was simulated by limiting displacement of the plate in vertical direction along the plane of plate. This condition closely resembled the experimental used in which the plate was hung vertically using strings of negligible stiffness.

3. Experimental Programme

The experimental investigation describes in detail of the materials and its fundamental constituents, the fabrication of composite plates, and the test methods according to standards.

3.1 Fabrication Method

Specimens were cast using hand layup technique as shown in Figure 3. In hand lay-up method, The percentage of fiber and matrix was taken as 50:50 by weight for fabrication of the plates. Lamination started with the application of a gel coat (epoxy and hardener) deposited on the mould by brush. Layers of reinforcement were placed on the mould at top of the gel coat and gel coat was applied again by brush. Any air which may be entrapped was removed using steel rollers. After completion of all layers, again a plastic sheet was covered the top of last ply by applying polyvinyl alcohol inside the sheet as releasing agent. Again one flat ply board and a heavy flat metal rigid platform were kept top of the plate for compressing purpose. The plates were left for a minimum of 48 hours in room temperature before being transported and cut to exact shape for testing.

Figure 3 - Hand Lay Up method used for fabrication

3.2 Determination of Physical Properties

The physical properties of fabricated composite plates such as density and thickness, represented in Table 1, were measured up to the required degree of accuracy. The thickness was measured using vernier caliper with a least count of 0.1 mm. The weight of the specimen was measured using digital weighing balance with an accuracy of 0.1 grams.

Table 1- Physical properties of the casted specimens

Sl. No.	No of layers	Length in m	Width in m	Thickness in m	Mass in g	Density in kg/m3
1	4	0.24	0.24	0.0021	174	1438.49
2	8	0.24	0.24	0.0042	345	1426.09
3	12	0.24	0.24	0.0065	519	1386.22

3.3 Tensile tests on CFRP plates

The Young's modulus was obtained experimentally by performing unidirectional tensile tests on specimens cut in longitudinal and transverse directions as described in ASTM Standard [19] for the FRP plates fabricated earlier. Strips of specimens having a constant rectangular cross-section, say 250 mm long × 25mm width are prepared from the plates. Three or more sample specimens were prepared from each plate of CFRP in this experiment. The specimen is gradually loaded up to failure, which was abrupt and sudden as the FRP material was brittle in nature. The INSTRON 1195 machine as shown in figure 4 directly indicated the Young's Modulus, ultimate strength.

Figure 4: Tensile testing of CFRP plates using INSTRON 1195

3.4 Setup and Test Procedure for Free Vibration Test:

The connections of FFT analyzer, laptop, transducers, modal hammer, and cables to the system were done as per the guidance manual. The pulse lab shop software key was inserted to the port of laptop. The plate was excited in a selected point by means of Impact hammer (Model 2302-5). The resulting vibrations of the specimens on the selected point were measured by an accelerometer (B&K, Type 4507) mounted on the specimen by means of bees wax. The plates were placed as per the required boundary conditions of free-free (FFFF), fully clamped (CCCC), simply supported (SSSS), cantilevered (CFFF) and CSCS conditions. Fully clamped and free free conditions were simulated as shown in figure 5(a) and 5(b).

(a) (b)

Figure 5: Carbon fibre composite plate during testing for different boundary conditions. (a) Fully Clamped condition. (b) Free free condition for aspect ratio 4.

4. Results and Discussion

The predictions of natural frequency of vibration using finite element analysis and experimental results are presented. Comparison with existing literature is done for the validation of the results obtained from finite element analysis. The above results are compared with that of finite element package, ANSYS. The experiments were conducted to study the modal frequencies of industry driven woven carbon fibre composite plates. The variation of the fundamental frequencies with boundary conditions, number of layers, aspect ratio and type of fibre were studied.

4.1 Material properties

The material properties of the carbon/epoxy composite are presented in Table 2.

Table 2 – Material properties of epoxy/carbon composite

E_1 (GPa)	40.32 GPa
E_2 (GPa)	40.32 GPa
G_{12} (GPa)	3.78 GPa
G_{13} (GPa)	3.5 GPa
ν_{12}	0.3
ρ(kg/m$^{3)}$	1426

4.2 Validation of results

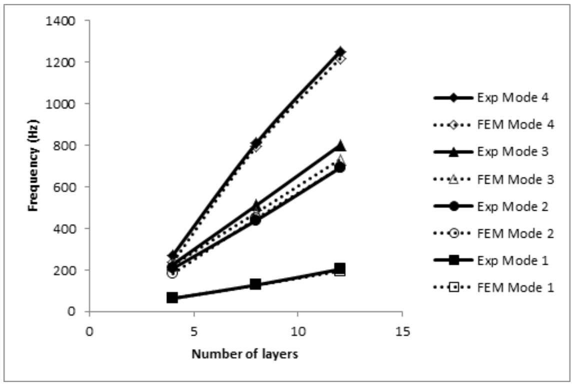
The present formulation is validated for vibration analysis of composites panels in free-free boundary conditions as shown in Table 3. The four lowest non dimensional frequencies obtained by the present finite element are compared with numerical solution published by Ju *et al.* [7].The experimental results were compared with analytical results as well as results from ANSYS, finite element package. The comparison has been presented in the subsequent sections. A good agreement was observed between the results with a maximum deviation of 20 % between experimental and FEM program results and 7 % between FEM program ANSYS.

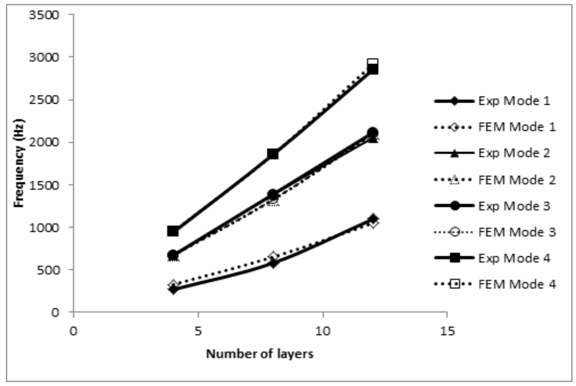
Table 3 - Comparison of natural frequencies (Hz) from FEM with the frequencies for 8 layers for fully free boundary condition

Studies	Mode 1	Mode 2	Mode 3	Mode 4
Ju *et al*. [7]	73.309	202.59	243.37	264.90
Present FEM	72.71	202.06	244.22	264.14

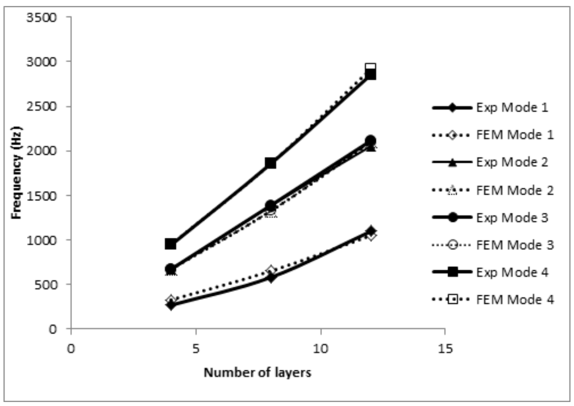
4.3 Modal Testing of Composite plates for different boundary conditions

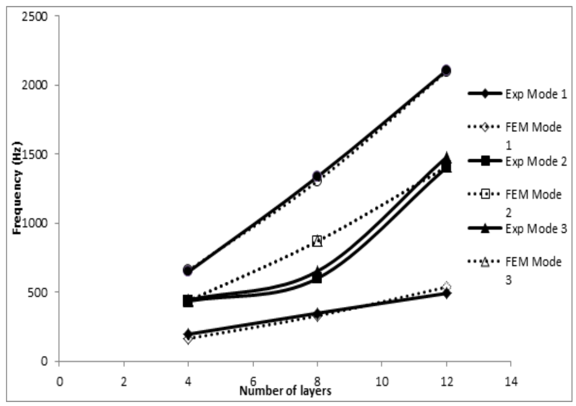
Natural frequencies of the first four modes obtained experimentally and using FEM analysis for various boundary conditions are represented in figures 6(a)-(e). The experimental values are in good agreement with the predicted values with a maximum deviation 18.03%. There is a marked increase in the modal frequencies with the increase in the number of layers of carbon fibre used for a particular boundary condition. This can be accounted for due to bending, stretching and coupling. It is also observed that the effect of plate thickness is most evident in case of FFFF boundary condition.



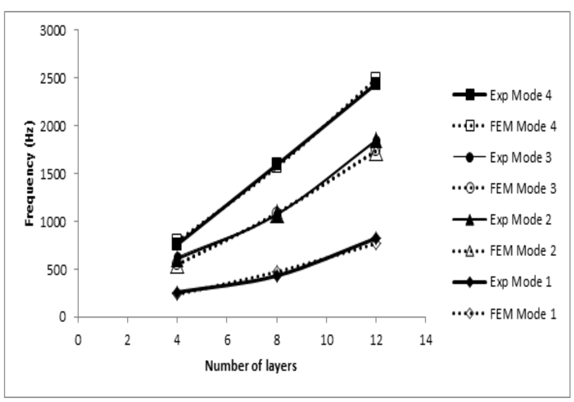


(a) (b)





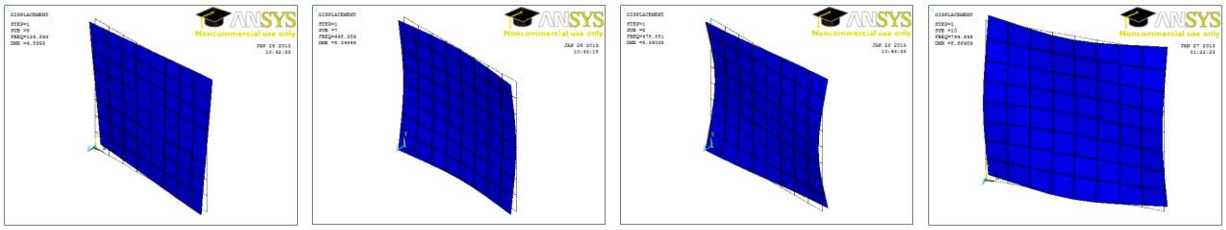
(c) (d)



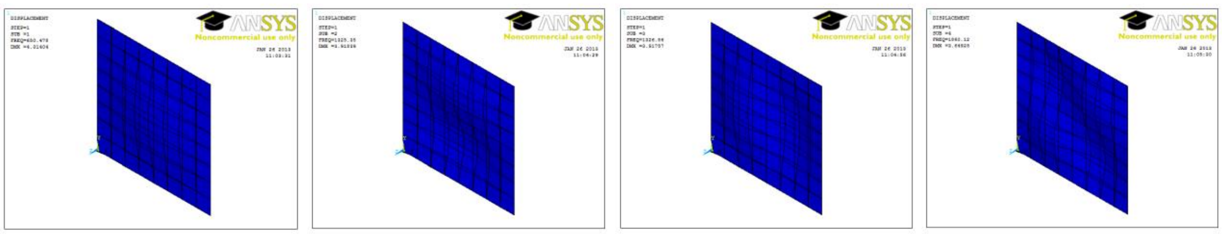
(e)

Figure 6 – Variation of natural frequency with number of layers for (a) free-free (b) fully clamped (c) cantilever (d) simply supported (e) CSCS boundary conditions.

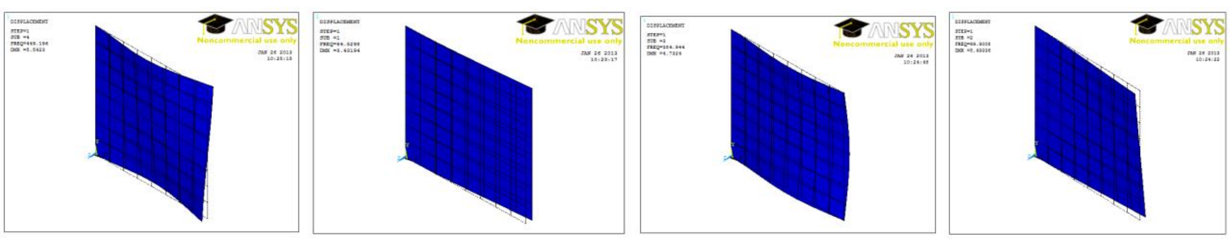
The first four mode shapes for 8 layered plates were obtained from ANSYS 13.0 and are illustrated in figures 7(a)-(e). It is observed that the frequencies for second and third modes are quite close for FFFF, CCCC, SSSS, and CSCS boundary conditions since they represent conjugate modes as evident from the mode shapes. A deviation from such behavior is noted in case of CFFF boundary condition which can be attributed to asymmetry in boundary condition.



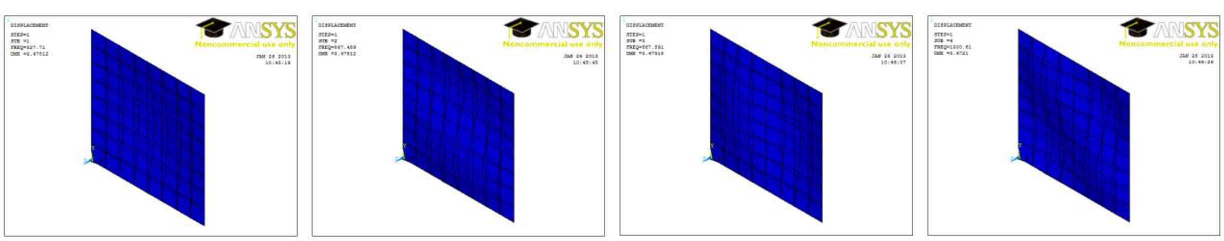
(a)



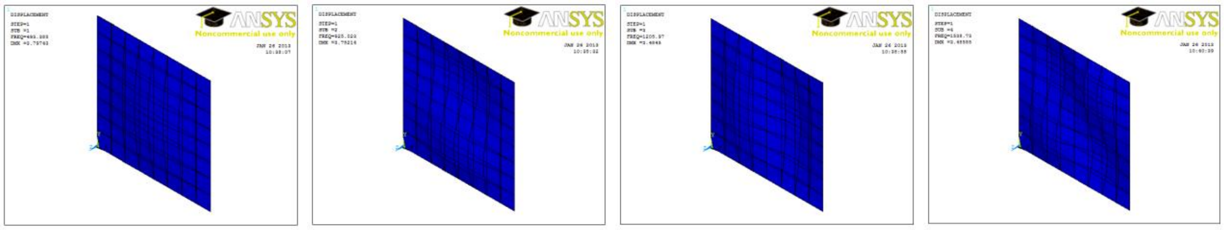
(b)



(c)



(d)



(e)

Figure 7 – Mode shapes for first four modes for 8 layered CFRP plate in (a) FFFF (b) CCCC (c) CFFF (d) SSSS (e) CSCS boundary conditions.

The comparison of the natural frequencies of 8 layered CFRP plates for different boundary conditions is shown in figure 8. The natural frequencies of vibration for CCCC condition are observed to be higher than that of other boundary conditions. This is followed by CSCS, SSSS, FFFF and CFFF in descending order. The greatest frequency in fully clamped condition can be attributed to greater stiffness of supports. With decrease in restraints the modal frequencies decrease.

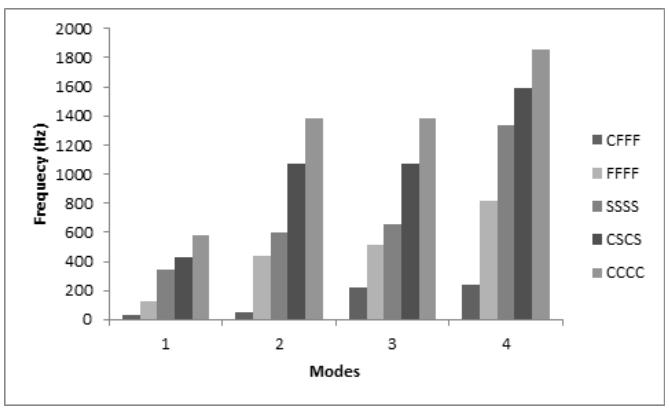


Figure 8 – Comparison of natural frequency of 8- layer CFRP plates for different boundary condition

The natural frequencies of vibration in FFFF boundary condition for aspect ratio 1, 2 and 4 are presented in figure 9. Figure 10(a)-(b) shows the mode shapes for the first four modes for different aspect ratios. It is observed that the modal frequencies increase with the increase in aspect ratio. The frequencies are increased by nearly 47% as the aspect ratio is increased and an almost linear variation was observed.

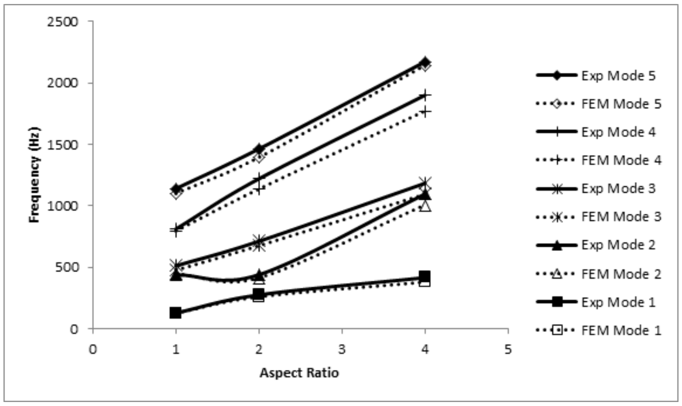
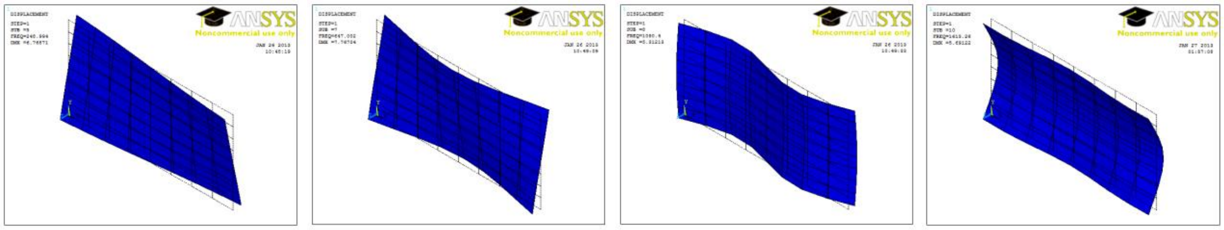
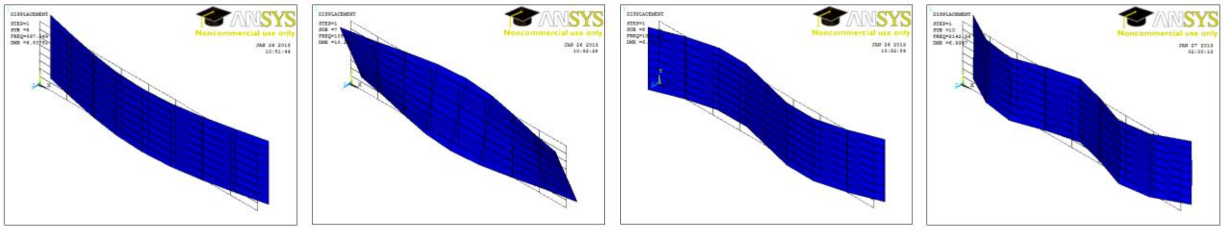


Figure 9 - Variation of natural frequency with aspect ratio



(a)



(b)

Figure 10 - Mode shapes for first four modes for 8 layered CFRP plate in FFFF boundary condition with (a) a/b = 2 (b) a/b = 4.

The variation of natural frequency with type of fibre is shown in figure 11. The present values obtained for CFRP plates have been compared with GFRP plates of equal dimension, reinforced with E Glass Fibre having E= 7.8GPa, υ=0.33 and σ= 2160 kg/m^2 obtained from Basa and Dwibedi [21]. The natural frequencies obtained for CFRP plates are significantly greater than those obtained for GFRP plates showing higher specific stiffness.The increase in frequencies is more pronounced at higher modes.

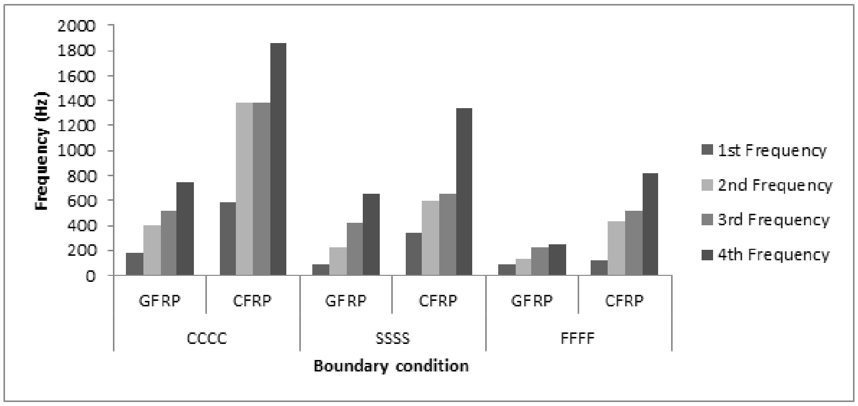


Figure 11 Variation of natural frequency with type of fibre

5. Conclusion

Based on the discussions of results, the conclusions are:

- Benchmark solutions on the natural frequencies of the first four modes are reported for simply supported, fully clamped, cantilever, free-free and CSCS boundary conditions.
- The mode shapes are plotted for CFRP plates supported on different boundaries.
- The frequencies of woven fiber CFRP plates increase with increase of aspect ratio.
- From the experiments conducted it was observed that the frequency of vibration of composite plates increase with increase in the number of layers of fiber for all the support conditions due to bending stretching coupling..
- The frequency of vibration was noted to be highest for fully clamped condition due to the increased stiffness.
- When compared with the results reported for GFRP, it was observed that the modal frequencies for CFRP were considerably higher than that of GFRP accounting for its better performance.

From the present studies, it is concluded that the vibration behavior of woven fiber laminated composite plates and shells is greatly influenced by the geometry and lamination parameter. The figures dealing with variation of the frequencies are recommended as design aids for flat panels. The above recommendations for design of composite plates are valid within the range of geometry and material considered in this study. So the designer has to be cautious while dealing with woven fiber composite plates. This can be utilized to the advantage of tailoring during design of laminated composite structures. The vibration studies can also be used a non destructive tool for damage detection and structural health monitoring of structures.

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