Carrier Frequency Offset Estimation in OFDM Systems

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***Abstract*—This paper presents a basic useful technique for carrier frequency offset (CFO) estimation in orthogonal frequency division multiplexing (OFDM) over frequency selective fading channel. The performance of OFDM system is very sensitive to CFO, which introduces inter-carrier interference (ICI). In cyclic prefix (CP) based estimation, the CFO can be found from the phase angle of the product of CP and corresponding rear part of the OFDM symbol. In CFO estimation using training symbol, the CFO estimation range can be increased by reducing the distance between two blocks of samples for correlation. This was made possible by using training symbol that are repetitive with shorter period. An analytic expression in form of mean square error(MSE) of frequency offset synchronization is reported, and simulation results verify theoretical analysis.**

***Index Terms*—Orthogonal Frequency Division Multiplexing (OFDM), Carrier Frequency Offset (CFO), Inter-Carrier Interference (ICI).**

I. Introduction

OFDM system is widely used in multi-carrier modulation schemes. In this modulation all sub-carriers are orthogonal to each other, which increases the bandwidth efficiency of the system. OFDM transmission frequency channel converts in the group of narrow band flat fading channel, one channel across each sub-channel. OFDM modulation and de-modulation is implemented efficiently by inverse discrete Fourier transform and discrete Fourier transform at the transmitter and receiver respectively [1]. Cyclic prefix (CP) is used for extension of OFDM symbol in time domain which increases the robustness of OFDM system against inter symbol interference (ISI). OFDM has been used in great extent application like wireless local area network IEEE802.11a/g standard, wireless metropolitan network, digital audio broadcasting and terrestrial video broadcasting standard.

OFDM is very sensitive to time and frequency synchronization. The synchronization problem consists of two major parts: carrier frequency offset (CFO) and symbol time offset (STO). This is due to Doppler shift and a mismatch between the local oscillator at the transmitter and receiver. In STO, time domain δ sample and phase shift offset is affected in the frequency domain. Frequency synchronization error destroys the orthogonality among the sub carriers which causes inter carrier interference (ICI) [2]. Therefore the CFO synchronization is essential to OFDM system. The CFO estimation has been extensively investigated for single input single output (SISO) and for multiple input multiple output (MIMO) OFDM based system. The normalized CFO can be divided into two parts which are integral CFO (IFO) ξ_i and fractional CFO (FFO) ξ_f. IFO produce a cyclic shift by ξ_i in receiver side to corresponding sub carrier it does not destroy orthogonality among the sub carrier frequency component and FFO destroys the orthogonality between the sub carriers.

For CFO estimation in time domain, cyclic prefix (CP) and training sequence are used. CP based estimation has analyzed assuming negligible channel effect. CFO can be found from the phase angle of the product of CP and the corresponding part of an OFDM symbol, the average has taken over the CP intervals and in training sequence estimation using training symbol that is repetitive with some shorter period.

In CFO estimation using frequency domain, this technique involves the comparison of the phase of the each sub carrier to successive symbol, the phase shift in symbol due to the carrier frequency offset. Two different estimation modes for CFO estimation in pilot based estimation method is used which are acquisition and tracking mode. In the acquisition mode large range of CFO estimation is done and in tracking mode only the fine CFO is estimated.Initially we assume that acquisition estimation is already performed and hence fine CFO estimation is performed in this paper. All simulation results show mean square error (MSE) with respect to different signal to noise ratio (SNR) in db and comparied for training sequence with ratio of OFDM symbol to repetitive sequence length with respect to different CFO value.

The rest of paper is organized as follows: OFDM system model is guven in section 2, CFO estimation methods in section 3, and simulation results shown in section 4.

II. System Model

In OFDM transmission scheme a wide-band channel divided into N orthogonal narrow-band sub-channels. N Point IFFT and FFT are used to implement OFDM Modulation and Demodulation. The transmitter maps the message bits X_m into a sequence of BPSK or QAM symbols which are subsequently converted into an N parallel bit stream. Each of N symbols from the serial-to-parallel (S/P) conversion is modulated on the different sub-carriers.

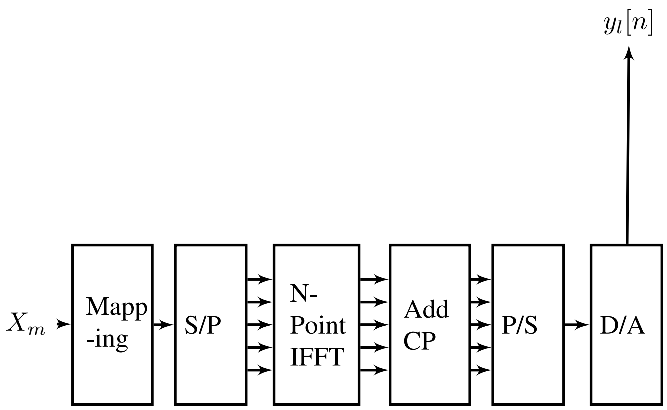

Fig. 1. OFDM Transmitter block

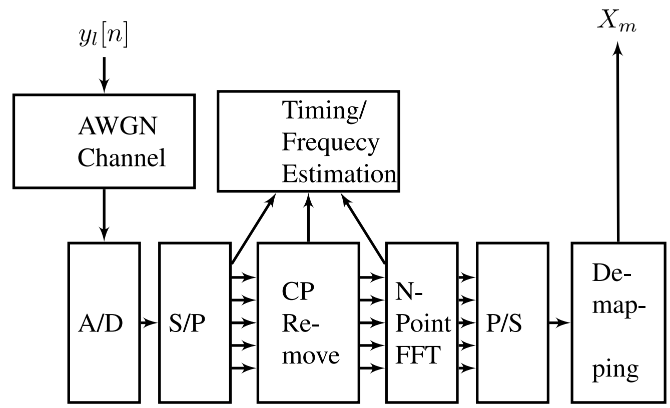

Fig. 2. OFDM Receiver block

Let $X_l[k]$ denote the l^{th} transmit symbol at k^{th} sub-carrier l = 0, 2, ... ∞ . k = 0, 1, 2, ... N-1, T_{sym} = NT_s OFDM symbol length.[3]

OFDM signal at the k^{th} sub-carrier,

$$\psi_{lk}(t) = \begin{cases} e^{2\pi j f_k (t - lTsym)} & 0 < t \leq T_{sym} \\ 0 & \text{eleswhere} \end{cases} \quad (1)$$

The passband and baseband OFDM in the continuous time domain.

$$x_l(t) = Re\left\{\frac{1}{T_{sym}}\sum_{l=0}^{\infty}\left\{\sum_{k=0}^{\infty} x_l[k]\psi_{lk}(t)\right\}\right\} \quad (2)$$

The continuous time baseband OFDM signal is sampled at t = lT_{sym}+nT_s with T_s = T_{sym}/N and f_k = k/T_{sym} to corresponding discrete time OFDM signal.

$$x_l[n] = \sum_{k=0}^{N-1} X_l[k] e^{2\pi jkn/N} \quad (3)$$

for n = 0, 1, ... N-1 the received baseband symbol with considering the effect of channel and noise at the receiver $\{y_l[n]\}_{n=0}^{N-1}$ the sample value of the received ODFM symbol $y_l(t)$ at t = lT_{sym}+nT_s is

$$y_l[k] = \sum_{n=0}^{N-1} H_l[n] y_l[n] e^{-2\pi jkn/N} + W_l[n] \quad (4)$$

The received baseband symbols under the presence of CFO ξ and STO δ

$$y_l[n] = \frac{1}{N}\sum_{k=0}^{N-1} H_l[k] X_l[k] e^{2\pi j(k+\xi)(n+\delta)/N} + W_l[k] \quad (5)$$

Where ξ is the normalized frequency offset (the ratio of actual frequency offset to the inter carrier spacing Δf) and $w_l[n]$ is the complex envelope of additive white Gaussian noise (AWGN).

the k^{th} element of DFT sequence consist of three component. [4]

$$y_k = (X_k H_k)\left\{\frac{sin\pi\xi}{Nsin(\pi\xi/N)}\right\} e^{\pi j(N-1)/N} + I_k + W_k \quad (6)$$

Here the first component is modulation and second component is ICI caused by the frequency offset.

$$I_k = \sum_{l=0, l\neq k}^{N-1} (X_l H_l)\left\{\frac{sin\pi\xi}{Nsin(\pi\xi(l-k+\xi)/N)}\right\}.e^{j\phi} \quad (7)$$

$$e^{j\phi} = e^{\pi j\xi(N-1)/Ne^{-\pi j(l-k)/N}} \quad (8)$$

In order to evaluate the statistical properties for estimation of the ICI, some further assumptions are necessary. Specifically, it will be assumed that E [I_k] = 0 and E [$X_k X_l^*$] = $|x|^2\delta_{lk}$ the modulation values have zero mean and are uncorrelated. With this provision E[I_k] = 0

III. CFO Estimation

A. CP Based:

With perfect symbol synchronization, a CFO of ξ results in a phase rotation of $2\pi n\xi/N$ in the received signal. Under the assumption of negligible channel effect, the phase difference between CP and the corresponding rear part of an OFDM symbol (spaced N samples apart) is 2πNξ/N = $2\pi\xi$. Then, the CFO can be found from the phase angle of the product of CP and the corresponding rear part of an OFDM symbol,

CFO estimation using CP based.

$$\widetilde{\xi} = (1/2\pi) arg\left\{y_l^*[n] y_l[n+N]\right\} \quad (9)$$

n = -1, -2, ... -N_g. In order to reduce the noise effect, its average can be taken over the samples in a CP interval.

$$\widetilde{\xi} = (1/2\pi) arg\left\{\sum_{n=-N_g}^{-1} y_l^*[n] y_l[n+N]\right\} \quad (10)$$

Arg() performed tan^{-1}(), the range of the CFO estimation is [-0.5+0.5] and mean square error performed by $\widetilde{\xi} - \xi$

B. Symbol Based:

Two identical training symbols are transmitted consecutively and the corresponding signals with CFO of ξ are related with each other. For an OFDM transmission symbol at one receiver with an assumption of the absence of noise the 2N Point sequence is [4]

$$r_n = \frac{1}{N}\sum_{k=0}^{N-1} H_k X_k e^{2\pi j(k+\xi)/N} \tag{11}$$

n = 0, 1, ... 2N-1,
The k^{th} element of the N Point DFT of the first N points (11) is

$$R_{1k} = \sum_{n=0}^{N-1} r_n e^{-2\pi jkn/N} \tag{12}$$

k = 0, 1, 2, ... N-1,
The second half of the sequence is-

$$R_{2k} = \sum_{n=0}^{N-1} r_n + N e^{-2\pi jkn/N} \tag{13}$$

$r_n + N = r_n e^{2\pi j\xi}, R_{2k} = R_{1k}e^{2\pi j\xi}$, including the AWGN noise $Y_{1k} = R_{1k} + W_{1k}$
$Y_{2k} = R_{1k}e^{2\pi j\xi} + W_{2k}$; k = 0, 1, 2, ... N-1.

Observe that between the first and second DFT symbols, both ICI and signal are altered in exactly the same way, by a phase shift proportional to frequency offset. Therefore, if frequency offset ξ is estimated using above observations, it is possible to obtain accurate estimation even when the offset is too large for satisfactory data demodulation [4].

$$\widetilde{\xi} = (\frac{1}{2\pi})tan^{-1}\left\{\frac{\sum_{k=0}^{N-1} Im[Y_{2k}Y_{1k}^*]}{\sum_{k=0}^{N-1} Re[Y_{2k}Y_{1k}^*]}\right\} \tag{14}$$

The limit for accurate estimation by (14) is $|\xi| \leq 0.5$

C. Training Sequence Based:

CFO only within the range$|\xi| \leq 0.5$, Since CFO can be large at initial synchronization stage, we may need estimation techniques that can cover wider CFO range. The range of CFO estimation can be increased by reducing the distance between two blocks of samples for correlation. This is made possible by using training symbols that are repetitive with some shorter period. Let D represents the ratio of the OFDM symbol length to the length of a repetitive pattern. Let the transmitter sends the training symbols with D repetitive patterns in the time domain, which generated combo-type signal in the frequency domain after taking IFFT.

$$X_l[k] = \begin{cases} A_m & \text{if}, k = D.i, i = 0, 1, ...(N/D-1) \\ 0 & \text{eleswhere} \end{cases} \tag{15}$$

where A_m represents an M-ary symbol and N/D is an integer and $x_l[n]$ and $X_l[n+N/D]$ are identical. After receiving repetitive length data sequence, receiver can make CFO estimation as [6]

$$\widetilde{\xi} = (D/2\pi)arg\left\{\sum_{n=0}^{N/D} y_l^*[n]y_l[n+N/D]\right\} \tag{16}$$

The estimation range in this technique is $|\xi| \leq D/2$, which becomes wider as D increases and number of samples for the computation of correlation is reduced by 1/D, which degrade the MSE performance of OFDM system. In other words, the increase in estimation range is obtained at the sacrifice of MSE (mean square error) performance. Figure (6) shows the estimation range of MSE vs. CFO performance for D = 2 and 4. simulation generates the plot which shows that the range of CFO is increased when the value of D is increasing.

$$\widetilde{\xi} = (D/2\pi)arg\left\{\sum_{m=0}^{D-2}\sum_{n=0}^{N/D-1} y_l^*[n + mN/D]y_l[n+(m+1)N/D]\right\} \tag{17}$$

The MSE performance can be improved without reducing the estimation range of CFO by taking the average of the estimates with the repetitive patterns of the shorter period.

D. Pilot Based:

Pilot tones inserted in the frequency domain and transmit every OFDM symbol for CFO tracking.The signals are transformed into $Y_l[k]_{k=0}^{N-1}$ and $Y_{l+D}[k]_{k=0}^{N-1}$ though FFT, from which pilot tones are extracted. After estimating CFO from pilot tones in the frequency domain, the signal is compensated with the estimated CFO in the time domain. In this process, two different estimation modes for CFO estimation are Implemented: acquisition and tracking modes. In the acquisition mode, a large range of CFO including an integer CFO is estimated and in the tracking mode, only fine CFO is estimated. The integer CFO is estimated by [5].

$$\widetilde{\xi} = (\frac{1}{2\pi T_{sub}})max(\xi)\left\{|\sum_{j=0}^{L-1} Y_{l+D}[p[j], \xi]Y_l^*[p[j],\xi]X_{l+D}^*[p[j]]X_l[p[j]]|\right\} \tag{18}$$

where L, p[j], and $X_l[p[j]]$ denote the number of pilot tones, location of the j^{th} pilot tone, and the pilot tone located at p[j] in the frequency domain at the l^{th} symbol period [3].

IV. SIMULATION RESULTS

CFO estimation is done by using four different techniques, first one by using Equation (10), the phase difference between CP and the corresponding rear part of an OFDM symbol. Second by using Equation (14), the phase difference between two repetitive preambles. Third by using Equation (16), training

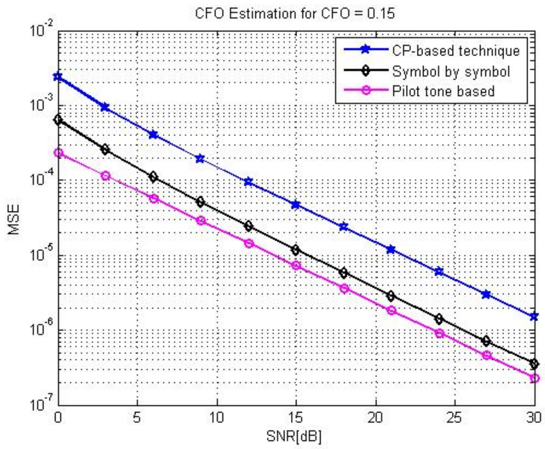

Fig. 3. Simulation With CFO = 0.15

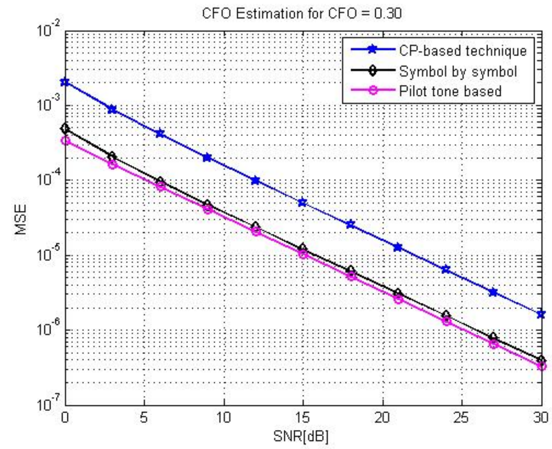

Fig. 4. Simulation With CFO = 0.30

sequence with D integer i.e. ratio of the OFDM symbol length to the length of a repetitive pattern, taking D = 1, 2 and 4, in this estimation range of CFO increases but MSE performance decreases with increasing the value of D. Simulation figure (5) shown for D = 1, 2 and 4, for MSE vs CFO performance shows in figure (6), in this figure D = 2 and D = 4, the range of CFO is increasing for D = 4 comparisons with D = 2, taking signal to noise ratio 6 dB. Fourth one by using Equation (18) estimation between pilot tones in two consecutive OFDM symbols. Figure (3) and Figure (4) show MSE performance for three different techniques with taking CFO = 0.15 and 0.30. Pilot tone based estimation is better then CP and Preamble based estimation. Performances of estimation techniques vary depending on the number of samples in CP, the number of samples in preamble, and the number of pilot tones, used for CFO estimation. Simulations are performed to verify the accuracy of MSE analysis.

The OFDM system parameters are CFO = 0.15 and CFO = 0.30, N = 128, Ng = 16, Nps = 4 (Pilot spacing), Number of pilots Np = 32, signal to noise ratio (SNR) 0 to 30 db, D = 1, 2 and 4. For OFDM mapping QAM modulation used and taking signal energy $E_s = 1$.

V. Discussion and Conclusion

In this paper, frequency synchronization in an OFDM system is studied. The simulation results show the superior performance of our proposed scheme in AWGN channel. Pilot based mean square estimation (MSE) performance is superior then compare to CP based and symbol based.By using repeated sequences with different value of D, CFO has been estimated.

Further intensive research is needed in MIMO-OFDM system considering the generalized system model.Where the CFO and propagation delay between each transmit antenna and receive antenna are possibly different.

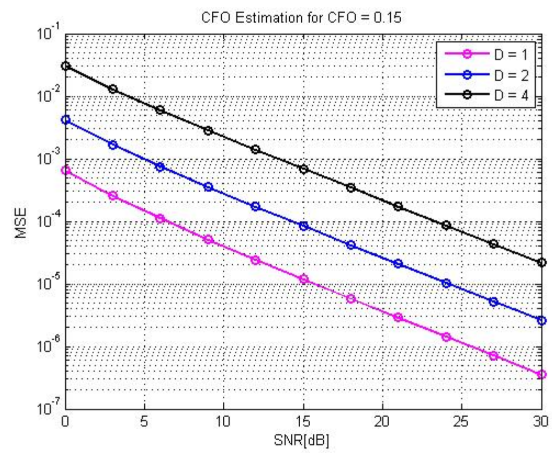

Fig. 5. Training Sequence Based

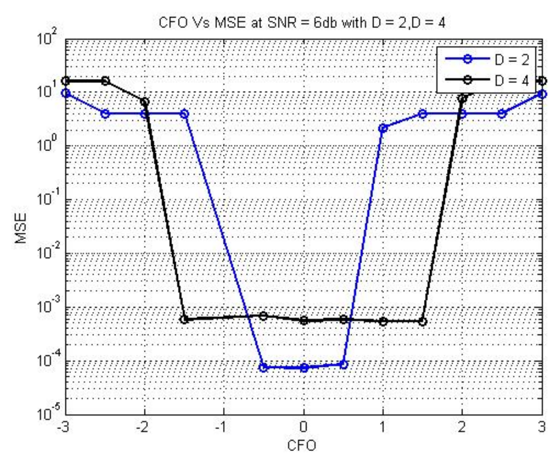

Fig. 6. MSE vs CFO

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