Geometric Nonlinear Vibration Analysis of Heated Laminated Cylindrical Panels

S. K. Panda (call2subrat@gmail.com)

Department of Mechanical Engineering, National Institute of Technology Rourkela, Odisha, India

T. R. Mahapatra

School of Mechanical Engineering, KIIT University, Bhubaneswar, India

ABSTRACT: Geometrically nonlinear free vibration behaviour of laminated composite cylindrical panel in thermal environment is analysed. The mid-plane kinematics is based on the higher order shear deformation theory and the geometric nonlinearity is introduced in the strain displacement relation using Green-Lagrange type nonlinearity. The sets of governing equations are obtained using Hamilton's principle and discretised using a nonlinear finite element approach. The results obtained using the proposed model is compared with those available in open literature.

1 INTRODUCTION

Laminated composites are important not only their load carrying capacity is higher but they have higher strength/stiffness to weight ratio and are also capable to carry certain amount of thermal load. In general, when the structures are exposed to thermal environment, the original geometry of the structure gets distorted and this has an adverse effect on its behaviour. Many researchers have been reported in literature on nonlinear vibration of laminated plates/panels in thermal environment. The mathematical models are developed based on various theories such as classical laminated plate theory (CLPT), first order shear deformation theory (FSDT) and higher order shear deformation theory (HSDT) considering the nonlinearity in von-Karman nonlinearity. Liu & Huang (1996) studied the nonlinear free vibration behavior of laminated plates. The mathematical model is developed based on the FSDT and the nonlinearity is considered in von-Karman sense. Lal & Singh (2009) studied the stochastic vibration behavior of laminated plates resting elastic foundation in thermal environment. The model is developed based on the HSDT and the nonlinearity is taken in von-Karman sense. Sairam & Sinha (1992) reported the effect of temperature and moisture on the vibration behavior of laminated plates. Ohnabe (1995) investigated heated orthotropic sandwich plates and shallow shells due to the temperature difference between the upper and lower face with von-Karman nonlinearity. Nanda & Pradyumna (2011) reported the nonlinear free and transient responses of the cylindrical/spherical panels with imperfection under hygrothermal loading. The model is developed based on the FSDT and the von-Karman type nonlinearity. Naidu & Sinha (2007) studied the nonlinear free vibration behavior of laminated composite shells in hygrothermal environment. The model is developed based on the FSDT and the Green-Lagrange type nonlinearity. Bhimraddi & Chandrasekhar (1993) reported buckling, postbuckling and nonlinear vibration of heated angle-ply laminated plates based on the parabolic shear deformation theory and von-Karman type nonlinearity. Panda & Singh (2009) reported nonlinear free vibration response of laminated composite shells. The authors are the first to develop a novel model based on the HSDT and Green-Lagrange type nonlinearity. In their study they have considered all the nonlinear higher order terms to predict the original flexure of the structure. A mathematical model for cylindrical panel based on the higher order shear deformation theory is used in their approach. The geometric nonlinearity of Green-Lagrange type is introduced through the strain displacement relation of the panel and all the nonlinear higher order terms are included in the formulation. A uniform temperature load is considered through the thickness. It is assumed that the material properties are invariant with temperature.

2 MATHEMATICAL MODELLING

2.1 *Geometry and midplane kinematics*

A cylindrical shell panel is assumed, which is composed of *N* number of anisotropic layers of uniform thickness (Figure 1). The panel has length *a*, width *b* and thickness *h*. The orthogonal curvilinear coordinate system is assumed for this study such that $\xi_1=R$ is the curvature with mid surface, $\xi_2=\infty$ and $\zeta = 0$ and ζ -curves are straight line perpendicular to the surface $\zeta = 0$ for the curved panels the lines of curvature coincide with the coordinate lines (Reddy 1997). The ζ_k and ζ_{k-1} be the top and bottom ζ-coordinates of the k^{th} lamina. The displacement field for laminated panel considered to derive the mathematical model is based on the HSDT (Panda & Singh, 2009).

$$\left.\begin{aligned}\bar{u}(\xi_1,\xi_2,\zeta,t)&=u(\xi_1,\xi_2,t)+\zeta\phi_1(\xi_1,\xi_2,t)+\zeta^2\psi_1(\xi_1,\xi_2,t)+\zeta^3\theta_1(\xi_1,\xi_2,t)\\ \bar{v}(\xi_1,\xi_2,\zeta,t)&=v(\xi_1,\xi_2,t)+\zeta\phi_2(\xi_1,\xi_2,t)+\zeta^2\psi_2(\xi_1,\xi_2,t)+\zeta^3\theta_2(\xi_1,\xi_2,t)\\ \bar{w}(\xi_1,\xi_2,t)&=w(\xi_1,\xi_2,t)\end{aligned}\right\} \quad (1)$$

where, $(\bar{u},\bar{v},\bar{w})$ are the displacements along the (ξ_1,ξ_2,ζ) coordinates at any point of the panel, (u,v,w) are the displacements of a points on the mid-plane of the panel and ϕ_1 and ϕ_2 are the rotations at $\zeta=0$ of normal to the mid-plane with respect to the ξ_2 and ξ_1 -axes, respectively. ψ_1, ψ_2, θ_1 and θ_2 are higher order terms of Taylor series expansion defined at the mid-plane. The assumed displacement field in the above equation represents the transverse shear strains as quadratic function of thickness coordinate, ζ and satisfies the requirement of zero transverse normal strain.

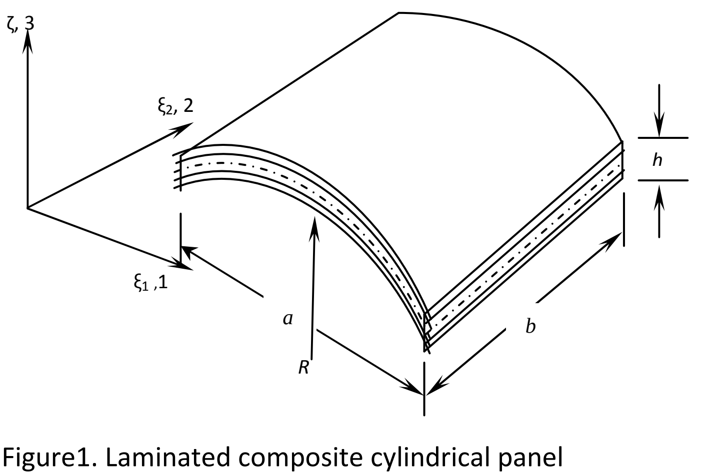

Figure1. Laminated composite cylindrical panel

2.2 *Strain displacement relation*

The nonlinear strain displacement relation for general laminated curved panel is same as Panda & Singh (2009).

$$\{\varepsilon\}=\{\varepsilon_L\}+\{\varepsilon_{NL}\} \quad (2)$$

where, $\{\varepsilon_L\}$ and $\{\varepsilon_{NL}\}$ are the linear and the nonlinear strain vectors, respectively. Individual strain terms are obtained by substituting Eq. (1) into Eq. (2) and expressed as:

$$\{\varepsilon\}=[\mathrm{H}]_L\{\bar{\varepsilon}_L\}+\frac{1}{2}[\mathrm{H}]_{NL}\{\bar{\varepsilon}_{NL}\} \quad (3)$$

where, $\{\bar{\varepsilon}_L\}=\begin{Bmatrix}\varepsilon_1^0 & \varepsilon_2^0 & \varepsilon_6^0 & \varepsilon_5^0 & \varepsilon_4^0 & k_1^1 & k_2^1 & k_6^1 & k_5^1 & k_4^1\\ k_1^2 & k_2^2 & k_6^2 & k_5^2 & k_4^2 & k_1^3 & k_2^3 & k_6^3 & k_5^3 & k_4^3\end{Bmatrix}^T$ and

$$\{\bar{\varepsilon}_{NL}\}=\frac{1}{2}\begin{Bmatrix}\varepsilon_1^4 & \varepsilon_2^4 & \varepsilon_6^4 & \varepsilon_5^4 & \varepsilon_4^4 & k_1^5 & k_2^5 & k_6^5 & k_5^5 & k_4^5 & k_1^6 & k_2^6 & k_6^6 & k_5^6 & k_4^6 & k_1^7 & k_2^7\\ k_6^7 & k_5^7 & k_4^7 & k_1^8 & k_2^8 & k_6^8 & k_5^8 & k_4^8 & k_1^9 & k_2^9 & k_6^9 & k_5^9 & k_4^9 & k_1^{10} & k_2^{10} & k_6^{10} & \end{Bmatrix}^T$$

The terms contained in $\{\bar{\varepsilon}_L\}$ and $\{\bar{\varepsilon}_{NL}\}$ having superscripts 0, 1, 2, 3, and 4, 5, 6, 7, 8, 9, 10 are the bending, curvature and higher order terms (Panda & Singh, 2009). $[\mathrm{H}]$ is the function of thickness coordinate.

2.3 *Stress-strain relation*

Thermo-elastic constitutive relation under temperature field for any general k^{th} orthotropic composite lamina having an arbitrary fibre orientation angle with reference to the coordinate axes (1, 2 & 3).

$$\{\sigma\}^k=\left[\bar{Q}\right]^k\{\varepsilon-\alpha\Delta T\}^k \quad (4)$$

where, $\{\sigma\}^k=\{\sigma_1 \quad \sigma_2 \quad \sigma_6 \quad \sigma_5 \quad \sigma_4\}^T$ is the total stress vector measured at the stress free state, $\{\varepsilon\}^k=\{\varepsilon_1 \quad \varepsilon_2 \quad \varepsilon_6 \quad \varepsilon_5 \quad \varepsilon_4\}^T$ is the strain vector, $\left[\bar{Q}\right]^k$ are the transferred reduced stiffness matrix and $\{\alpha\}_m^k=\{\alpha_{1m} \quad \alpha_{2m} \quad \alpha_{12m}\}^T$ the transformed thermal expansion coefficient vector for the k^{th} layer respectively. ΔT is the uniform temperature difference.

Thermal in-plane forces can be obtained by integrating the equation (5) over the thickness of the shell panel and can be expressed in matrix form as follows

$$\begin{Bmatrix} \{N_{\Delta t}\} \\ \{M_{\Delta t}\} \\ \{P_{\Delta t}\} \end{Bmatrix} = \left[\sum_{k=1}^{N} \int_{\zeta_{k-1}}^{\zeta_k} [\bar{Q}]_m^k \{\alpha\}^k \left(1, \zeta, \zeta^3\right) \Delta T d\zeta \right] \tag{5}$$

where, $\{N_{\Delta t}\}$, $\{M_{\Delta t}\}$ and $\{P_{\Delta t}\}$, are the resultant vectors of compressive in-plane forces, moments and the higher order terms due to the temperature difference (ΔT) in composite matrix.

2.4 *Strain energy of the laminate*

The strain energy of the heated curved panel can be written as:

$$U = \frac{1}{2} \int_v \{\varepsilon\}^T \{\sigma\} dV \tag{6}$$

The energy will be obtained by substituting the values of stresses and strains from Eq. (3) and (4) in Eq. (6).

2.5 *Kinetic energy of the laminate*

The kinetic energy expression of a vibrated panel can be written as

$$T = \frac{1}{2} \int_V \rho \left\{\dot{\bar{\delta}}\right\}^{\mathrm{T}} \left\{\dot{\bar{\delta}}\right\} dV \tag{7}$$

where, ρ, $\{\bar{\delta}\}$ and $\left\{\dot{\bar{\delta}}\right\}$ are the density, displacement vector and the first order differential of displacement with respect to time, respectively.

The global displacement vector can be expressed as

$$\{\bar{\delta}\} = \{\bar{u} \quad \bar{v} \quad \bar{w}\}^T = [f]\{\delta\} \tag{8}$$

Where, $[f]$ is the function of thickness coordinate. The kinetic energy for 'N' number of orthotropic layered composite shell panel will be

$$\begin{aligned} T &= \frac{1}{2} \int_A \left(\sum_{k=1}^{N} \int_{\zeta_{k-1}}^{\zeta_k} \{\dot{\delta}\}^{\mathrm{T}} [f]^{\mathrm{T}} \rho^k [f] \{\dot{\delta}\} d\zeta \right) dA \\ &= \frac{1}{2} \int_A \{\dot{\delta}\}^{\mathrm{T}} [\mathrm{m}] \{\dot{\delta}\} dA \end{aligned} \tag{9}$$

where, $[\mathrm{m}] = \sum_{k=1}^{N} \int_{\zeta_{k-1}}^{\zeta_k} \left([f]^{\mathrm{T}} \rho^k [f] \right) d\zeta$ is the inertia matrix.

2.6 *Work done due to the temperature field*

The work done (W) due to the in-plane compressive thermal force resultants $\{N_{\Delta t}\}$ in Green-Lagrange sense for the curved panel can be obtained in similar fashion as in Eq. (6)

$$W = \int_V [\bar{\varepsilon}_G] \{N_{\Delta t}\} dV \tag{10}$$

where, $\{\bar{\varepsilon}_G\}$ is the geometric strain vector.

The expression as given in Eq. (10) can be rearranged to the following form as given in Cook et al. (2000)

$$W = \frac{1}{2} \int_A \{\varepsilon_G\}^T [\mathrm{D}_G] \{\varepsilon_G\} dA \tag{11}$$

The values of the geometric strain vector $\{\varepsilon_G\}$, the material property matrix $[\mathrm{D}_G]$ and the evaluation steps can be seen in the Panda & Singh (2010).

3 SOLUTION TECHNIQUE

3.1 *Finite element formulation*

A nine noded isoparametric Lagrangian element with eighty one degrees of freedom (DOFs) per element is employed for the present analysis. The details of the element can be seen in the reference Cook et al. (2000)

The displacement vector

$\left(\delta^*\right) = \begin{bmatrix} u & v & w & \phi_1 & \phi_2 & \psi_1 & \psi_2 & \theta_1 & \theta_2 \end{bmatrix}^{\mathrm{T}}$ Can be presented to the form by employing FEM

$$\left\{\delta^*\right\} = [N_i] \left\{\delta_i^*\right\} \tag{12}$$

where, $[N_i]$ and $\{\delta_i\}$ is the nodal interpolation function and displacement vector for i^{th} node, respectively.

Substituting Eq. (12) into equations (6) and (11) the strain energy and the work done expressions can be rewritten as

$$U=\frac{1}{2}\int_A\begin{pmatrix}\{\delta^*\}_i^T[\mathrm{B}_L]_i^T[\mathrm{D}_1][\mathrm{B}_L]_i\{\delta^*\}_i\\ +\frac{1}{2}\{\delta^*\}_i^T[\mathrm{B}_L]_i^T[\mathrm{D}_2][\mathrm{A}]_i[\mathrm{G}]_i\{\delta^*\}_i\\ +\frac{1}{2}\{\delta^*\}_i^T[\mathrm{G}]_i^T[\mathrm{A}]_i^T[\mathrm{D}_3][\mathrm{B}_L]_i\{\delta^*\}_i\\ +\frac{1}{4}\{\delta^*\}_i^T[\mathrm{G}]_i^T[\mathrm{A}]_i^T[\mathrm{D}_4][\mathrm{A}]_i[\mathrm{G}]_i\{\delta^*\}_i\end{pmatrix}dA-\{\mathrm{F}_{\Delta t}\}_i \tag{13}$$

where, $\{\varepsilon_L\}_i=\left[\mathrm{B}_L\right]_i\left\{\delta^*\right\}_i$

$\{\varepsilon_{NL}\}_i=\frac{1}{2}\left[\mathrm{B}_{NL}(\delta)\right]_i\left\{\delta^*\right\}_i=\frac{1}{2}\left[\mathrm{A}(\delta)\right]_i[\mathrm{G}]_i\left\{\delta^*\right\}_i$,

and $\{\mathrm{F}_{\Delta t}\}_i=\int_A\left[\mathrm{B}_L\right]_i^T\{N_{\Delta t}\}dA$.

$$W_{\Delta t}=\frac{1}{2}\int_A\left(\{\delta\}_i^T\left[\mathrm{B}_G\right]_i^T\left[\mathrm{D}_G\right]\left[\mathrm{B}_G\right]_i\{\delta\}_i\right)dA \tag{14}$$

where, $\left[\mathrm{B}_L\right]$ and $\left[\mathrm{B}_G\right]$ are the product form of the differential operator and nodal interpolation function in the linear strain terms and geometric strain terms, respectively. $[\mathrm{A}]$ is function of the displacements and $[\mathrm{G}]$ is the product form of differential operator and shape function in the nonlinear strain terms. The expressions of $[\mathrm{A}]$ and $[\mathrm{G}]$ arising due to the Green-Lagrange nonlinearity in the nonlinear stiffness matrices are given in details in the literature Panda and Singh (2009). $\mathrm{F}_{\Delta t}$ is the thermal load vector due to the temperature difference ΔT.

3.2 *Governing equation*

The governing equation for the free vibrated composite panel under thermal loading is obtained by using Hamilton's principle. This result in

$$\delta\int_{t_1}^{t_2}L\mathrm{dt}=0 \tag{15}$$

where, $L=(T+U-W_{\Delta t})$

Using Eqs. (6)- (14), Eq. (15) can be expressed following the procedure as given in Sundermurthy et al. (1973)

$$[\mathrm{M}]\{\ddot{\delta}\}+\left[\mathrm{K_L}\right]+\frac{1}{2}\left[\mathrm{K_{NL}}\right]_1+\frac{1}{3}\left[\mathrm{K_{NL}}\right]_2=\{\mathrm{F}_{\Delta t}\} \tag{16}$$

where, [M] is the global mass matrix, $[\mathrm{K_L}]$ is the global linear stiffness matrix, $[\mathrm{K_{NL}}]_1$ and $[\mathrm{K_{NL}}]_2$ are the nonlinear mixed stiffness matrices which depend on the displacement vector linearly and quadratically, respectively.

In order to obtain an eigenvalue solution of the nonlinear responses the Eq. (16) is modified as follows:

$$\left\{\left([\mathrm{K}]_\mathrm{L}-\lambda_{cr}\left[\mathrm{K_G}\right]+\frac{1}{2}\left[\mathrm{K_{NL}}\right]_1+\frac{1}{3}\left[\mathrm{K_{NL}}\right]_2\right)-\omega^2[\mathrm{M}]\right\}\{\delta\}=0 \tag{17}$$

where, $[\mathrm{K_G}]$ is the global geometry stiffness matrix. It is obtained by dropping the thermal force in the Eq. (16) and the effect due to that is associated with the governing equation in terms of geometry matrix. The above matrix equations (16) have been solved by employing a direct iterative method to obtain the desired output. The steps to obtain the output of for any general case of nonlinear analysis are discussed point wise fashion in the reference (Panda & Singh 2009).

4. RESULTS AND DISCUSSION

In this section the nonlinear free vibration behaviour of laminated composite cylindrical panel is performed for different geometric and material properties. A computer code has been developed in MATLAB 7.0 based on the proposed nonlinear model. In order to probe the efficacy of the present nonlinear model different comparisons have been made with von-Karman nonlinearity based on the FSDT/HSDT midplane kinematics. In this study the material properties of composites are assumed to be invariant with temperature.

For the comparison purpose, the geometry, the material and the stacking sequences are taken as same as the references. The non-dimensional linear frequency $\left(\varpi_L=\omega_L a^2(\rho/E_2)^{1/2}/h\right)$ and the nonlinear frequency parameters are presented in the Table 1. It can be easily viewed from the table that the difference exists within one percent for both linear and nonlinear cases. It can be seen in the difference column that the results shows in between point i.e., it is higher than Lal & Singh (2009) and lower than Liu & Huang (1996). It is because of the fact that, the former one is based on the FSDT with von-Karman nonlinearity; however, the latter one is based on the HSDT with von-Karman nonlinearity. Hence, the present model which is developed on based on the HSDT with Green-Lagrange type nonlinearity is necessary to predict the real responses. Some new results are also generated for different parameters and discussed in detailed.

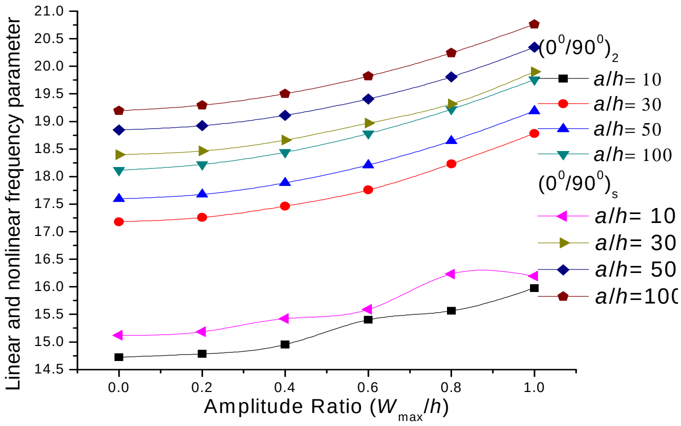

Figure 2. Effect of thickness ratio on nonlinear vibration behavior of a square simply supported cylindrical panel.

Example: 1

A simply supported square symmetric and anti-symmetric cross ply cylindrical panel problem is solved using the material properties E_2 = 1GPa, E_1/E_2 = 40, G_{12}/E_2 = 0.6, G_{23}/E_2 = 0.5, G_{12} = G_{13}, ν_{12} = 0.25, α_0 = $10^{-6}/^0$ C, α_1/α_2 = 0.3 and geometrical property R/a = 100 for a temperature gradient ΔT = 50^0 C. The effect of thickness ratios, lamination schemes and amplitude ratios on the nonlinear free vibration responses are shown in Figure 2. The non-dimensional fundamental linear frequency increases with increase in thickness ratio which is obvious. It can be observed from the figure that the nonlinear frequency parameters are increasing with increase in amplitude ratios and thickness ratios. It is also interesting to observe that both the linear and the nonlinear frequency parameters are showing higher values for symmetric cross-ply lamination in comparison to anti-symmetric lamination.

Example: 2

In this example, effect of aspect ratio on the nonlinear frequency parameter on antisymmetric angle ply cylindrical panel is analysed and presented in Table 2. The material properties are same as in Example 1. The geometrical properties are a/h = 100, R/a = 20 and **all four sides of the panel is considered as simply supported.** The set of results are obtained for two temperature gradients ΔT = 0^0C and 100^0C. Results demonstrate that, the linear and the nonlinear frequency parameters are increasing with increase in the temperature gradient, amplitude ratio and the aspect ratios. The responses are also increasing when the numbers of layers are increasing. It can be depicted from the table that, the non-dimensional linear frequencies are higher for anti symmetric laminates and the nonlinear frequency parameters are higher for symmetric laminates. The responses for a/b = 1.0 shows a mixed type of behavior i.e., the nonlinear frequency parameter increases with increase in W_{max}/h= 0.5 to 1.0 and then decreases at W_{max}/h =1.5 for both the temperature gradient. This is happens due to the present nonlinear model where all the higher order term is considered in the mathematical formulation.

5. CONCLUSIONS

The nonlinear free vibration behaviour of laminated composite cylindrical panel has been studied using the proposed nonlinear model. The geometrical nonlinearity is modeled in Green-Lagrange sense incorporating all the nonlinear higher order terms arising in the mathematical formulation based on the HSDT. The validation study shows the necessity of present nonlinear model for the more accurate analysis of nonlinear responses of the structures for large amplitude free vibration under uniform thermal loading. The nonlinear frequency parameter of simply supported cylindrical panel increases with thickness ratio, amplitude ratio and aspect ratios. The results also show mixed type of behavior for the amplitude ratios and angle ply laminations.

REFERENCES

Liu, C. F. and Huang, C. H. 1996. Free vibration of composite laminated plates subjected to temperature changes, Computers and Structure, Vol. 60, No. 1: 95-101

Lal, A. and Singh, B. N. 2009. Stochastic nonlinear free vibration laminated composite plates resting on elastic foundation in thermal environment, Computational Mechanics, Vol. 40: 15-29

Sairam, K. S. and Sinha, P. K. 1992. Hygrothermal effects on the free vibration of laminated composite plates, Journal of Sound and Vibration, Vol. 158, No. 1: 133-148

Ohnabe, H. 1995. Nonlinear vibration of heated orthotropic sandwich plates and shallow shells, Int. J. of Nonlinear Mechanics, Vol. 30, No.4: 501-508

Nanda, N. and Pradyumna, S. 2011. Nonlinear dynamic response of laminated shells with imperfections in hygrothermal environment, Journal of Composite Materials, Vol. 45, No.20: 2103-2112

Naidu, N.V.S. and Sinha, P. K. 2007. Nonlinear free vibration analysis of composite shells in hygrothermal environment, Composite Structures, Vol. 77: 475-483

Bhimraddi, A. and Chandrasekhara, K. 1993. Nonlinear free vibration of heated antisymmetric angle ply laminates, International Journal of Solids and Structures, Vol. 30, No.9: 1255-1268

Panda, S. K. and Singh, B. N. 2009. Nonlinear free vibration of spherical shell panel using higher order shear deformation theory – a finite element approach, International Journal of Pressure Vessels and Piping, Vol. 86, No.6: 373-383

Panda, S. K. and Singh. B. N. 2010. Thermal post-buckling analysis of laminated composite spherical shell panel embedded with SMA fibres using nonlinear FEM, Proc. IMechE Part C: Journal of Mechanical Engineering and Science, Vol. 222, No.4: 757-769.

Cook, R. D. Malkus, D. S. and Plesha, M. E. (2nd edition). 2000. Concepts and applications of finite element analysis, John Willy and Sons Pte. Ltd. Singapore.

Reddy J. N. (2nd edition). 1997. Mechanics of Laminated Composite Plates and Shells Theory and Analysis.

Sundaramoorthy, R. David, W. and Murray, M. 1973. Incremental finite element matrices, *ASCE*, Journal of Structural Division, Vol. 99, No.ST12: 2423-2438.

Table 1. Comparison study of nondimensionalised linear and nonlinear vibration frequencies

W_{max}/h	Present	Liu & Huang (1996)	Lal & Singh (2009)	% Difference	
				Liu & Huang (1996)	Lal & Singh (2009)
0.1	15.1188	15.160	15.0985	-0.27	0.13
0.2	15.1738	15.195	15.1543	-0.64	0.12
0.3	15.2217	15.272	15.2424	-0.33	-0.14
ϖ_L	15.1003	15.160	15.0794	-0.39	0.14

Table 2. Effect of aspect ratio on the nonlinear vibration ratio on symmetric and antisymmetric angle ply laminates.

Lay up	ΔT^0C	W_{max}/h	Aspect Ratio (a/b)		
			1.0	1.5	2.0
$[\pm45^0]_1$	0	0.5	24.0919	30.8757	39.7122
		1.0	29.7149	36.2996	46.0560
		1.5	26.5391	47.2760	64.1817
		ϖ_L	21.0030	30.8827	44.0043
	100	0.5	24.4319	31.4703	40.6245
		1.0	29.9841	36.7995	46.8549
		1.5	26.9507	38.7423	64.7501
		ϖ_L	21.3947	31.4948	44.8648
$[\pm45^0]_2$	0	0.5	30.3743	41.4607	56.7525
		1.0	36.5799	49.1014	68.1200
		1.5	44.5544	61.2754	87.7403
		ϖ_L	26.8079	39.6505	56.3830
	100	0.5	30.6196	41.8917	57.3967
		1.0	36.7799	49.4156	68.5605
		1.5	44.6830	61.4026	88.1625
		ϖ_L	27.0864	40.1023	57.0313
$[\pm45^0]_s$	0	0.5	31.1601	43.0576	60.3092
		1.0	39.0749	53.5663	75.9593
		1.5	37.8409	67.6010	97.8913
		ϖ_L	25.4835	37.4831	53.5773
	100	0.5	31.3710	43.4380	60.8435
		1.0	39.2413	53.9219	76.3923
		1.5	38.0470	58.4842	98.2201
		ϖ_L	25.7359	37.9072	54.2003