Orissa Journal of Physics
ISSN 0974-8202

Vol. 20, No.1
February 2013
pp. 99-111

Vortex Shedding in the Wake Region of a Cylinder

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Received : 1.12.2012 ; Accepted : 19.01.2013

Abstract : During flow past a blunt body, such as a circular cylinder, usually experiences boundary layer separation and very strong flow oscillations in the wake region behind the body. In certain Reynolds number range, a periodic flow motion will develop in the wake as a result of boundary layer vortices being shed alternatively from either side of the cylinder. It creates an oscillating flow at a discrete frequency that is correlated to the Reynolds number of the flow. The objective of this project work is to plot the relations between frequency of vortex shedding and Reynold’s number as well as Strouhal’s number and Reynold’s number. A 2-D model is constructed using GAMBIT and transported to FLUENT, where it is simulated in unsteady state. An animation movie is made by capturing glimpses of the flow field at periodic intervals. Later, this movie is played to manually determine frequency of vortex shedding.

Keywords: Vortex, Strouhal, Reynold, Cylinder

1. Introduction

Vortex shedding is an unsteady flow occurring in special flow velocities (depending the size and shape of the cylindrical body). In this flow, vortices are created at the back of the body and detach periodically from either side of the body. This regular pattern of vortices in the wake is called a Karman vortex street [1] and is caused when a fluid flows past a blunt object. The fluid flow past the object creates alternating low-pressure vortices on the downstream side of the object. The object tends to move toward the low-pressure zone.

In addition to the presence of a strong viscous force, the fluid particles also have to move against the increasing pressure force. Therefore, the fluid particles could be arrested or reversed, causing the neighboring particles to move away from the surface. This phenomenon is called the boundary layer separation [2]. The boundary layer separating from the surface forms a free shear layer and is highly unstable. This shear layer will eventually roll into a discrete vortex and detach from the surface (a phenomenon called vortex shedding). Another type of flow instability emerges as the shear layer vortices being shed from both the top and bottom surfaces interact with one another. They shed alternatively from the cylinder and generate a regular vortex pattern.

Williamson [3] gave a very comprehensive review on vortex dynamics in the wake region of a circular cylinder. In his paper, he discussed three-dimensional vortex patterns in the laminar shedding regime (Re = 49 to 140–194 [3]), and three-dimensional vortex dynamics in the transition regime (Re ≈ 190 to 260 [3]). Norberg [4], on the other hand, looked into the influence of aspect ratio on flow past a circular cylinder.

The formation of the Kármán vortex street [1] for two-dimensional symmetric bluff bodies has been discussed in several works ([5]; [6]). Opposite sign circulation, generated at the flow separation points on the opposite faces of the obstacle, leads to equal intensity, but opposing sign shear layers in the wake. The coupled instabilities of these shear layers bring out the formation of two staggered rows of regularly spaced, counter-rotating vortices of equal intensity moving at the same convective speed. Existing models ([7]; [6]) rely heavily on this intrinsic symmetry for predicting the shedding frequency, the induced drag and relating base pressure to the strength of the shed vortices.

It is seen that the structure of the Kármán vortex street is characterized by the convective speed of the vortices, Uc, their circulation, γ, and the streamwise-to-transverse vortex spacing ratio, b/a. Von Kármán [1] showed that for two parallel rows of vortices of equal but opposing circulation a stable arrangement of vortices exists only if b/a=0.281, which implies a universal Strouhal number as the streamwise spacing is the product of the convective speed and shedding frequency. Several studies ([8];[6];[9]; [10]) have asserted that b/a is obstacle-geometry dependent. However, Roshko [7] showed that it was possible to define a wake Strouhal number, that was constant for several obstacle geometries. Instead of using U_c, the velocity scale could be taken as that of the free streamline at the separation points, U_s, and the length scale as the distance between the free streamlines. Using this approach, U_s could be related directly to the base pressure and the average circulation generated at the separation point and thus a direct estimates of b/a becomes redundant.

Roshko's universal wake Strouhal number has been shown to be sensitive to the free stream turbulence level and its utility is regarded doubtful ([5]; [7]; [11]). Bearman [6] proposed to use Us and the transverse vortex spacing, b, as scales. This approach also produces a distinct relationship between Us and the product of the drag and shedding frequency. Lee [11] showed that both this new Strouhal number and the drag–velocity relationships are satisfied for a larger range of turbulence levels and obstacle geometries.

It is well acknowledged that vortex shedding occurs when the Reynolds number, Re, based on the cylinder diameter (D) and the free-stream velocity (U_0), is above 40. For Reynolds numbers up to about 150, the vortex shedding flow stays laminar [12]. Transition to three-dimensional flow begins at a Reynolds number of about 180–194 depending on experimental conditions, and ends at about Re=260, at which fine scale three-dimensional eddies appear [3]. The flow is classified in the sub-critical regime for Reynolds numbers from 300 to 1.4×105 [13], where, the boundary layer along the surface of the cylinder is laminar throughout the circumference till separation. The vortex shedding is regular, and the Strouhal number, St, that indicates the non-dimensional vortex shedding frequency in terms of D and U0, remains constant at approximately 0.2. Taniguchi and Miyakoshi [14] assumed that Karman vortex streets were formed by concentrations of vorticity due to the rolling-up of separated shear layers that issued from both sides of the cylinder. It is normally understood that three-dimensionality develop for flow around an isolated circular cylinder for Re>260 [3],

The frequency at which vortex shedding takes place for a cylinder is related to the Strouhal number by the following equation:

$$St=(f*D)/V$$ [15]

where St is the Strouhal number, f is the vortex shedding frequency, D is the diameter of the cylinder, and V is the flow velocity. The Strouhal number depends on the body shape and Reynold's number.

The velocity U of the oncoming flow is assumed to increase linearly with height z and the level of shear dU/dz is denoted by the non-dimensional shear-rate parameter $K \equiv D(dU/dz)/U_c$, where U_c is the velocity at the level of the cylinder axis and D denotes the diameter of the cylinder. The other non-dimensional parameter which governs a planar shear flow is the Reynolds number $Re \equiv U_cD/\nu$, where ν is the kinematic fluid viscosity. Experiments by Kiya et al. [16] were performed for Reynolds numbers in the range $43 < Re < 1000$. They observed that the Strouhal number $St \equiv fD/U_c$, where f is the shedding frequency, increased with increasing shear rate for $K > 0.1$. For $Re = 100$ the vortex shedding vanished when K exceeded 0.13. Kwon et al. [17] reached the same conclusions on the basis of their experiments.

More recently, however, Sumner and Akosile [18] and Cao et al.[19] and [20] reported that the Strouhal number remained unchanged as the shear rate increased at sub-critical Reynolds numbers. Lei et al. [21] performed two-dimensional simulations with shear rates up to $K = 0.25$ for Reynolds numbers ranging from 80 to 1000. They found that the Strouhal number slightly decreased as the shear-parameter increased from 0 to 0.25 and vortex shedding could still be detected even at $K = 0.25$. Kang [22] also on the basis of his two-dimensional simulations concluded that the Strouhal number remained nearly constant or slightly decreased with increasing shear rates. Contrary to the earlier observations by Kiya et al.[16], he saw that the vortex shedding persisted for K = 0.2 at Re = 100.

2. Experimental Procedure

Grid generation and simulation

- The dimensions used for various parameters of the model are as follows:-
  Length of the upper boundary = 14 units
  Length of the lower boundary = 14 units
  Width of the inlet = 6 units
  Width of the outlet = 6 units
  Diameter of the cylinder = 2 units
- For structured meshing, a square of side 4 units is constructed around the cylinder.

  The cylinder circumference is uniformly divided into 4 parts, such that each part faces one side of the square forming 4 pairs.
- The end points of the split parts are connected to the nearest square corners. All the square sides, circumference parts and connecting lines are edge meshed into equal number of nodes. (consecutive meshing into 100 nodes each)
- Each of the 4 faces enclosed by a square side, a cylinder part and 2 connecting lines are separately face meshed .(Quad mesh of map type and 0.1 spacing)
- On the remaining region, triangular mesh of pave type and 0.7 node spacing is applied.
- The different zones and their respective boundary types are :-
  Upper boundary - Symmetry
  Lower boundary - Symmetry
  Inlet-Velocity inlet
  Outlet - Pressure outlet
  Cylinder wall – Wall
  Interior - Default interior
- The area outside the cylinder is declared to be fluid.
- The grid is then exported to a given directory and imported to FLUENT. The scale is set in cm and the gauge pressure is fixed at 101325 pa.

- The solver settings are as follows:-
  Solver - pressure based Formulation - Implicit
  Unsteady formulation - 1st order upwind
  Porous formulation - Superficial velocity
  Time - Unsteady
  Space - 2-D
  Velocity formulation - Absolute
  Gradient option - Green Gauss cell based

- The fluid material chosen is air with density 1.225 kg/m3 and viscosity 1.7894e-05 kg/m-s. In boundary conditions, the flow velocity at the inlet is specified.
- An animation sequence is set to monitor the velocity contours (stream functions) at every time step by defining the path of the directory where the graphic files will be stored.
- By playing the recorded animation, the time period is observed and hence the frequency of vortex shedding is calculated. ,
- The Re number and St number for a particular flow velocity can be calculated by :-

$$\text{Re num} = \rho v\, D/\mu \quad , \quad \text{St num} = f\, D/v$$

where, ρ = density , v = flow density
D = Diameter of cylinder , f = frequency of vortex shedding
μ = viscosity

3. Results and Discussions

Given below is a picture of the 2-D mesh generated in GAMBIT and the values of vortex shedding frequency and Strouhal's number determined for each of the flow velocities.

Table 1

Velocity in m/s (Re no)	Frequency of shedding	Strouhal's number
0.3136 (400)	66667	0.42517
0.392(500)	7.142857	0.36443
0.4704 (600)	7.6923	0.32705
0.5488 (700)	8.3333	0.30369
0.6272 (800)	9.0909	0.28988

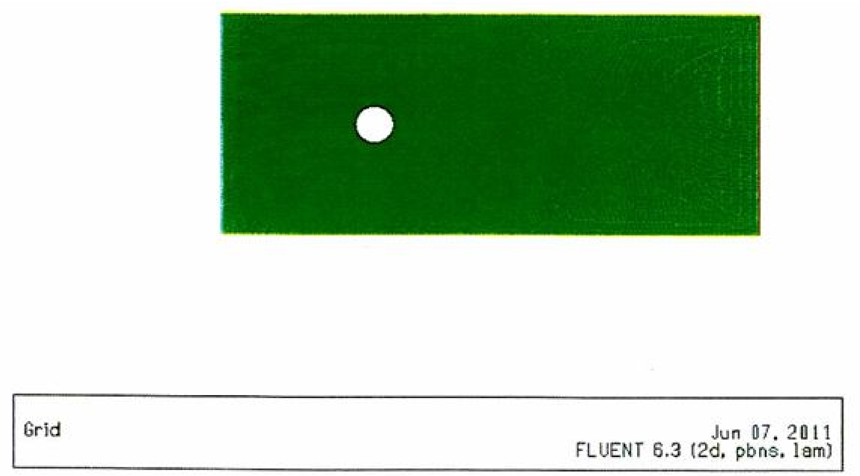

Fig 1(a): Grid generated in GAMBIT

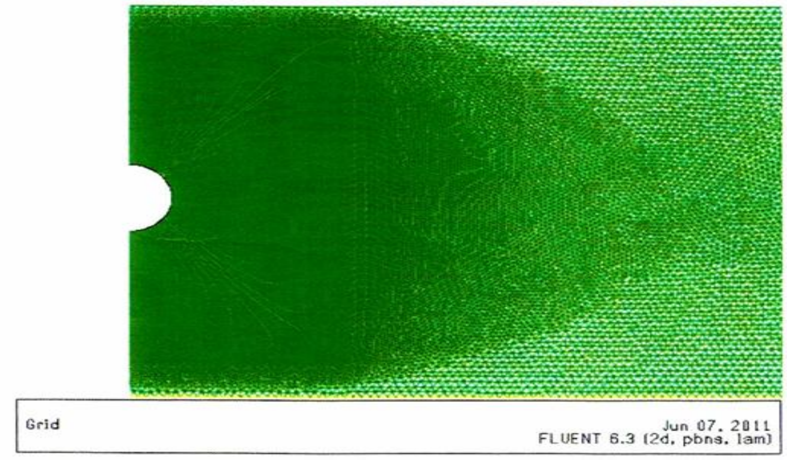

Fig. 2(b): Magnified image of the grid generated

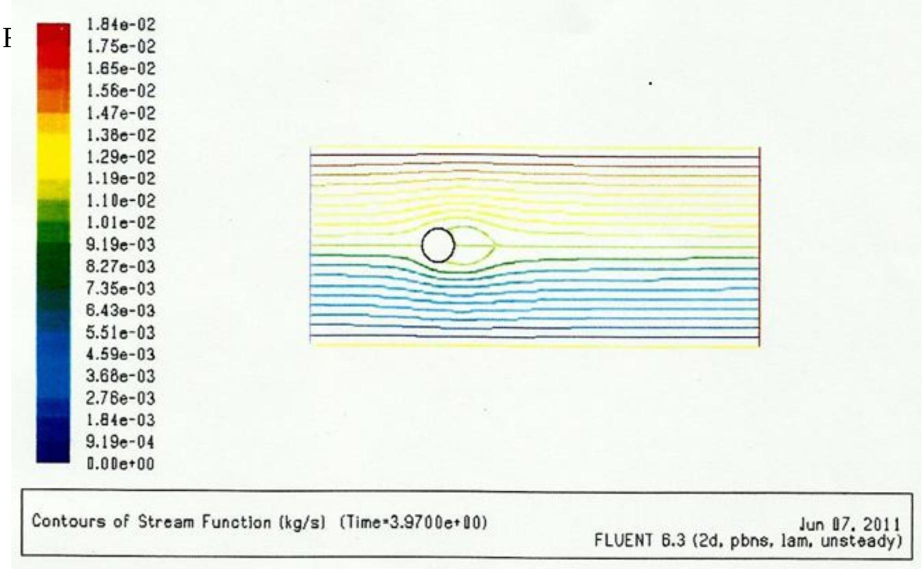

Fig. 2 : STAGE 1

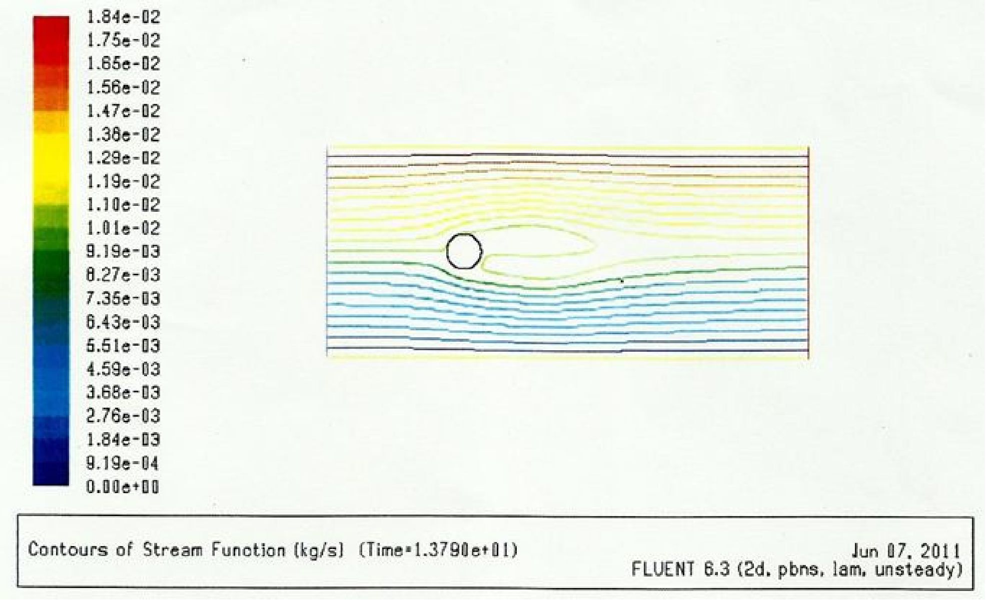

Fig. 3 : STAGE 2

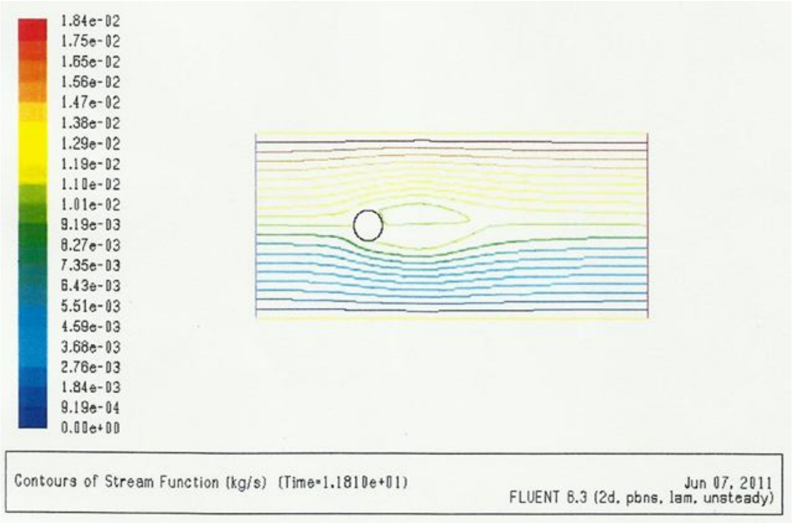

Fig. 4 : Stage 3

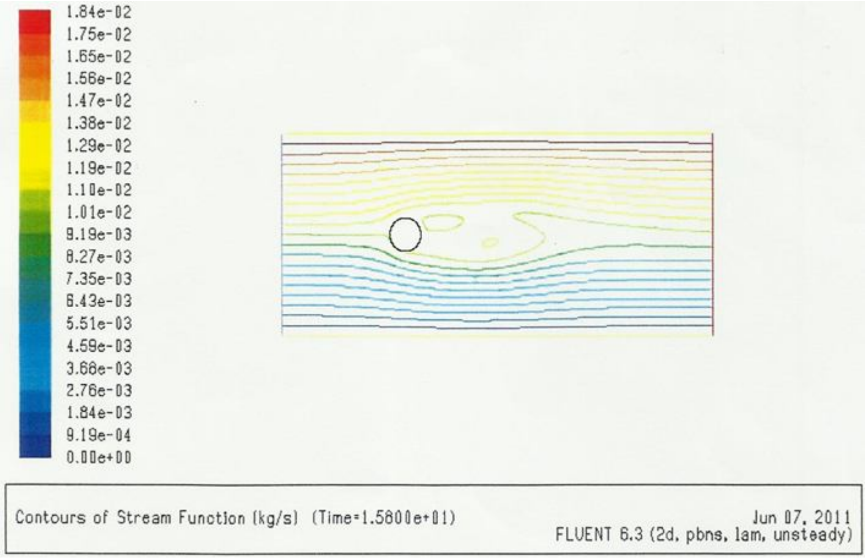

Fig. 5 : Formation of low pressure vortices

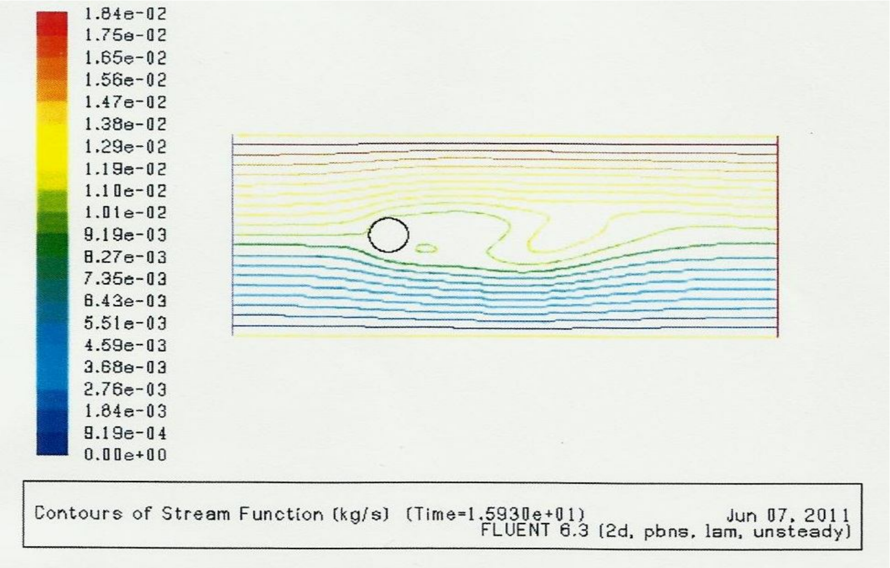

Fig. 6 : Stage 5 (Karman Vortex Street)

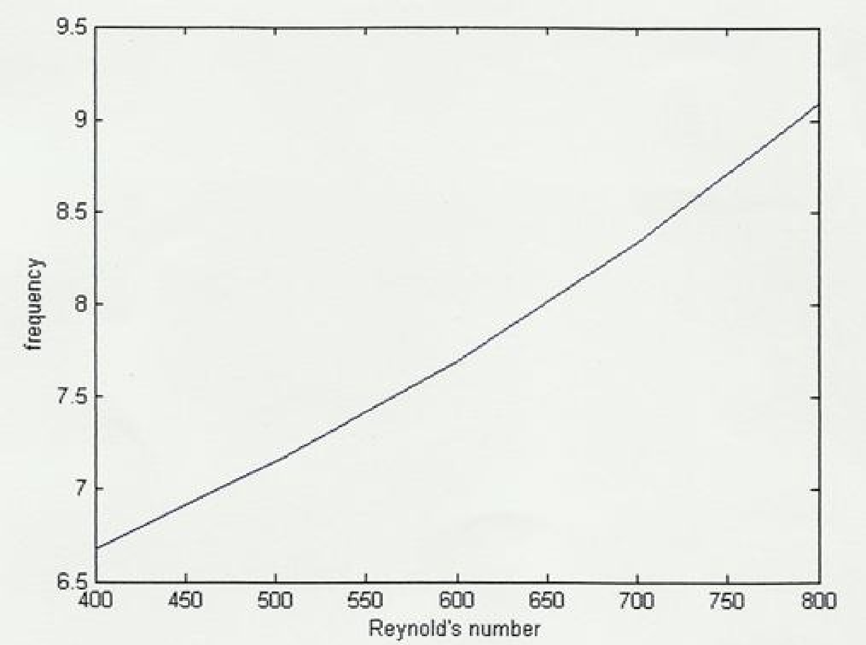

Fig. 7 : Relation between frequency of vortex shedding and Reynold's number of inlet flow (Plotted using MATLAB)

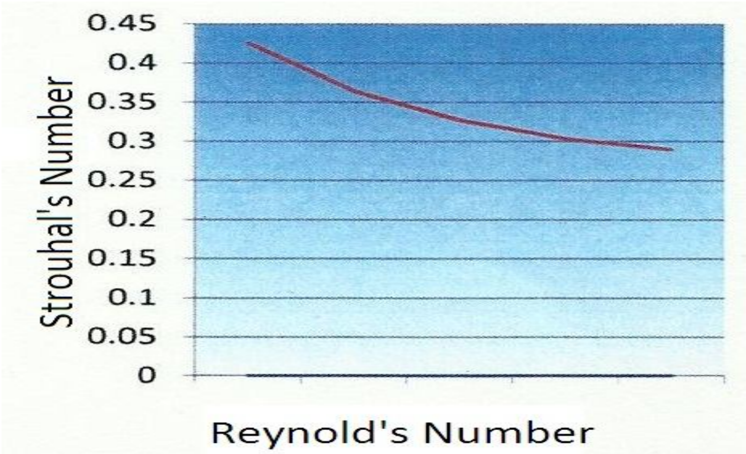

Fig.8: Relation between Strouhal's number and Reynold's number

The figures 2,3,4,5 and 6 depict the different stages of the boundary layer separation and vortex formation process in the wake region of the cylinder. It is clear from the plot of frequency of vortex shedding as a function of Reynold's number, shown in figure 7 that the frequency is increases with increase in inlet flow velocity. It is also evident from figure 8 that the dimensionless Reynold's and Strouhal's numbers are inversely proportional to each other when the geometry and dimensions of the body remain unchanged.

Kovasznay [23] performed some of the earliest quantitative studies on the regular vortex street pattern behind a circular cylinder using a hot-wire technique, that covered Reynolds numbers up to 10^4. He obtained the critical Reynolds number of 40 at which vortices are shed, and the correlation between the Strouhal number and Reynolds number from the critical value of 40 to around 10,000. Roshko [24] also used standard hot-wire techniques to study the wake development behind the circular cylinder at low Reynolds number in a low-speed wind tunnel. Two distinct Reynolds number ranges according to different periodic wake phenomena behind circular cylinders were recorded. The first is the stable Reynolds number range from 40 to 150, in which the classical von Karman vortex street is formed and no turbulence develops. In the second range (above Reynolds number of around 300), the periodic shedding is accompanied by irregular or turbulent velocity fluctuations. The correlations between the Strouhal number and Reynolds number in two Reynolds number ranges were given. Roshko also pointed out that the effect of other geometrical parameters, such as the blockage ratio, on the Strouhal number must be taken into account.

Williamson [25] carried out a study on the transition range between the "stable" and "irregular" regions and confirmed that there exists a complex relationship between the Strouhal number and Reynolds number in the range from 150 to around 300. West and Apelt [26] investigated the effects of tunnel blockage on the mean flow past a circular cylinder. In a range of blockage from 2% to 16%, its effect on the pressure distribution, drag coefficient and Strouhal number was revealed. Okajima [27] conducted experiments on the vortex-shedding frequencies and corresponding Strouhal numbers of various rectangular cylinders in a wind tunnel and in a water tank. The hot-wire anemometer was used for measuring shedding frequencies, and its signal and the corresponding frequency spectra were analyzed. Relationships between Strouhal and Reynolds numbers for different rectangular cylinders were obtained. Al-Asmi and Castro [28] investigated the flow past bluff bodies of different geometries including flat plate, tee-shaped body, as well as triangular and rectangular cross-sections. The variations of Strouhal number with Reynolds number and wind tunnel blockage were obtained. In a relatively recent research, Ferreira and Vieira [29] investigated the flow around a modified circular cylinder having a longitudinal notch using hot-film anemometry. The relationship between Strouhal and Reynolds numbers up to Reynolds number of 1000 was obtained.

Studies of Strouhal number in oscillatory flows are relatively limited, for which there are two main reasons. Firstly, oscillatory flows past bluff bodies typically lead to much more complex vortex shedding phenomena than in steady flows and therefore their phenomenological description is not as straightforward as the classical von Karman vortex street. For instance, Williamson and Roshko [30] observed the vortex formation in the wake of an oscillating cylinder in stationary fluid. Various vortex patterns behind the oscillating cylinder were found, such as the formation of a single vortex, vortex pairs or combination of single vortices and vortex pairs and so on. Similar studies were conducted by Okajima et al. [31] who assayed the morphology of vortex patterns for circular and square cylinders in an oscillatory flow by employing flow visualizations and attributed their appearance to the unsteady structural loading. Chung and Kang [32] and Barbi et al. [33] carried out studies of the vortex shedding and its "lock-on" effects behind a cylinder in the oscillatory incoming flow using a numerical analysis.

4. Conclusion

- The frequency of vortex shedding was found to be proportional to Reynold's number or inlet flow velocity.
- The Strouhal number was found to decrease with increase in Reynold's number.

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