Convective flow over a cone in the presence of thermal and solutal dispersion

CH. RAMREDDY

Department of Mathematics, National Institute of Technology, Rourkela-769008, Odisha.

Outline

1. Basic concepts
2. Mathematical formulation of the proposed problem
3. Results and Discussion

Abstract: The effects of thermal and solutal dispersion on natural convection about an isothermal vertical cone with fixed apex half angle, pointing downwards in a Newtonian fluid are analyzed. The non-linear governing equations and their associated boundary conditions are initially cast into dimensionless forms by local-similarity variables. The resulting system of equations is then solved numerically. A parametric study of the physical parameters involved in the problem is conducted, the results of this parametric study are shown graphically and the physical aspects of the problem are highlighted and discussed.

Mathematical Formulation

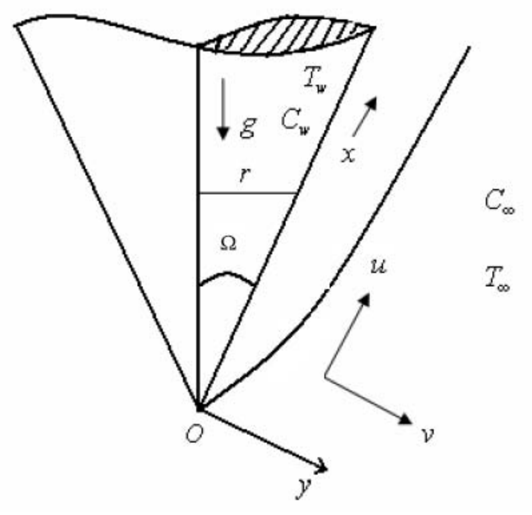


Figure: *Physical model and coordinate system*

Governing equations with associated B.C.

Under the above assumptions, the governing viscous fluid equations can be written as:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g(\beta_T(T - T_\infty) + \beta_C(C - C_\infty))\cos\Omega \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\alpha_e\frac{\partial T}{\partial y}\right) \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = \frac{\partial}{\partial y}\left(D_e\frac{\partial C}{\partial y}\right) \tag{4}$$

The boundary conditions are

$$u = 0, \quad v = 0, \quad T = T_w(x), \quad C = C_w(x) \quad at \quad y = 0 \tag{5a}$$

$$u \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad as \quad y \to \infty, \tag{5b}$$

Non-Dimensional Quantities

The non-dimensional quantities x, y, r, u, v, T and C are related to their dimensional counterparts $X, Y, R, U, V, \overline{T}$ and $\overline{C}$ by

$$X = \frac{x}{L}, Y = \frac{y}{L} Gr^{1/4}, R = \frac{r}{L}, U = \frac{u}{U_0}, V = \frac{v}{U_0} Gr^{1/4}, \tag{6a}$$

$$\overline{T} = \frac{T - T_\infty}{T_w - T_\infty}, \overline{C} = \frac{C - C_\infty}{C_w - C_\infty}. \tag{6b}$$

The continuity equation (1) is automatically satisfied by defining a stream function ψ such that

$$U = \frac{1}{R}\frac{\partial \psi}{\partial Y}, \quad V = -\frac{1}{R}\frac{\partial \psi}{\partial X} \tag{7}$$

Substituting (7) in (2)-(4) and then using the following transformations

$$\psi(X, Y) = XRf(Y), \ \overline{T}(X, Y) = X\theta(Y), \ \overline{C}(X, Y) = X\phi(Y) \tag{8}$$

$$U_0 = [g\beta_T L(T_w - T_\infty)\cos\Omega]^{1/2}, \quad Gr = \left(\frac{U_0 L}{\nu}\right)^2$$

Reduced Equations with B.C.

We obtain the following nonlinear system of differential equations.

$$f''' + 2ff'' - (f')^2 + \theta + N\phi = 0, \tag{9}$$

$$\frac{1}{Pr}\theta'' + D_s X(f''\theta' + f''\theta'') + 2f\theta' - f'\theta = 0, \tag{10}$$

$$\frac{1}{Sc}\phi'' + D_c X(f''\phi' + f''\phi'') + 2f\phi' - f'\phi = 0. \tag{11}$$

Boundary conditions in terms of f, θ, ϕ become

$$Y = 0: \;\; f = 0, \;\; f' = 0, \;\; \theta = 1, \;\; \phi = 1 \tag{12a}$$

$$Y \to \infty: \;\; f' \to 0, \;\; \theta \to 0, \phi \to 0 \tag{12b}$$

where D_s and D_c are the is the thermal and solutal dispersion parameters.

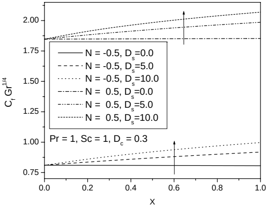
Results and Discussion

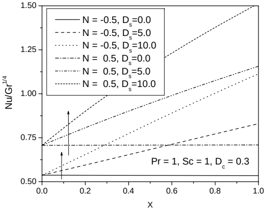
The flow Eq.(9) coupled with the energy and concentration Eqs. (10) and (11) constitute a set of nonlinear non-homogeneous differential equation for which closed-form solution cannot be obtained. Hence the problem has been solved numerically using shooting technique along with fourth order Runge-Kutta integration.

Table: Comparison of results for a laminar free convection flow over a cone [Eco (2005)] with $N = 0$, $D_s = D_c = 0$.

	$f''(0)$		$-\theta'(0)$	
Pr	Eco (2005)	Present results	Eco (2005)	Present results
1.0	0.681482	0.681482	0.638859	0.638859
10.0	0.433269	0.433272	1.275548	1.275554

Cont...Results and Discussions





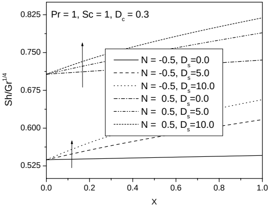


Figure: *(a)Skin friction coeff., (b)Heat transfer rate and (d)Mass transfer rate*

Cont...Results and Discussions

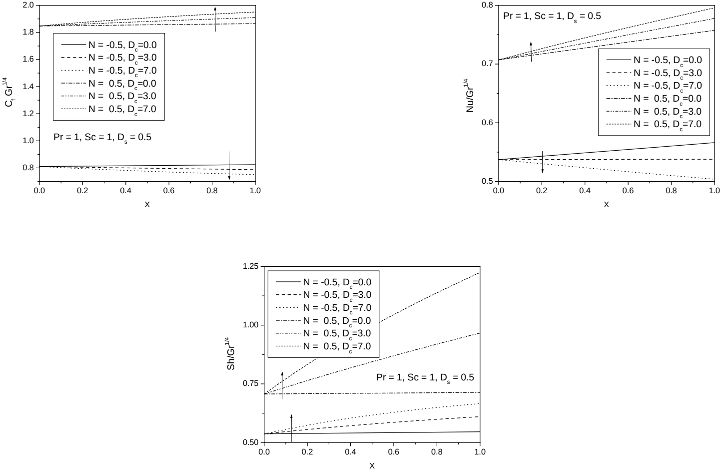


Figure: *(a)Skin friction coeff., (b)Heat transfer rate and (d)Mass transfer rate*

Conclusions

In this paper, the effects thermal and solutal dispersion on natural convection heat and mass transfer flow over a cone has been analyzed. The major conclusion are

- The skin friction, heat and mass transfer rates increase with the thermal dispersion effect in both aiding and opposing buoyancy cases.
- The skin friction and heat transfer rate decrease while mass transfer rate increases with solutal dispersion effect in opposing buoyancy case but skin friction, heat and mass transfer rates increase with solutal dispersion effect in aiding buoyancy case.

Review of Literature

- Alamgir (1989): Investigated the overall heat transfer in laminar natural convection from vertical cones using the integral method.
- Pop *et al.* (1991): Studied the compressibility effects in laminar free convection from a vertical cone.
- Murthy and Singh (2000): Obtained the similarity solution for non-Darcy mixed convection about an isothermal vertical cone with fixed apex half angle, pointing downwards in a fluid saturated porous medium.
- Chamkha (2001): Analyzed the coupled heat and mass transfer by natural convection of Newtonian fluids about a truncated cone in the presence of magnetic field and radiation effects.
- Eco (2005): Performed a similarity analysis to investigate the laminar free-convection boundary-layer flow in the presence of a transverse magnetic field over a vertical down-pointing cone with mixed thermal boundary conditions.
- Srinivasacharya *et al.* (2012): Considered the effects of magnetic field and double dispersion on free convection along a vertical plate embedded in a doubly stratified non-Darcy porous medium saturated with power-law fluid.

THANK YOU