# Effect of Tuning Parameters of a Model Predictive Binary Distillation Column

[1]Rakesh Kumar Mishra,[2]Rohit Khalkho,[3]Brajesh Kumar,[4]Tarun Kumar Dan
Department of Electronics and Communication Engineering
National Institute of Technology
Rourkela, India
[1]rakeshbbu@gmail.com,[2] rohitkhalkho70@gmail.com, [3] brajesh.nitrkl@gmail.com

*Abstract*— **Model Predictive Control (MPC) is used mainly for specific handling of constraints. MPC is implemented mainly by microprocessors. So, before implementation it is converted to discrete time. This paper presents about the design of dynamic linear controller for a binary distillation column. The design is based on MPC, which is based on prediction of control variable. We have used Wood and Berry 2×2 function for the distillation column. Firstly, we have implemented an ideal MPC by taking unit step input. Secondly, we have implemented general MPC for binary distillation column by taking the unit step input with and without disturbance. We have analyzed the manipulated and controlled variables for the distillation column using MPC. We have also find how to remove the ringing effect in manipulated variables for MPC.**

***Keywords- Model Predictive Control; Wood and Berry; Ringing Effect; Binary Distillation Column; MIMO***

## I. INTRODUCTION

Distillation is the separation method in the petroleum and chemical industries for purification of final products. Distillation columns are used to enhance mass transfer or for transferring heat energy. A general distillation column consists of a vertical column, where plates or trays are used to increase the component separations. A condenser is used to cool and condense the vapour and a reboiler is used to provide heat for the necessary vaporization from the bottom of the column. A reflux drum is used to hold the condensed (liquid) vapour to recycle the liquid reflux to back from top of the column [15]. Modern day distillation control systems are based on common or advanced techniques by assuming the column at a constant pressure. Due to Pressure fluctuations, control of several parameters is difficult to handle and this reduces the performance. The L-V (Liquid-Vapour) structure [1] is known as the energy balance structure and can be considered as the standard control structure for a dual composition control distillation.

The goal of this paper is to understand the model predictive controller for constraint and unconstraint input/output.

The control structure for the distillation column is based on L-V structure or the energy balance method. In this control configuration the vapour flow rate V and the liquid flow rate L are the control inputs. The main objective [1] is to maintain the specification of the product concentration outputs $X_B$ and $X_D$ (controlled variable) due to disturbance F (feed flow) and $X_F$ (feed concentration). Fig-1 presents the distillation column [1].

The MPC has been selected for controlling the distillation column [15]. MPC is based on open loop control and close loop control method. It generates an online feedback control by using the open-loop optimization [15].

The basic concept involved in MPC design is to predict the future plant response by the help of a process model and always trying to minimize a finite horizon objective function which consists of a sum of future predicted errors and control moves.

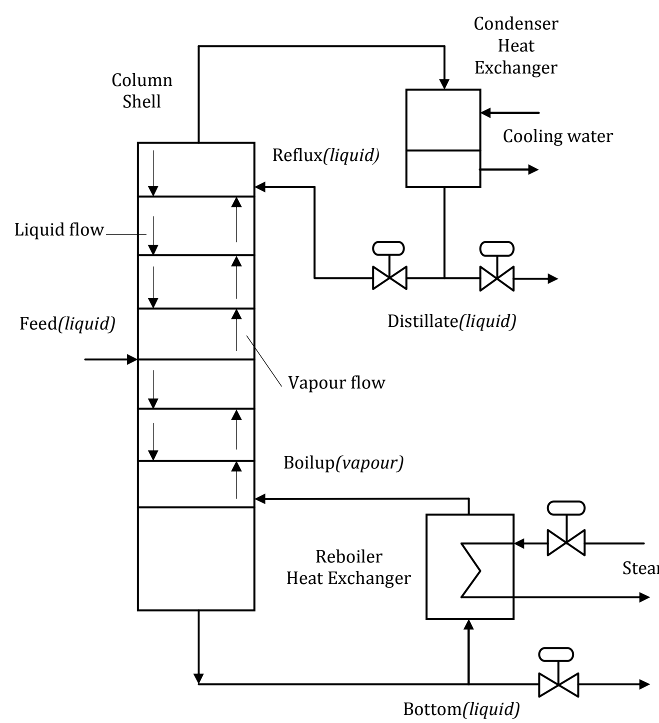


Fig-1 Distillation Column

## II. MODEL PREDICTIVE CONTROL

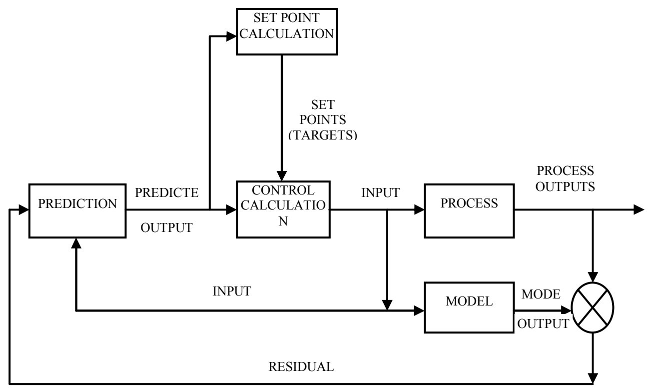


Fig.2: Block Diagram of MPC

To predict the current values of output variables a process model is use [2]. The differences between the actual and predicted outputs, serve as the feedback signal to the prediction block which is known as residuals. The set point for the control calculation is called as target. The control calculations are based on current measurements and predictions of the future values of the outputs [4]. The basic MPC concept can be summarized as follows: we can use the model and current measurement to predict future value of the outputs [9], [10]. Then the appropriate changes in the input variable can be calculated by both prediction and measurement. In MPC applications [4], the output variable also referred to as controlled variables (CV), while the input variables are also called manipulated variables (MV). Measured disturbance variables are called DV or feed forward variables. Model predictive control offers several important advantages: The process model captures the dynamic and static interaction between input, output, and disturbance variables. Constraints on input and outputs are considered in a systematic manner. The control calculations can be coordinated with the calculation set points and at last accurate model predictions can provide early warning of potential problems. So these are the advantages of MPC. Fig-2 shows the block diagram of MPC [2].

### *A. Fundamental of MPC*

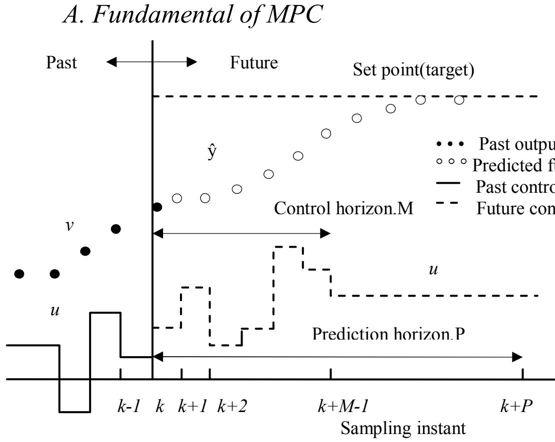


Fig. 3: Basic Concept of MPC

The actual output y, predictive output y^, and manipulated input are shown in fig-3 [5], [13]. At the current sampling instant, denoted by k, MPC strategy calculates a set of M values of the input {u(k+i-1), i = {1, 2, 3,….,P}}.The set consists of the current input u(k) and M-1future inputs. The input is held as constant after the M control moves. The inputs are calculated so that a setoff P predicted outputs {y^(k+i), I = {1,2,……,P}} reaches the set point in an optional manner. The control calculations are based on optimizing an objective function. The number of prediction P is referred as prediction horizon while the number of control moves M is called control horizon [3].

A distinguishing feature of MPC [7] is its receding horizon approach [3]. Although a sequence of M control moves is calculated at each sampling instant, only the first move is implemented. After new measurements become available only the first input move is implemented. This procedure is repeated at each sampling instant.

### *B. MPC Based Step Response Model*

With the help of Step response, models are based on the following ideas. The system is taken to be at rest. For a single input single output (SISO) [8] linear time invariant system, let the output change for a unit input change Δv be given by {0,S1, S2,...,Sn} [14]. Here we have assumed that the system settles down exactly after n steps. The step response {s1, s2... sn} constitutes a complete model of the system [14]. This equation 1 helps in getting the output for any input sequence:

$$Y(k) = \sum_{i=1}^{n} S_i \Delta V(k-i) + S_n V(k-n-1) \tag{1}$$

Step response model is sometimes used for stable and incorporating processes [14]. For an incorporating process the response of the slope remains constant after n steps, i.e. equation 2 shows about the incorporating process.

$$S_n - S_{n-1} = S_{n+1} - 1 = S_{n+2} - S_{n+1} \tag{2}$$

For a multi input multi output (MIMO) method [12], nv inputs and ny outputs are used. Series of step response coefficient matrices is obtained as [14]:

$$S_n = \begin{bmatrix} S_{1,1,i} & S_{1,2,i} & \cdots\cdots\cdots & S_{1,nv,i} \\ S_{2,1,i} & S_{2,2,i} & \cdots\cdots\cdots & S_{2,nv,i} \\ \vdots & \vdots & \vdots\vdots\vdots & \vdots \\ S_{ny,1,i} & S_{ny,2,i} & \cdots\cdots\cdots & S_{ny,nv,i} \end{bmatrix}$$

Where Sl, m, i is the ith step response coefficient relating to mth input to the 1th output [14]. The step response can be achieved directly by identification experiments, or generated from a continuous or discrete transfer function or state-space model. Fig-4 presents the step response model.

For example, if discrete system description (sampling time=0.1) is (equation 3):

$$Y(k) = -0.5y(k-1) + v(k-3) \quad (3)$$

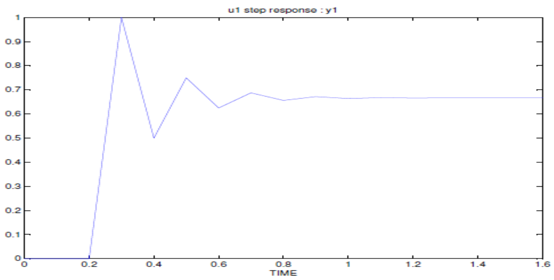

Fig. 4: Step Response of MPC

## III. MODEL DESCRIPTION

The MPC control law [5] can be most easily derived by referring to the above fig-3. For any assumed set of present and future control moves(k), $_3$u(k +1),...,$_3$u(k + M – 1) the future behavior of the process outputs y(k +1.k),y(k +2.k),...,y(k +P.k) are predicted over a horizon P [14]. The present and future control moves (M≤ P) are calculated to reduce the quadratic objective of the form shown in equation (4):

$$\min_{\Delta U(k)\cdots\Delta U(k+m-1)} = \sum_{l=1}^{p} \left\| \Gamma_l^y ([y(k+l|k) - r(k+l)]) \right\|^2 + \sum_{l=1}^{m} \left\| \Gamma_l^u [\Delta U(k+l-1)] \right\|^2 \quad (4)$$

Here $\Gamma_l^y$ and $\Gamma_l^u$ are weighting matrices to punish particular components of y or u at certain future time intervals. r (k +l) is the (possibly time varying) Vector of future reference values (set points). Though m control moves are calculated, only the first one (Δu(k)) is implemented. At the next sampling interval, new values of the measured output are obtained, the control horizon is shifted forward by one step, and the same calculation is repeated. The resulting control law is referred to as "moving horizon" or "retiring horizon".

The predicted process outputs y(k+1.k),...,y(k+ p.k)depend on the current measurement and assumptions we make about the unmeasured disturbances and measurement noise affecting the outputs. To under the stated assumptions, it can be shown that a linear time-invariant feedback control law results

$$\Delta u(k) = \text{KMPC Ep}(k+1.k) \quad (5)$$

Where, EP (k +1.k) is the vector of predicted future errors

over horizon P.

For open-loop stable plants, nominal stability of the closed-loop system depends only on KMPC which in turn is affected by the horizon p, the number of moves m and the weighting matrices and No precise conditions on m, p, and exist which guarantee closed-loop stability. In general, decreasing m relative to p makes the control action less fast growing and tends to stabilize a system. For p = 1, nominal stability of the closed-loop system is guaranteed for any finite m, and time-invariant input and output weights. More commonly, is used as a tuning parameter.

For SISO Process

$$Y = 5.72\exp(-14s)/(60s+1)\,u(s) + 1.52\exp(-15s)/(25s+1)\,d(s) \quad (6)$$

$$\text{Plant transfer function} = 5.72\exp(-14s)/(60s+1) \quad (7)$$

$$\text{Disturbance transfer function} = 1.52\exp(-15s)/(25s+1) \quad (8)$$

For above SISO process taking tuning parameters default

*M*=5, *P*=10, *T*=1

Equation 6, 7 and 8 shows the output, plant transfer function and disturbance function respectively. For above SISO process, taking tuning parameters [8] as M=5, P=10, T=1. Fig-5 shows the response of unconstraint MPC of manipulated variables and controlled variable.

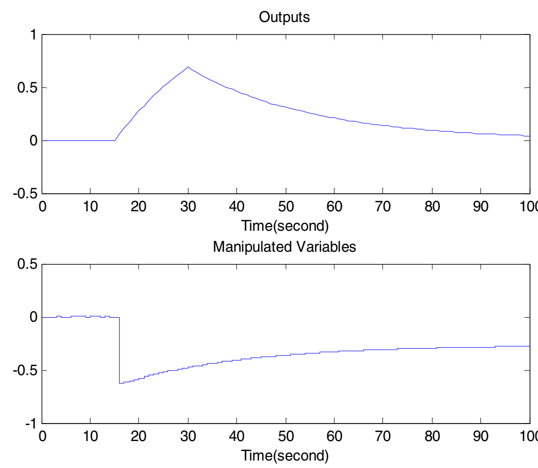

Fig. 5: Response of Unconstraint MPC

### B. *Constrained Model Predictive Control*

Constrained Model Predictive Control [6] satisfies three conditions, where U is the manipulated input and $_3$U is the rate of manipulated variable and Y predicted process output. The three conditions are described below:

Manipulated variable constraints:

$$U\min(l) \le U(k+l) \le U\max(l)$$

Manipulated variable rate constraints:

$$|U(k+l)| \le U\max(l)$$

Output variable constraints:

$$Y\min(l) \le Y(k+l.\,k) \le Y\max(l)$$

For above SISO process taking tuning parameters as P=10, M=5 and T=1. Here we have imposed constraints in manipulated variable as u [-0.4to 0.1]. Fig-6 shows the response of constraint MPC.

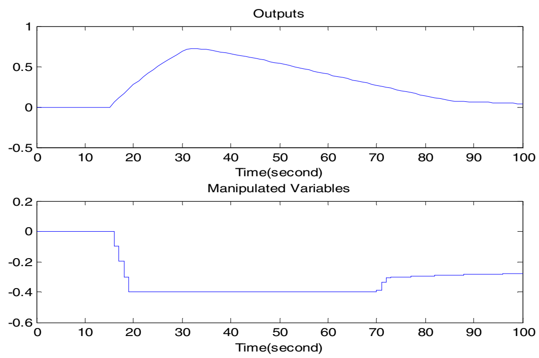

Fig. 6: Response of Constraint MPC

## IV. MPC FOR DISTILLATION COLUMN

Wood and Berry 2×2 process [11].

$$\begin{bmatrix} x_D(s) \\ x_B(s) \end{bmatrix} = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} R(s) \\ S(s) \end{bmatrix} + \begin{bmatrix} \frac{3.8e^{-8.1s}}{14.9s+1} \\ \frac{4.9e^{-3.4s}}{13.2s+1} \end{bmatrix} F(s) \quad (9)$$

Where, F = feed of distillation column

$X_D$ = composition of distillation

$X_B$ = composition of bottom

R = reflux flow

S = steam flow

Equation 9 shows the function of Wood Berry process of the distillation column. By taking design parameters as M=17, P=73 and T=1.67 and setting other values as default.

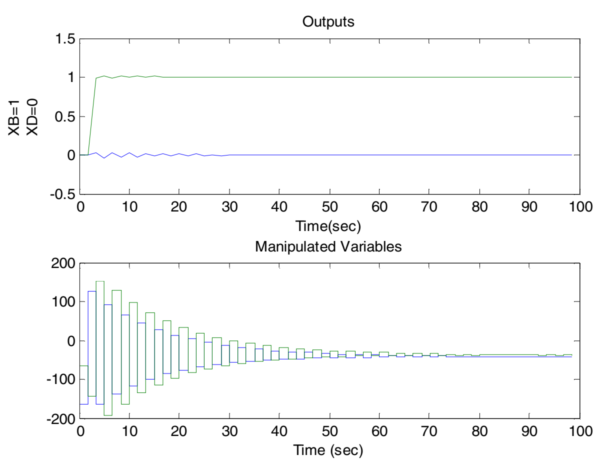

Fig. 7: Response of Wood Berry without Disturbance

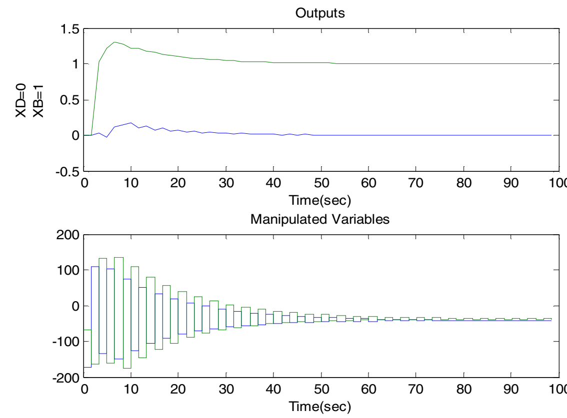

Fig. 8: Response of Wood Berry with Disturbance

Fig- 7 and fig-8 shows the response of Wood Berry distillation column model with disturbance and without disturbance respectively.

It shows that when disturbance is applied, peak overshoot and settling time increased. But in both the case ringing effect occurs in the manipulated variable. So we are trying to reduce ringing with the change of design parameters. Generally when we take P>M, but close to M (M=5, P=10,and T=2).

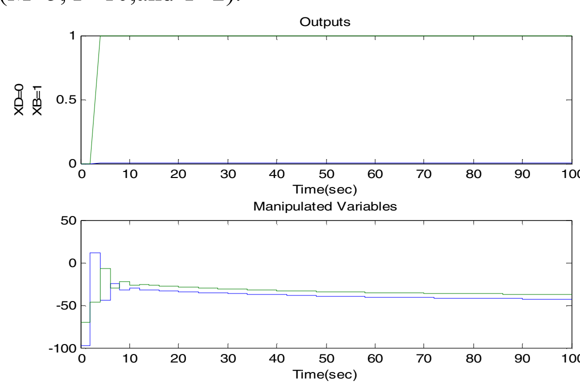

Fig. 9: Response Using Design Parameter Default Without Disturbance

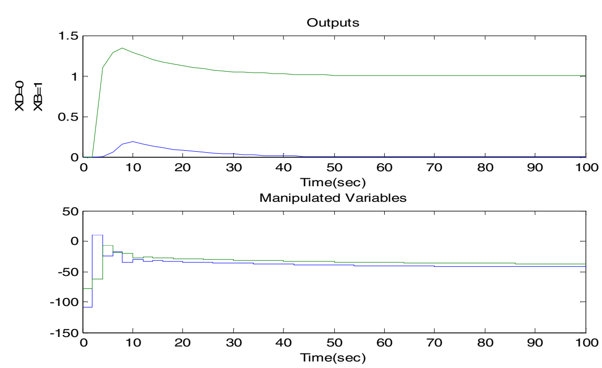

Fig. 10: Response Using Design Parameter Default With Disturbance

In above fig-9 and fig-10 ringing effect reduces compared to fig-7 and fig-8 in a manipulated variable using different design parameters. It shows that if we change the design parameters we can reduce ringing of the manipulated variable.

*A. Design of Ideal Controller*

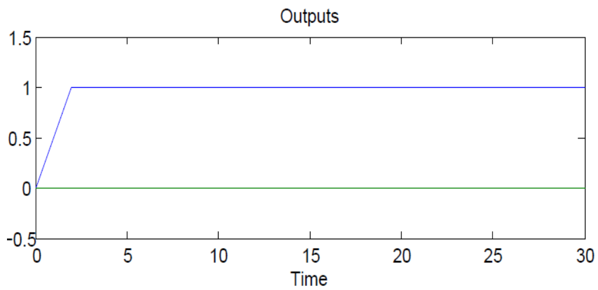


Fig. 11: Output Response of Ideal Controller

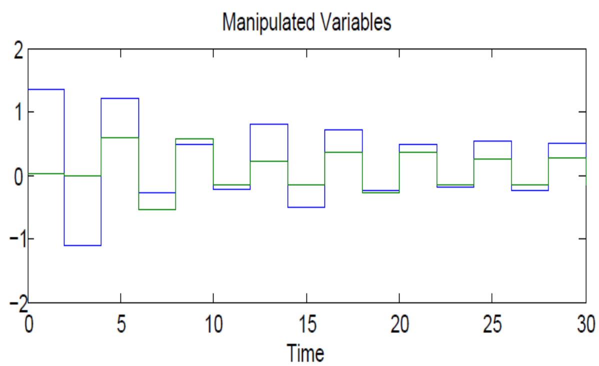


Fig. 12: Manipulated Variables of Ideal Controller

Here we have taken M=5=P, but in this case manipulated variable is affected by ringing effect. Fig-11 shows the control variables of the distillation column using ideal controller and fig-12 shows the manipulated variable of the ideal controller.

*B. Removal of Ringing by Taking P>M*

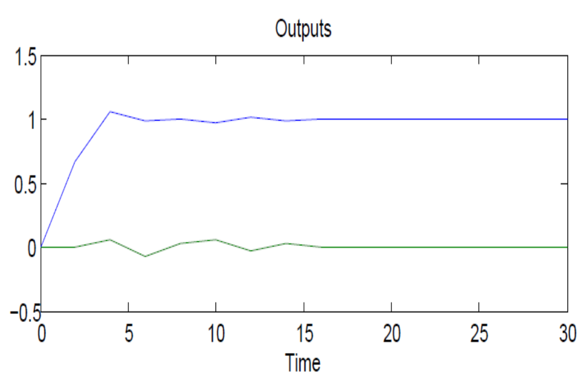


Fig. 13: Response of Controlled variable (XD/XB vs. Time)

By taking P=10, M=3.

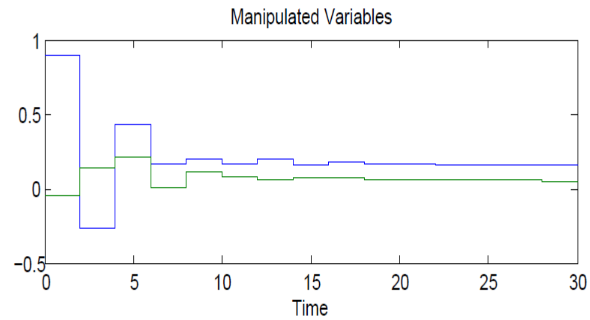


Fig. 14: Manipulated Variable Response

Fig-13 shows the response of control variable of a distillation column using MPC and fig-14 shows the manipulated variable response of a MPC. Here we can see ringing effect is less compared to fig-12.

*C. Removal of Ringing by Blocking*

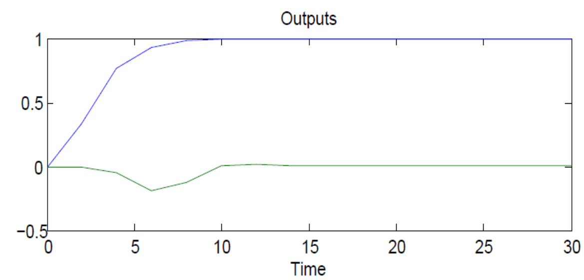


Fig. 15: Response of Controlled variable (XD/XB vs Time)

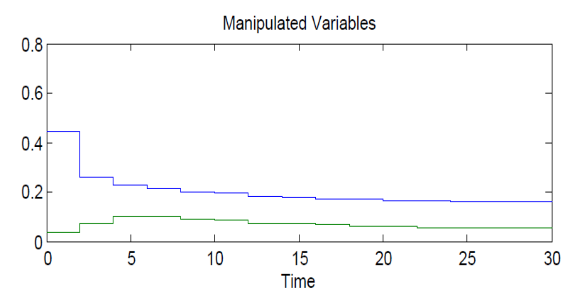


Fig. 16: Manipulated Variable Response

Fig-15 shows the response of the control variable and fig-16 shows the response of MPC of a distillation column when the input is send as block by block. By this ringing reduces, but disturbance affects to the composition of bottom $X_B$/ top $X_D$.

*D. Removal of Ringing by Increasing the Input Weights*

Fig-17 shows the response of the control variable and fig-18 shows the response of MPC of a distillation column. By the changing the manipulated input (weight) we can reduce the ringing effect.

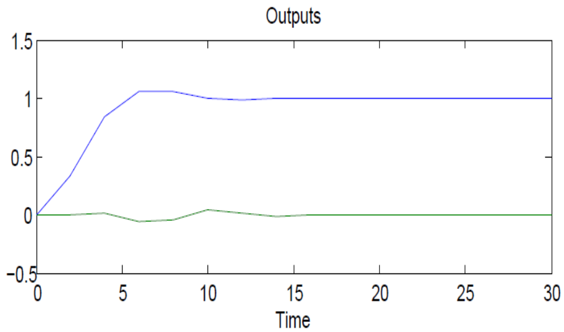


Fig. 17: Response of Controlled variable (XD/XB vs Time)

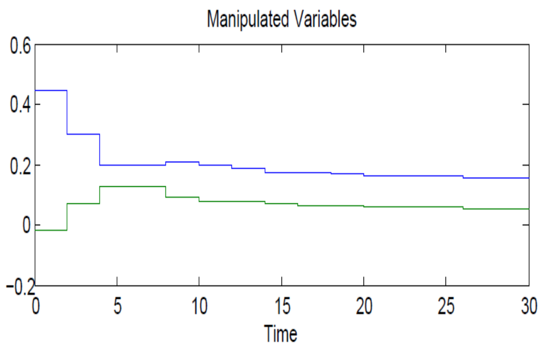


Fig. 18: Manipulated Variable Response

## IV. CONCLUSION

In this paper we have studied about the effects of tuning parameter of distillation column model with the help of MPC. We analyze SISO distillation column performance. Effect of tuning parameter had been studied and it is found that at certain tuning parameter the performance of control system is better than any classical control system. This paper studies only about the composition of control system. However, these strategies can be applied to other control variable in a distillation column. Further it can be extended to MIMO. With the help of MPC we can analyze the output of distillation column and can find the ringing effect in the manipulated variables and find out several techniques to remove it.

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