Nonlinear Self-Tuning PID Controller for a Flexible Manipulator based on NARMAX Model

Santanu Kumar Pradhan
Center for Industrial Electronics and Robotics,
Dept. of Electrical Egg, National Institute of Technology,
Rourkela-769008, INDIA
santanupradhan.nitrkl@gmail.com

Bidyadhar Subudhi, *Senior Member, IEEE*
Center for Industrial Electronics and Robotics,
Dept. of Electrical Egg, National Institute of Technology,
Rourkela-769008, INDIA
bidyadhar@nitrkl.ac.in

Abstract— **This paper describes the design of a new self-tuning controller to control tip trajectory and tip deflection of a two-link flexible manipulator handling variable payloads based on nonlinear autoregressive with moving average and exogenous input (NARMAX) model. This proposed adaptive controller consists of a multivariate proportional integral and derivative (PID) feedback loop. The gains of the PID controllers are calculated in real-time based on the kinship between PID and generalized minimum variance control laws. Parameters of the NARMAX model are estimated in real-time using an adaptive filter based on the recursive least square (RLS) algorithms. Simulation results envisage that the NARMAX based self-tuning controller tracks a desired tip trajectory while suppressing the tip deflection under load pick-up and release operation.**

Keywords- Flexible robot, self-tuning PID control, NARMAX, RLS

I. INTRODUCTION

Flexible-link manipulators offer several advantages such as high-speed operation, lower energy consumption, and increase in payload carrying capacity for some specific applications like space robots where rigid-link robots are unsuitable [1]. However, controlling a flexible-link robot is difficult owing to its distributed link flexure thus the dynamics of the manipulator becomes distributed parameter system, other control complexity encountered in controlling the robot are due to its non-minimum phase behavior, under actuation, non-collocation [1]. Further, control of a flexible-link robot becomes more challenging when it has to handle variable payloads. In order to achieve good tip trajectory tracking while suppressing tip deflection with varied payloads in a flexible-link robot, adaptive controller are employed, which provide appropriate control torques to the actuators to achieve the above two-control tasks (good tip trajectory tracking and suppression of tip deflection).

Different identification and control methods have been applied to estimate flexible robot dynamics. Yurkowich et.al [1] developed identification and control strategy of a single-link flexible robot using on-line frequency domain linear model, and an ARMA model with weighted recursive least square (RLS) algorithm is used for parameter adaptation in [2]. Yazdizadeh et.al proposed a dynamic neural network using dynamic neurons for identification of a two-link flexible robot in [3]. However, disadvantages of the above methods are that linear model is considered, also which is in appropriate to capture the non-linear dynamics of the flexible-link robot.

Recent works by Kukreja et.al [4] and Tsai et.al [5] proposed NARMAX model based identification for nonlinear ankle dynamics systems and manipulator dynamics respectively which show good approximation of the models. Also self-tuning control methods applied to process control for cement plants in [7] and [8] show outstanding results. Also there is no work that emphasis NARMAX modeling based flexible robot adaptive control design. Motivated by the above mentioned reasons, an attempt has been made in this paper to control the tip trajectory tracking and suppression of tip deflection for a flexible using NARMAX model as proposed by Chen and Billings in [6].

II. DYNAMIC MODEL OF THE TLFM

The schematic diagram of a planar TLFM is shown in Fig.1,

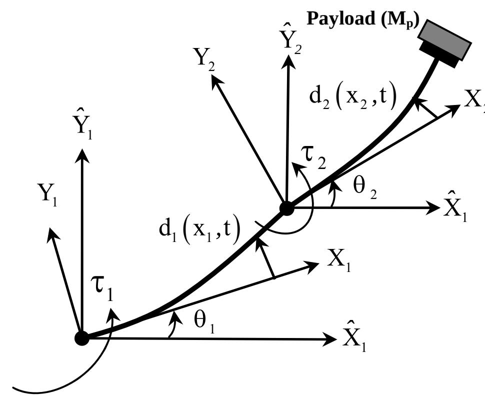

Fig.1 Schematic diagram of a planar TLFM

where τ_i is the actuated torque of the i^{th} link, θ_i is the joint angle of the i^{th} joint and $d_i(l_i, t)$ represents the deflection along i^{th} link. The outer free end of the TLFM is attached with payload mass, $\mathbf{M_p}$. The dynamics of the TLFM is given by [14]

$$\mathbf{M}(\theta_i,\delta_i)\begin{bmatrix}\ddot{\theta}_i\\ \ddot{\delta}_i\end{bmatrix}+\begin{bmatrix}\mathbf{c_1}(\theta_i,\delta_i,\dot{\theta}_i,\dot{\delta}_i)\\ \mathbf{c_2}(\theta_i,\delta_i,\dot{\theta}_i,\dot{\delta}_i)\end{bmatrix}+\mathbf{K}\begin{bmatrix}0\\ \delta_i\end{bmatrix}+\mathbf{D}\begin{bmatrix}0\\ \dot{\delta}_i\end{bmatrix}=\begin{bmatrix}\tau_i\\ 0\end{bmatrix} \quad (1)$$

where **M** is the positive-definite symmetric inertia matrix, $\mathbf{c_1}$ and $\mathbf{c_2}$ are the vectors containing of Coriolis and Centrifugal forces respectively, **K** is the stiffness matrix and **D** is the damping matrix. If the output is taken as tip position, the overall manipulator system becomes non-minimum phase [4]; hence, the redefined output is given by

$$y_{pi} = \theta_i + \left[\frac{d_i(l_i, t)}{l_i}\right] \tag{2}$$

where l_i is length of the i^{th} link. TLFM dynamics (1) can be rewritten in state space form as

$$\dot{x} = f_i(x) + g_i(x)u_i \tag{3}$$

with x as the state vector i.e. $x = \left[\theta_i, \dot{\theta}_i, \delta_i, \dot{\delta}_i\right]^T$ and

$$f_i(x) = \mathbf{M}(\theta_i, \delta_i)^{-1}\left(-\begin{bmatrix}\mathbf{c}_1(\theta_i, \delta_i, \dot{\theta}_i, \dot{\delta}_i)\\ \mathbf{c}_2(\theta_i, \delta_i, \dot{\theta}_i, \dot{\delta}_i)\end{bmatrix} - \mathbf{K}\begin{bmatrix}0\\ \delta_i\end{bmatrix} - \mathbf{D}\begin{bmatrix}0\\ \dot{\delta}_i\end{bmatrix}\right)$$

$g_i(x) = \mathbf{M}(\theta_i, \delta_i)^{-1}$ and $u_i = \begin{bmatrix}\tau_i\\ 0\end{bmatrix}$

δ_i and $\dot{\delta}_i$ being the modal displacement and modal velocity for the i^{th} link respectively and the actual output vector, **y** is given by

$y = \left[\theta_i, \dot{\theta}_i\right]$

To express the dynamics of the TLFM in terms of the redefined tip position and tip velocity, the states are redefined as $\zeta = \left[y_{pi}, \dot{y}_{pi}\right]$

The new state space representation of the TLFM using redefined output can be expressed as

$$\dot{\zeta} = \hbar_i(\zeta) + \lambda_i(\zeta)v_i \tag{4}$$

where v_i is the i^{th} torque input with respect to the redefined output ζ

$$\hbar_i(\zeta) = M(\theta_i)^{-1}\left(-\mathbf{c}_1(\theta_i, \delta_i, \dot{\theta}_i, \dot{\delta}_i) - K\delta_i - D\dot{\delta}_i\right) + \ldots$$
$$M(\delta_i)^{-1}\left(-\mathbf{c}_2(\theta_i, \delta_i, \dot{\theta}_i, \dot{\delta}_i)\right)$$
$$\text{and } \lambda_i(\zeta)v_i = M(\theta_i, \delta_i)^{-1}\tau_i$$

where δ_i and $\dot{\delta}_i$ are the modal displacement and modal velocity for the i^{th} link respectively.

Equation (4) can be rewritten as

$$\dot{x}_i = L_i(x_i, v_i) \tag{5}$$

where

$L_i(x_i, v_i) = f_i(x_i) + g_i(x_i)v_i$

III. PROPOSED SELF-TUNING CONTROLLER BASED ON NARMAX MODEL OF THE TLFM

A. *Representation of the TLFM using NARMAX model*

The NARMAX model for describing the input-output relationship of the nonlinear multi-input multi-output (MIMO) systems can be written as [9]

$$y_i(k) = F_i^n\begin{pmatrix} y_i(k-1), \ldots, y_i(k-n_y),\\ y_i^2(k-1), \ldots, y_i^2(k-n_y),\\ u_i(k-1), \ldots, u_i(k-n_u),\\ u_i^2(k-1), \ldots, u_i^2(k-n_y),\\ \xi_i(k-1), \ldots, \xi_i(k-n_\xi),\\ y_i(k-1)u_i(k-1), \ldots, y_i(k-n_y)u_i(k-n_u),\\ y_i^2(k-1)u_i(k-1), \ldots, y_i^2(k-n_y)u_i(k-n_u),\\ y_i(k-1)u_i^2(k-1), \ldots, y_i(k-n_y)u_i^2(k-n_u),\\ y_i^2(k-1)u_i^2(k-1), \ldots, y_i^2(k-n_y)u_i^2(k-n_u), \end{pmatrix} \tag{6}$$

where

$y_i(k)$	Nonlinear autoregressive (NAR) vector
$u_i(k)$	Exogenous (X) vector
$\xi_i(k)$	Moving average (MA) variable vector
F_i	Nonlinear function with i^{th} input and output
n	Order of the nonlinearity
n_y, n_ξ, and n_u	Order of NAR, MA, and X respectively

In order to represent the TLFM dynamics as a NARMAX model we need to discretized the derivative terms of $\dot{x}_i$ given in (4) using the forward difference method at t=kT by

$$\dot{x}(kT) = \frac{x(kT+T) - x(kT)}{T}$$

where T is the sampling time at k^{th} instant. Using the above relati
-onship the discrete time representation of equation (4) is

$$x(k+1) = x(k) + TL_i\left(x(k), v(k)\right) \tag{7}$$

To express the dynamics of the TLFM in terms of the redefined tip position and tip velocity, the states are redefined as $\zeta_i = \left[y_{pi}, \dot{y}_{pi}\right]$ using equation (2). Now, the input-output representation of TLFM dynamics using redefined output ζ_i is given as [4]

$$\zeta_i(k+2) = A_i(x) + B_i(x)v(k) \tag{8}$$

where

$$A_i(x) = (\theta_i, \delta_i) + 2T(\dot{\theta}_i, \dot{\delta}_i) - T^2\mathbf{M}(\theta_i, \delta_i)^{-1}\cdots$$
$$\left(\begin{bmatrix}\mathbf{c}_1(x)\\ \mathbf{c}_2(x)\end{bmatrix} + \mathbf{K}\begin{bmatrix}0\\ \delta_i\end{bmatrix} + \mathbf{D}\begin{bmatrix}0\\ \dot{\delta}_i\end{bmatrix}\right)$$

$B_i(x) = T^2\mathbf{M}(\theta_i, \delta_i)^{-1}$

Representing the difference equation given in (7) as NARMAX representation defined in (5), with order of the nonlinearity as n=2, $y_i=\xi_i$, $u_i=v_i$ and n_y, n_u, n_ξ are found to be 2. Using the above representation equation (7) can be rewritten as

$$y_i(k) = \phi_i^T(k)w_i(k) + e_i(k) \tag{9}$$

where

$$\phi_i^T(k) = \big(y_i(k-1),\ldots,y_i(k-n_y),u_i(k-1),\ldots$$
$$\ldots,u_i(k-n_u),\xi_i(k-1),\ldots,\xi_i(k-n_\xi)\big)^T$$
$$w_i(k) = \big(w_{i1}(k),w_{i2}(k),w_{i3}(k)\cdots w_{iN}(k)\big)$$

B. *Identification of the TLFM based on NARMAX model*

Fig.2 shows the structure for estimation of NARMAX parameters using RLS algorithm. The principle of least square is to estimate w_i defined in eq. (13) to the true parameter such that it minimizes the sum of squared errors $J(\theta_i)$ [9]:

$$J_i(w_i) = \sum_{k=1}^{N} e_i^2(k,w_i),\ i=1,2 \tag{10}$$

The NARMAX parameters are estimated using the RLS algorithm given as [9]

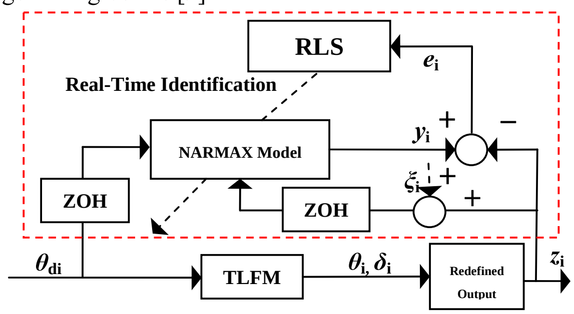

Fig.2 Structure for estimation of NARMAX parameters for TLFM

$$\hat{w}_i(k) = \hat{w}_i(k-1) + \frac{P_i(k-1)\phi_i(k)}{\lambda + \phi^T{}_i(k)P_i(k-1)\phi_i(k)} e_i(k) \tag{11}$$

$$P_i(k) = \frac{1}{\lambda}\left\{ P_i(k-1) - \frac{P_i(k-1)\phi_i(k)\phi^T{}_i(k)P_i(k-1)}{\lambda + \phi^T{}_i(k)P_i(k-1)\phi_i(k)} \right\} \tag{12}$$

$$y_i(k) = \phi^T{}_i(k)\hat{w}_i(k-1) + e_i(k),\ i=1,2 \tag{13}$$

where $P_i(k) \in \Re^{(nnum+nden-1)\times(nnum+nden-1)}$ with $\lambda \le 1$ is the forgetting factor, $P_i(0) = \Gamma_i I_{nmum+nden-1}$ and Γ is large value.

To be able to estimate the system parameters uniquely the number *N* in eq. (14) must not be less than the number of unknown parameters and thus can be gaining using PID. The result of the estimation based on N^{th} iteration the recursive parameter estimate given by eq. (15) with any initial condition is equal to the optimal least squares estimation based on the data provided i.e $P_i(0) = \Gamma_i I$.

C. *Multivariate PID (MPID) control design*

The multivariate PID control law with $e_i(t) = y_i(t) - \theta_{di}(t)$ and the output $u_i(t)$ is given as

$$u_i(t) = K_i\left[e_i(t) + \frac{1}{T_I}\int_0^t e_i(t)dt + T_D\frac{de_i(t)}{dt} \right],\ i=1,2 \tag{14}$$

where Ki is the gain, T_I is the integral time and T_D is the derivative time. Now, rewriting the eq. (18) in discrete domain we get

$$u_i(k) = K_i\left[1 + \frac{1}{T_I}\frac{T_s}{2}\frac{z+1}{z-1} + \frac{T_D}{T_s}\frac{z-1}{z} \right] e_i(k) \tag{15}$$

where T_s is the sampling time and z is the Z-transform parameter. The trapezoidal approximation used for the integral mode and backward difference formula is used for derivative mode in (18). On cross multiplying (19) we get

$$(z^2-z)u_i(k) = K_i\begin{bmatrix} (z^2-z) + \frac{T_s}{2T_I}(z^2+z) + \\ \frac{T_D}{T_s}(z-1)^2 \end{bmatrix} e_i(k) \tag{16}$$

We divide by z^2 and invert (20), to obtain

$$\Delta u_i(k) = K_i\begin{bmatrix} e_i(k) - e_i(k-1) \\ +\frac{T_s}{2T_I}\{e_i(k) - e_i(k-1)\} \\ +\frac{T_D}{T_s}\{e_i(k) - e_i(k-1) + e_i(k-2)\} \end{bmatrix} \tag{17}$$

where $\Delta u_i(k) = u_i(k) - u_i(k-1)$. Let $L_i(z^{-1})$ be a polynomial defined as

$$L_i(z^{-1}) = K_i\left(1 + \frac{T_s}{T_I} + \frac{T_D}{T_s}\right) - K_i\left(1 + \frac{2T_D}{T_s}\right)z^{-1} + \frac{K_iT_D}{T_s}z^{-2} \tag{18}$$

Then (21) can be rewritten as

$$L_i(z^{-1})y_i(k) + \Delta u_i(k) - L_i(z^{-1})\theta_{di}(k) = 0 \tag{19}$$

The polynomial $L_i(z^{-1})$ for the MPID control law is tuned vial minimum variance control law derived as follows.

D. *Tuning of PID parameters using minimum variance control law*

Fig.3 shows the NARMAX based self-tuning controller for TLFM.

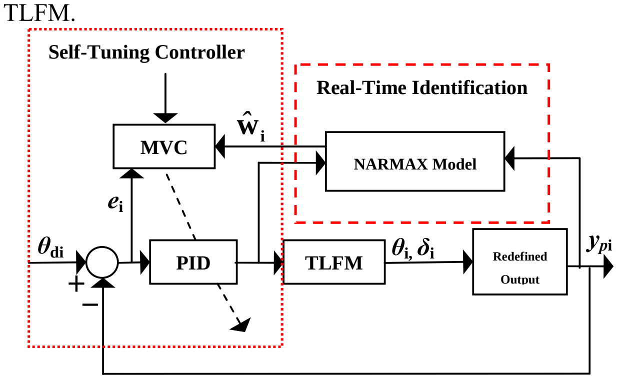

Fig.3 NARMAX model based self-tuning controller for TLFM

In order to tune the PID parameters based on the principle of minimum variance a performance index is considered as follows

$$J = E\left[P_i(z^{-1})y_{pi} + \Gamma_i u_i - R_i(z^{-1})y_{di} \right] \tag{20}$$

where Γ_i in eq. (20) is the weighting factor with respect to the control input, $P_i(z^{-1})$ is the user defined polynomial of the form

$$P_i\left(z^{-1}\right)=1+p_{i,1}z^{-1}+p_{i,2}z^{-2} \quad (21)$$

and $R_i(z^{-1})$ defined in eq. (20) is determined using eq. (17) and eq. (19).

The control law minimizing the performance index J eq. (20) is given by the following equation [8]

$$F_i\left(z^{-1}\right)\theta_{pi}+\left\{E_i\left(z^{-1}\right)C_i\left(z^{-1}\right)+\mu_i\right\}u_i-P_i\left(z^{-1}\right)\theta_{di}=0 \quad (22)$$

where the E_i (z^{-1}) and F_i (z^{-1}) are found out by solving the following Diophantine equation

$$P_i\left(z^{-1}\right)=A_i\left(z^{-1}\right)E_i\left(z^{-1}\right)+z^{-(-k_{mi}+1)}F_i\left(z^{-1}\right) \quad (23)$$

And

$$\left.\begin{aligned} E_i\left(z^{-1}\right)&=1+e_{i,1}z^{-1}+\ldots+e_{i,km_i}z^{-km_i}\\ F_i\left(z^{-1}\right)&=f_{i,0}+f_{i,1}z^{-1}+f_{i,2}z^{-2}\end{aligned}\right\} i=1,2 \quad (24)$$

Next, in eq. (22) the term $E_i(z^{-1})$ $C_i(z^{-1})$ μ_i be assumed to be

$$E_i\left(z^{-1}\right)C_i\left(z^{-1}\right)+\mu_i=v_i \quad (25)$$

then eq. (23) can be written as

$$\frac{F_i\left(z^{-1}\right)}{v_i}y_{pi}+u_i-\frac{P_i\left(z^{-1}\right)}{v_i}\theta_{di}=0 \quad (26)$$

Therefore using eq. (26) and eq. (19), PID parameters k_{c1}, k_{c2}, T_D, T_i can be calculated as follows.

$$\left.\begin{aligned} k_{ci}&=-\frac{\left(f_{i,1}+2f_{i,2}\right)}{v_i}\\ T_{Ii}&=-\frac{f_{i,1}+2f_{i,2}}{f_{i,0}+f_{i,1}+f_{i,2}}T_s\\ T_{Di}&=-\frac{f_{i,2}}{f_{i,1}+2f_{i,2}}T_s\end{aligned}\right\} \quad (27)$$

The parameters of $Pi(z^{-1})$ is designed based on the response considering the overshoot and settling time, and μ_i is chosen according to the system stability criteria. Thus $Pi(z^{-1})$ parameters are $p_{i1=-}e^{-\rho i}$ and $p_{i2=-}e^{-\rho i/£I}$, where ρ_i and $£_I$ are defined by

$$\rho_i = {Ts}/{\sigma_i}, £_i = 0.45 \quad (28)$$

and $£_I$ and ρ_i denote the rise-time and damping ratio.

E. Controller realization in steps

The proposed NARMAX based adaptive controller can be realized via following steps.

1) Chose $P_i(z^{-1})$ and μ_i
2) Estimate w_i by using RLS algorithm in eq. (15)
3) Solve the Diophantine eq. (24)
4) Calculate v_i based on (26)
5) Calculate the PID parameters using eq. (28)
6) Based on the eq. (28) calculate the control input given in eq. (17)

For each iteration k^{th} update the parameters and return to step-2

IV. Results and Discussions

A. Simulation Results

The numerical simulation of the NARMAX based controller is performed using MATLAB/SIMULINK®.

The proposed NARMAX based PID adaptive controller has been applied to the TLFM available in advanced robotics research Lab., NIT, Rourkela. To validate the tip trajectory tracking performances, the desired trajectory vector for two joints $\theta_{di}(t)$ i=1,2 are chosen as

$$\theta_{d_i}\left(t\right)=\theta_0\left(t\right)+\left[6\frac{t^5}{t_d^{\,5}}-15\frac{t^4}{t_d^{\,4}}+10\frac{t^3}{t_d^{\,3}}\right]\left(\theta_f\left(t\right)-\theta_0\left(t\right)\right) \quad (29)$$

where $\theta_{di}(t)=[\theta_{d1}, \theta_{d2}]^T$, $\theta_d(0)=\{0,0\}$ are the initial positions of the links and $\theta_f(0)=\{\pi/4, \pi/6\}$ are the final positions for link-1 and link-2, t_d is the time taken to reach the final positions which is taken as 4 sec and total simulation time is 10 sec.

The physical parameters of the studied TLFM are given in Table I.

Table I: Physical parameter of the TLFM

Parameter	Symbol	Value
Link length	L_1, L_2	0.201m, 0.2m
Elasticity	E_1=E_2	2.0684×10^{11}(N/m^2)
Rotor moment of Inertia	K_{s1}, K_{s2}	6.28×10^{-6}, 1.03×10^{-6} (kg m^2)
Drive moment of Inertia	J_{11}, J_{21}	7.361×10^{-4}, 44.55×10^{-6} (kg m^2)
Link moment of Inertia	J_{21}, J_{22}	0.17043, 0.0064387(kg m^2)
Gear ratio	N_1, N_2	100, 50
Maximum Rotation	R_1, R_2	(+/-90, +/-90) deg.
Drive Torque constant	K_{t1}, K_{t2}	0.119; 0.0234(Nm/A)

Using the above control parameters along with the TLFR parameters the simulations were carried out. The results are shown in Fig.3 to Fig.7. Fig.4 shows the NARMAX parameters estimation using the RLS algorithm defined in eqs. (11)-(13). It is clear from Fig.4 that the parameters are updated with the change in operating conditions i.e. load pick-up and load release operation. Next in Fig.5 to Fig.7, the performance of the adaptive controller are shown with respect to tip trajectory tracking, tip deflection and control signals with respect to the desired tip trajectory defined in eq. (29). Fig.5 shows the tip trajectory tracking error for link-1 and link-2 respectively. It is clear from Fig.5 that in spite of change in operating conditions the controller tracks the given trajectory with minimum error.

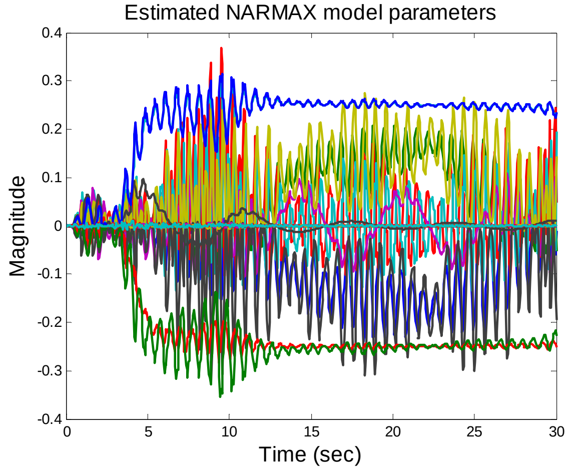

Fig.4 Estimated NARMAX parameters

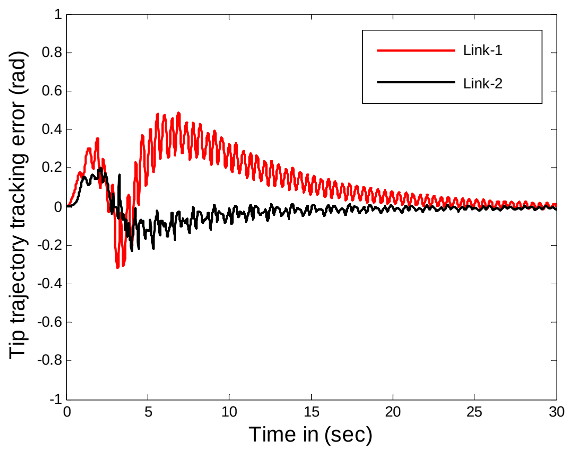

Fig.5 Tip trajectory tracking errors for TLFR

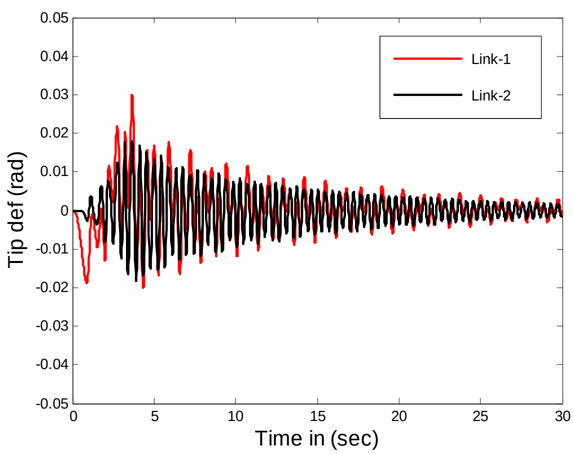

Fig.6 Tip deflection trajectories for TLFR

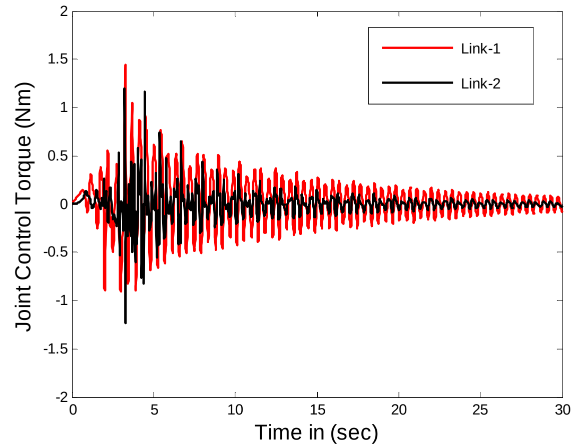

Fig.7 Control torque trajectories for TLFR

Fig.6 shows the link deflection during the robot operation, from the Fig.6 it is clear that the proposed controller considerably reduces the tip oscillation for both link-1 and link-2, thus suppressing the overall link vibration even in addition of a payload. Also in Fig.7 the control inputs to both the actuators are given. From the results shown in Fig.7 it is clear that the control input to the actuators are small. Thus it can be said that the proposed adaptive controller provides good tip trajectory tracking while suppressing tip deflection with varied payloads.

V. Conclusions

An adaptive controller using NARMAX representation of the two-link flexible robot is proposed. Unlike the adaptive controller formulated in [1] and [2] using a linear model, a nonlinear model is considered here. The proposed controller efficiently tracks the given tip trajectory while simultaneously damping the link vibration. Another advantage of the proposed method is that the PID parameters are tuned via a minimum variance performance index giving rise to a near optimal control input.

References

[1] S. Yurkovich, and A. P. Tzes, “Experiments in Identification and Control of a Flexible-Link Manipulator,” *IEEE Contr. Syst. Mag.*,pp. 41–47, Feb. 1990.

[2] S. Yurkovich, K.L. Hillsley, and A. P. Tzes, Identification and Control fora Manipulator with Two Flexible-Links,” Proc. *IEEE Conf.on Decision and Control, Honolulu, Hawaii*,pp. 1995–2000, Dec. 1990.

[3] A.Yazdizadeh, K. Khorsani and R. V. Patel, “Identificationof aTwo-link Flexible-Link Manipulator using Adaptive time delay Neural Networks,” *IEEE Trans. on Syst. Man and Cybernetics-Part B*, vol. 30, no. 1, pp. 165–172, Feb. 2000.

[4] S. L. Kukreja, H. L. Galiana and R. E. Kearney, “NARMAX Representation and Identification ofAnkle Dynamics,” *IEEE Trans. on Biomedical Egg.*, vol. 50, no. 1, pp. 70–81, Jan. 2003.

[5] J. S. H. Tsai, C. T. Yang, C. C. Kuang, S. M. Guo, L. S. Shieh and C. W. Chen, “NARMAX model-based state-space self-tuning control for nonlinear stochastic hybrid systems,” *Journal of Applied Mathematical Modelling*,vol. 34, pp. 3030–3054, Jan.2010.

[6] S. Chen and S. A. Billings, “Representations of nonlinear systems: the NARMAX model,” *Int. J. Contr.*, vol. 49, no. 3, pp. 1013–1032, 1989.

[7] M. Agarwal, and D.E. Seborg, “Self-tuning Controllers for Nonlinear Systems,” *Automatica.*, vol. 23, no.2, pp. 2009–2014, 1987.

[8] T. Yamamoto, and S. L. Shah, "Design and experimental evaluation of a multivariable self-tuning PID controller," *IEEE proc.-Control Theory Appl.*., vol. 151, no. 5, pp. 645–652, Sep.2004.

[9] A. de Luca and B. Siciliano, "Closed-Form Dynamic Model of Planar Multilink Lightweight Robots," *IEEE Trans. on Syst. Man and Cybernetics*, vol. 21, no. 4, pp. 826–839, July/Aug.1991.

[10] Z. Su, and K. Khorsani, "A Neural Network Based Tracking Controller for a Single-Link Flexible Manipulator using the Inverse Dynamic Approach," *IEEE Trans. on Industrial Elect.,* vol. 48, no. 6, pp. 1074–1086, Dec.2001.

[11] L. Ljung, *System Identification: Theory for the User*, 2nd ed. Englewood Cliffs, NJ: Prentice-Hall, 1999.