ANALYSIS OF AN UNDERACTUATED COMPLIANT FIVE-BAR LINKAGE

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Abstract: This paper presents the kinetostatic analysis and design of a two degree-of-freedom five-bar underactuated compliant mechanism using a pseudo-rigid body modeling. Two of the joints in the linkage are flexible and actuation is provided to the crank. The slider connected to coupler provides a grasping action. Equations of motion along with loop closure relations facilitate in obtaining the kinematic solution. Newton-Rapson's approach is used to solve the complex coupled nonlinear transcendental equations and results are shown for an assumed load-function pattern.

Keywords: Underactuated compliant linkage; Kinetostatic analysis; Load function; Virtual work principle; Loop closure equations.

1. Introduction

Underactuated mechanical systems, which possess fewer actuators than degrees of freedom (DOF), have received widespread attention in the last two decades [1–3]. Their typical applications include space robots, helicopters, surface/underwater vehicles, hopping robots, robotic hands and fingers and underactuated manipulators etc. The simplest underactuated mechanism is a single degree of freedom linkage without any actuation [4]. As degrees of freedom and inputs increase, the complexity increases. Underactuation has some advantages such as reduction in actuator cost and overall weight to save energy and increase the reliability of the system in case of actuator failure. Underactuated mechanisms generally use elastic elements in the design of their unactuated joints. Thus, one should rather think of these joints as uncontrollable or passively driven instead of unactuated. Here, the actuation wrench is applied to the input and is transmitted to the phalanges through suitable mechanical elements. Passive elements are used to kinematically constrain the linkage and ensure the shape-adaptation to the object grasped.

In conventional rigid-link mechanisms, the mobility of the system is only due to the movable joints. On the other hand compliant mechanisms get at least some of their mobility from the flexibility of their members (links) along with the movable pin joints or hinges. This can be used to store energy and to release at some other time. Fig.1 shows a partially compliant slider-crank linkage.

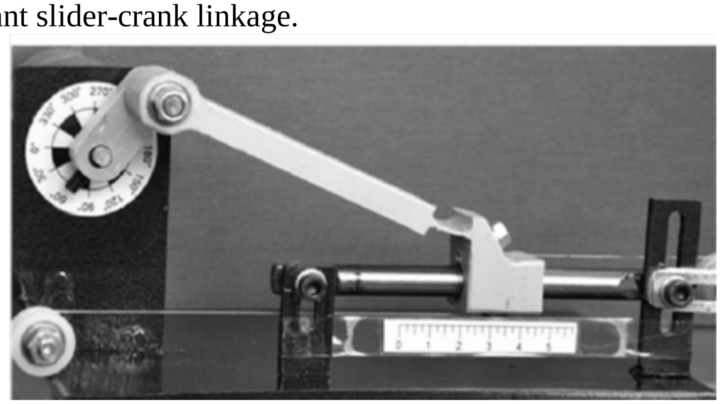


Fig.1 Partially compliant eccentric slider crank linkage

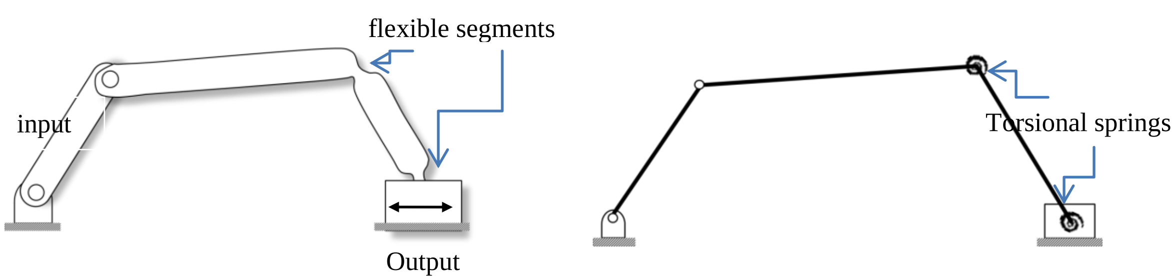
The main advantages of compliant mechanism over its rigid-body counterpart are reduced cost, lighter weight, lower maintenance, higher performance and high precession. The reduction in the number of joints also gives higher precession to the mechanism.

For multi degree of freedom underactuated mechanisms, kinematic analysis must be performed along with a static force analysis. Therefore, the analysis and design of these mechanisms can be performed by assigning a specific output-loading condition. Kinematics of underactuated mechanisms is related to their loading conditions as well as link proportions. Several authors illustrated the kinetostatic analysis of underactuated gripping fingers. Cheng et al.[5] proposed an underactuated mechanism with 1-DOF for finger operation. Design of this underactuated mechanism is based on spring elements of the structure. Kragten and Herder [6] defined two metrics for measuring the performance of underactuated hands to pick and move the objects. These metrics quantify the capability to achieve stable grasp equilibrium of a range of freely moving objects (ability to grasp), and the capability to keep hold of the grasped objects while disturbing forces are applied (ability to hold). The calculations and measurements of these metrics are shown for cable-pulley driven hands. Compliant mechanisms are often analyzed using pseudo rigid-body models (PRBM). Tanik and Soylemez [7] illustrated analysis and design of an underactuated compliant mechanism using pseudo rigid body model. The paper considers the mechanism for two conditions (i) given output loading and (ii) constant input torque and presents design charts for given output loading and constant input torque. More recently Petkovic et al.[8] explained kinetostatic analysis of a gripper structure using rigid body model with added compliance at every joint. Fuzzy logic approach was employed to find the corresponding gripper forces.

In present work, kinetostatic analysis of a 2-DOF slider crank underactuated linkage is presented. It has two compliant joints and one input actuation at the crank. The nonlinear equations of motion are solved using numerical approach for given output loading condition by assuming the output-link load as a function of the crank's position.

2. Kinetostatic Analysis of underactuated compliant mechanism

Fig.2 shows the linkage under consideration. It has two degrees of freedom, but only one actuation is provided at the crank. Using underactuation, the stroke variation according to loading is possible. In this mechanism input is provided across crank and output is obtained across the slider.



(a) Compliant linkage (b) Equivalent PRBM

Fig.1 Two degree of freedom five bar linkage

It is assumed that masses of the links are negligible and operating speeds are slow. Therefore, the entire analysis is based on the static equilibrium. For the analysis, the method of virtual work is used. In this technique, springs between links are removed and torques which are equal in magnitude and opposite in direction are employed at those points. From kinematic analysis two basic loop closure equations are obtained. Other equations can be obtained by force analysis, which is nothing but to giving some virtual angular displacement and equating the net work done (sum of virtual works of active force F_{15} and torque T_{12} (see Fig.3) along with those of two springs) equal to zero.

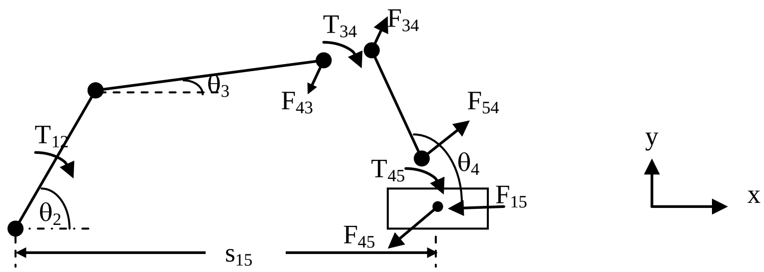


Fig.3 Force analysis of mechanism

If a_2, a_3 and a_4 are the link lengths, the resolution of position vector along x and y direction results in

$$a_2\cos\Theta_2 + a_3\cos\Theta_3 - a_4\cos\Theta_4 = s_{15} \tag{1}$$

$$a_2\sin\Theta_2 + a_3\sin\Theta_3 - a_4\sin\Theta_4 = 0 \tag{2}$$

Also, by applying principle of virtual work, we get:

$$\partial W = \partial W_1 + \partial W_2 + \partial W_3 + \partial W_4 \tag{3}$$

Equating it to zero, we get:

$$-T_{12}\,\partial\Theta_2 - F_{15}\,\partial s_{15} + T_{34}(\partial\Theta_4 - \partial\Theta_3) + T_{45}\partial\Theta_4 = 0 \tag{4}$$

Here, T_{34} and T_{45} are torques at the springs given by:

$$T_{34} = k_{34}\,(\Theta_3 - \Theta_4 + C_{34}) \text{ and } T_{45} = k_{45}\,(-\Theta_4 + C_{45}) \tag{5}$$

k_{ij} and C_{ij} represent the spring stiffness spring initial position constant between ith and jth link respectively. Differentiating Eqs.(1) and (2) and simplifying the expressions for $\partial\Theta_3$ and $\partial\Theta_4$as:

$$\partial\Theta_3 = \frac{a_2 \sin(\theta 2 - \Theta 4)\,\partial\theta 2 + \cos(\Theta 4)\,\partial s15}{a_3 \sin(\Theta 4 - \Theta 3)} \tag{6}$$

$$\partial\Theta_4 - = \frac{a_2 \sin(\theta 2 - \Theta 3)\,\partial\theta 2 + \cos(\Theta 3)\,\partial s15}{a_4 \sin(\Theta 4 - \Theta 3)} \tag{7}$$

Substituting Eqs.(6) and (7) into Eq.(4) we get:

$$-T_{12}\,\partial\Theta_2 - F_{15}\,\partial s_{15} + T_{34}\left(\frac{a_2 a_3 \sin(\theta_2 - \Theta_3)\,\partial\theta_2 + a_3\cos\Theta_3\,\partial s_{15}}{a_3 a_4 \sin\ (\Theta_4 - \Theta_3)} - \frac{a_2 a_4 \sin(\theta_2 - \Theta_4)\,\partial\theta_2 + a_4\cos\Theta_3\,\partial s_{15}}{a_3 a_4 \sin\ (\Theta_4 - \Theta_3)}\right)$$

$$+T_{45}\left(\frac{a2 \sin(\theta 2 - \Theta 3)\,\partial\theta 2 + \cos(\Theta 3)\,\partial s15}{a_4 \sin(\Theta 4 - \Theta 3)}\right) = 0 \tag{8}$$

On simplification, it takes the form:

$$\delta W = Q_1\delta\theta_2 + Q_2\delta s_{15} \tag{9}$$

According to the principle of virtual work, a necessary and sufficient condition for the generalized equilibrium is that each of the generalized forces Q_i must vanish. That is: Q_1=0 and Q_2=0. Hence, the two equilibrium equations can be obtained as:

$$T_{34}\left(\frac{a_2a_3\sin(\theta 2-\theta 3)-a_2a_4\sin(\theta 2-\theta 4)}{a_3a_4\sin(\Theta 4-\Theta 3)}\right)+T_{45}\left(\frac{a_2\sin(\theta 2-\theta 3)}{a_4\sin(\Theta 4-\Theta 3)}\right)=T_{12} \tag{10}$$

$$-F_{15}+T_{34}\,\frac{a_3\cos(\theta 3)-a4\cos(\Theta 4)}{a_3a_4\sin(\Theta 4-\Theta 3)}+T_{45}\left(\frac{\cos(\Theta 3)}{a_4\sin(\Theta 4-\Theta 3)}\right)=0 \tag{11}$$

If two of the four position variables (θ_2, θ_3, θ_4 and s_{15}) are known (two position inputs for a two-degree of freedom mechanism yields a constrained mechanism), F_{15} and T_{12} can be determined in closed-form. However, this is not the common case in practice since mechanisms are generally analyzed for a given output-loading condition. In present case, the known variation of output force F_{15} at each crank angle θ_2 is considered and the parameters θ_3, θ_4, s_{15} and T_{12} are predicted. Now equations (1),(2),(10) and(11) are to be solved simultaneously by numerical approach namely, Newton Raphson's method. The set of functions defined from each of these four equations are:

$$f=a_2\cos\Theta_2+a_3\cos\Theta_3-a_4\cos\Theta_4-s_{15} \tag{12}$$

$$g=a_2\sin\Theta_2+a_3\sin\Theta_3-a_4\sin\Theta_4 \tag{13}$$

$$m=T_{34}\left(\frac{a_2a3\sin(\theta 2-\theta 3)-a_2a_4\sin(\theta 2-\theta 4)}{a_3a_4\sin(\Theta 4-\Theta 3)}\right)+T_{45}\left(\frac{a_2\sin(\theta 2-\theta 3)}{a_4\sin(\Theta 4-\Theta 3)}\right)-T_{12} \tag{14}$$

$$n=T_{34}\,\frac{a_3\cos(\theta 3)-a_4\cos(\Theta 4)}{a_3aa_4\sin(\Theta 4-\Theta 3)}+T_{45}\left(\frac{\cos(\Theta 3)}{a_4\sin(\Theta 4-\Theta 3)}\right)-F_{15} \tag{15}$$

As f= 0,

$$f_0+\frac{\partial f}{\partial\Theta_3}\Delta\Theta_3+\frac{\partial f}{\partial\Theta_4}\Delta\Theta_4+\frac{\partial f}{\partial s_{15}}\Delta s_{15}+\frac{\partial f}{\partial T_{12}}\Delta T_{12}=0 \tag{16}$$

g=0,

$$g_0+\frac{\partial g}{\partial\Theta_3}\Delta\Theta_3+\frac{\partial f}{\partial\Theta_4}\Delta\Theta_4+\frac{\partial g}{\partial s_{15}}\Delta s_{15}+\frac{\partial g}{\partial T_{12}}\Delta T_{12}=0 \tag{17}$$

m=0,

$$m_0+\frac{\partial m}{\partial\Theta_3}\Delta\Theta_3+\frac{\partial m}{\partial\Theta_4}\Delta\Theta_4+\frac{\partial m}{\partial s_{15}}\Delta s_{15}+\frac{\partial m}{\partial T_{12}}\Delta T_{12}=0 \tag{18}$$

n=0,

$$n_0+\frac{\partial n}{\partial\Theta_3}\Delta\Theta_3+\frac{\partial n}{\partial\Theta_4}\Delta\Theta_4+\frac{\partial n}{\partial s_{15}}\Delta s_{15}+\frac{\partial n}{\partial T_{12}}\Delta T_{12}=0 \tag{19}$$

These can be written in vectorial notation as:

$$\{B\}+[A]\{\Delta X\}=0 \quad \text{(or)} \quad \Delta X=-A^{-1}B$$

The incremental solution is obtained using $\{X\}=\{X_0\}+\{\Delta X\}$, where $\{X_0\}$is initial guess vector. Now matrix [A] contains the values of first order differential for f, g, m and n at each $\{X_i\}$. With the help of a computer program written in C language, the inverse of matrix [A] is calculated in every cycle. Values for F_{15} for various values of Θ_2 are taken in a file format using an output load function.

3. Results and Discussion

Kinematic analysis of the linkage for clockwise rotation of the crank with the following data is considered: The link proportions $a_2=a_4=1$ unit and $a_3=3$ units. The spring variables are $k_{34}=k_{45}=k=100$ N/rad and $c_{34}=c_{45}=2.618$ rad. The maximum value of the output load (F=200 N) during the work stroke ($180°<\theta_2<360°$) is assumed to be five times that of the return stroke's maximum value as shown in Fig.4. The direction of load is resistive to the motion of the output-link. The unknown parameters θ_3, θ_4, s_{15}, and T_{12} are obtained with one degree increment. The same set of initial guess for the zero degree of crank angle may not be appropriate for another crank angle. Therefore, in order to obtain a proper solution quickly, the code used for the solution is modified so that initial guesses are obtained from the previous cycle's set of solutions.

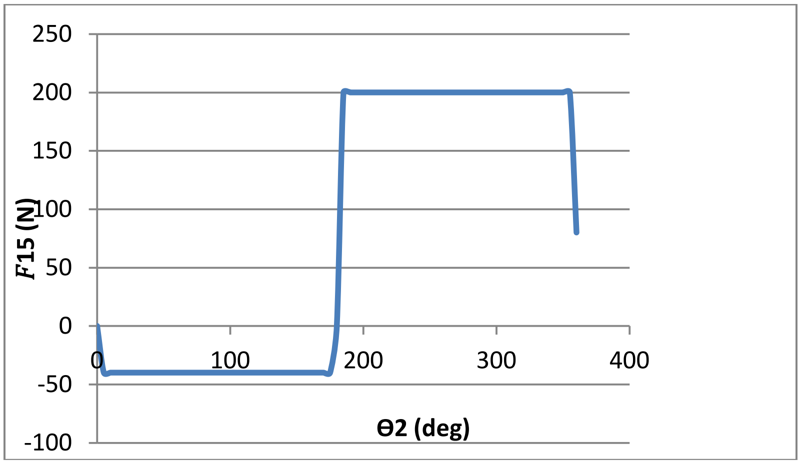


Fig. 4 Output load function selected

Fig. 5 shows variation of θ_3, θ_4 and T_{12} as a function of crank angle $\theta_{2.}$

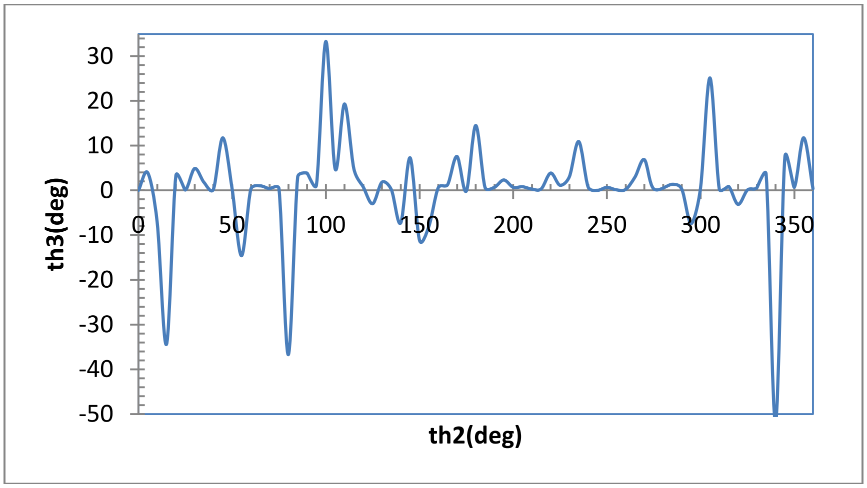


Fig.5 Variation of θ_3

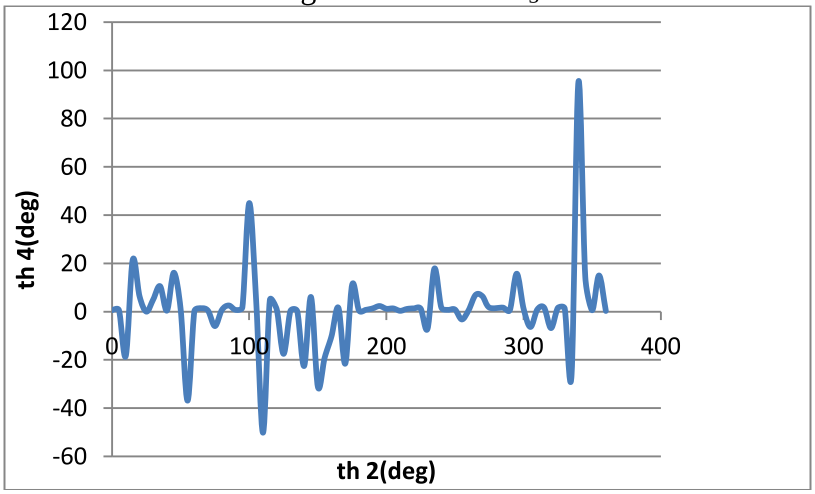


Fig. 6 Variation of θ_4

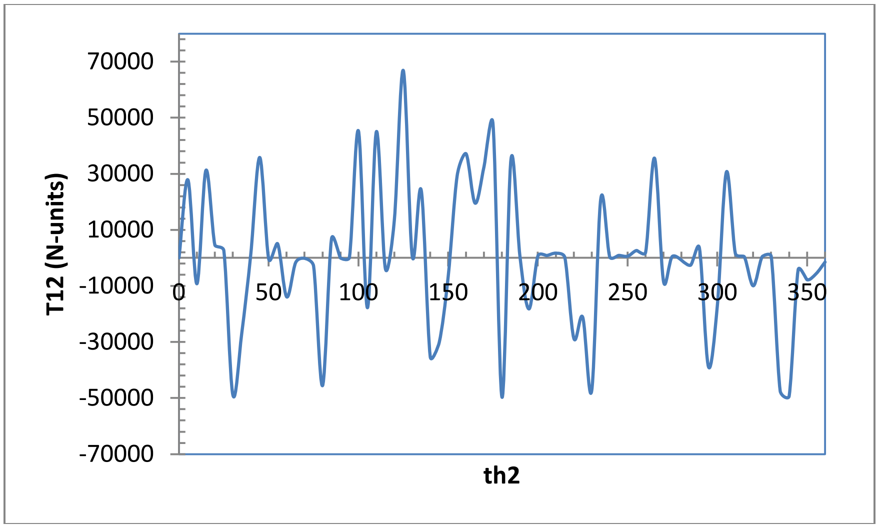


Fig. 7 Variation of T_{12}

It is seen that even the output load function is smooth the evaluated parameters are varying drastically in each interval.

4. Conclusions

The analysis of a partially compliant underactuated five bar slider-crank linkage was presented in this work. An illustration was shown to obtain the unknown variables using a numerical approach for assumed output load function. This mechanism can be used as an end effector that can grasp different sized objects with constant forces. The real time prototype of the linkage is to be prepared to know its workability.

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