Designing of Decoupler for a 4×4 Distillation Column Processes with PID Controller

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***Abstract*—This paper extend the concept of relative normalized gain array (RNGA) and an Equivalent Transfer Function (ETF) for each element in the transfer function matrix was derived for the closed-loop control system and proposes a systematic approach to designing decentralized PID control for distillation column process containing integrators and/or differentiators. The decoupler could be easily determined with each element in the First-Order-plus-Time-Delay (FOPTD) form and resulted in a stable, proper and causal decoupled matrix. The main advantage of this method is its simplicity; it reduces excessive calculation.**

I. INTRODUCTION

Despite the availability of complicate methods for designing multivariable control systems,most of loops are controlled by decentralized PI/PID controllers because of their relative effectiveness and their simple structure, which can be easily understood and implemented.the primary task in the design of decentralized control systems is to determine loop configuration, i.e. pair the manipulated variables and controlled variables.Since controllers interact with each other in MIMO processes, the performance of one loop cannot be evaluated without getting information about the controllers of other loops. To solve this problem, a decoupling control scheme has been proposed that introduces a decoupler to eliminate the presence of interactions. Then the MIMO process can be treated as an SISO loops. A general decoupling control system is represented decoupler matrix and the controller transfer function matrix respectively. $G_I(s)$ acts upon the process, G(s), such that the transfer function matrix as in Fig. 1, where, $G_I(s)$ and $G_C(s)$ are the n-dimensional process matrix

$$G_R(s) = G(s)G_I(s) \quad \text{…. (1)}$$

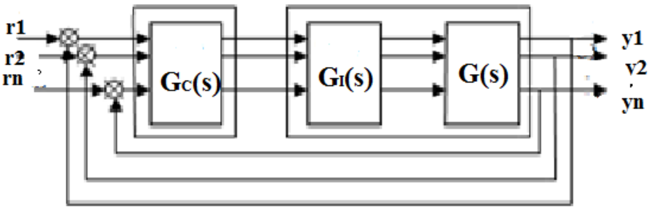


fig.1

There are basically two types of decoupling method used for solving MIMO process such as static and dynamic decoupling. If there is substantial differences in dynamic characteristics among the transfer function elements it is difficult to get the desire performance. The dynamic decoupler provides better result when the process dynamics comes into the process; however it is difficult to design practical dynamic decoupler for high dimensional MIMO system.

In this paper, the normalized decoupling control design methodology was extended from a 4×4 design to a high-dimensional MIMO design (A.Wahid and A.Ahmed (2008),Multivariable control of a (4×4) process in distillation column). The design was based on the concept of Relative Normalized Gain Array (RNGA) design (Xiong & Cai, 2006; Xiong, Cai& He,2007.He,Cai,Ni &Xie,2009). Using the gain and phase information revealed in RNGA design, the Equivalent Transfer Function (ETF) of each element in the transfer function matrix was derived for the closed-loop control system. Further, an equivalent transfer function matrix was approximated by the inverse of the process transfer function matrix used for calculating decoupling matrix. The decoupler could then be easily determined by multiplying the inverse of the ETF with a stable, proper and causal ideal-diagonal transfer function. Finally, a PI/PID controller was designed for the diagonal transfer function to meet the control system's performance target. Both the simulation and the real-time testing results of a distillation process demonstrated the effectiveness and the simplicity of the design components, incorporating the applicable criteria that follow.

II. CASE STUDY

The process is presented by Doukas and Luyban(1978).They studied dynamic of a distillation column producing a liquid sidestream product.The objective is to maintain four composition specification on the product stream.The transfer function matrix is of (4×4) model is given below. The control and manipulated variables are y1 (toluene impurity in bottom), y2 (toluene impurity at distillate), y3 (benzene impurity in side stream), y4 (xyene impurity at side stream),u1 (side stream flow rate), u2(reflux ratio), u3 (reboil duty) and u4 (side draw location).

$$G=\begin{pmatrix} \frac{2.22e^{-2.5s}}{(36s+1)(25s+1)} & \frac{-2.94(7.9s+1)e^{-0.05s}}{(23.7s+1)^2} & \frac{0.017e^{0.2s}}{(31.6s+1)(7s+1)} & \frac{-0.64e^{-20s}}{(29s+1)^2} \\ \frac{-2.33e^{-5s}}{(35s+1)^2} & \frac{3.46e^{-1.01s}}{32s+1} & \frac{-0.51e^{-7.5s}}{(32s+1)^2} & \frac{1.68e^{-2s}}{(28s+1)^2} \\ \frac{-1.06e^{-22s}}{(17s+1)^2} & \frac{3.511e^{-13s}}{(12s+1)^2} & \frac{4.41e^{-1.01s}}{16.2s+1} & \frac{-5.38e^{-0.5s}}{17s+1} \\ \frac{-5.73e^{-2.5s}}{(8s+1)(50s+1)} & \frac{4.32(25s+1)e^{-0.01s}}{(50s+1)(5s+1)} & \frac{-1.25e^{-2.8s}}{(43.6s+1)(9s+1)} & \frac{4.78e^{-1.15s}}{(48s+1)(5s+1)} \end{pmatrix}$$

A. INPUT-OUTPUT PAIRING

Consider an $n\times n$ system with a decentralized Feedback control structure as shown in Figure1,where $r=[r_1\, r_2 ....r_n]^T$, $u=[u_1\, u_2\, ...u_n]^T$ are vectors of references, inputs, and outputs, respectively $G(s)=[g_{i,j}]_{n\times n}$ is the system's transfer function matrix; and

$G_c(s) = diag\{g_{c1}(s)\, g_{c2}(s) \cdots g_{cn}(s)\}$ is the decentralized controller; i, j = 1, 2, ..., n, are integer indices".

B. LOOP PAIRING FOR PROCESSES CONTAINING INTEGRATORS/DIFFERENTIATORS

Here, it is assumed that **K** and T_{ar} are finite and nonzero to validate the definition of the RNGA. If a MIMO process contains integrators, T_{ar} value goes to infinity; on the other hand, if a MIMO process contains differentiators, **K** equals zero. For such a process, the RNGA cannot be computed directly using the definition. To deal with such processes, let a MIMO process be given by $G(s)=[g_{ij}(s)]_{n\times n}$,where $g_{ij}(s)=k_{ij}s^{m_{ij}}\bar{g}_{ij}(s)$, m_{ij} an integer, and does not contain any integrator or differentiator and satisfies $\bar{g}_{ij}(0)=1$. If $m_{ij}<0$, the transfer function contains integrator (s); if $m_{ij}>0$ the transfer function contains different differentiator(s); and if $m_{ij}=0$, it is a normal transfer function. Suppose that there exist diagonal output scaling matrix $S_1(s)\in R^{n\times n}$ and diagonal input scaling matrix $S_2(s)\in R^{n\times n}$ for $K\otimes\bar{G}(s)$, such that **G**(s) can be factorized as

$$G(s) = S_1(s)[K \otimes \bar{G}(s)]S_2(s) \quad \text{.... (2)}$$

where $\bar{G}(s)=[\bar{g}_{ij}(s)]_{n\times n}$. Because the RNGA is invariant to input/output scaling, the RNGA (denoted by Φ) for **G**(s) is the same as that for $K\otimes\bar{G}(s)$. Once the RNGA is obtained, the RGA-NI-RNGA rules for normal processes can be applied directly to determine the input output pairing for the MIMO process with integrators/differentiators. The above loop pairing is limited to the class of MIMO processes that can be factorized as in Eq 2. (A similar constraint occurs in the loop pairing using the RGA analysis) Indeed, many practical MIMO processes fall into this class. Typical examples will be shown in section

$$K_N = K \odot T_{ar}$$

$$K_N=\begin{pmatrix} k_{N,11} & \cdots & k_{N,1n} \\ \vdots & \ddots & \vdots \\ k_{N,n1} & \cdots & k_{N,nn} \end{pmatrix} \quad \text{....(3)}$$

$$\Phi = K_N \otimes K_N^{-T} \quad \text{....(4)}$$

$$\Phi=\begin{pmatrix} \Phi_{11} & \cdots & \Phi_{1n} \\ \vdots & \ddots & \vdots \\ \Phi_{n1} & \cdots & \Phi_{nn} \end{pmatrix} \quad \text{....(5)}$$

$$\Gamma=\begin{pmatrix} \gamma_{11} & \cdots & \gamma_{1n} \\ \vdots & \ddots & \vdots \\ \gamma_{n1} & \cdots & \gamma_{nn} \end{pmatrix}$$

III. DECOUPLING CONTROL SYSTEM DESIGN

The design of an ideal-diagonal decoupler problem was transformed to determine the decoupler $G_I(s)$,

$$G_I(s)=\hat{G}^T(s)G_R(s) \quad \text{....(6)}$$

$$G_I(s)=\begin{pmatrix} g_{I,11}(s) & g_{I,12}(s)\ldots g_{I,1n}(s) \\ g_{I,21}(s) & g_{I,22}(s)\ldots g_{I,2n}(s) \\ . & \\ . & \\ g_{I,n1}(s) & g_{I,n2}(s)\ldots g_{I,nn}(s) \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1/\hat{g}_{11}(s) & 1/\hat{g}_{21}(s) & \ldots 1/\hat{g}_{n1}(s) \\ 1/\hat{g}_{12}(s) & 1/\hat{g}_{22}(s) & \ldots 1/\hat{g}_{n2}(s) \\ . & & \\ . & & \\ 1/\hat{g}_{1n}(s) & 1/\hat{g}_{2n}(s) & \ldots 1/\hat{g}_{nn}(s) \end{pmatrix} \times \begin{pmatrix} g_{R,11}(s) & 0 & 0 \\ 0 & g_{R,22}(s & 0 \\ . & & \\ . & & \\ 0 & 0 & g_{R,nn}(s) \end{pmatrix} \quad \text{..7}$$

To observe how the problem's definition and the design method of a normalized decoupling control system was different from the existing methods, each element of the process transfer function matrix was represented by Eq(7) (Yuling Shen , Wen-JianCai , ShaoyuanLi ,(2010**)**) are from the ETF was represented, and the desired forward transfer function elements were of the form

$$g_{R,ii}(s) = \frac{e^{-\theta_{R,ii}s}}{\tau_{R,ii}s+1} \quad i, j = 1, 2, \ldots.n, \qquad \ldots. (8)$$

where $\tau_{R,ii}$ and $\theta_{R,ii}$ are the adjustable time constant and the dead time of $g_{R,ii}(s)$,respectively. By substituting Eq.()into(),the element in the ideal decoupler matrix has the form

$$g_{I,ij}(s) = \frac{e^{-(\theta_{R,ii}-\hat{\theta}_{ji})s}}{\hat{k}_{ji}} \times \frac{\tau_{ji}s+1}{\tau_{R,ii}s+1} \quad i, j = 1, 2, \ldots n \qquad (9)$$

A. CONTROLLER TUNING AND ROBUSTNESS ANALYSIS

After determination $G_I(s)$ the parameters in the controller, $G_C(s)$, could be tuned individually for the corresponding elements of $G_C(s)$ .Thus, the present SISO PID tuning methods could be directly applied to assurance the stability and performance of each loop. In this work, the GPM (gain and phase margin) method was implemented for putting into practice because of its simplicity and robustness. Since each element in $G_R(s)$ was represented by a FOPDT model, the standard PI controller of the forms A diagonal matrix, $G_R(s)$ was specified such that holds. Consequently, the design of the normalized decoupler started from the obtained $\hat{G}^T(s)$ determined the diagonal forward transfer function matrix, $G_R(s)$, such that the decoupler, $G_I(s)$, from Eq. satisfied realizable conditions.

$$g_{c,ii}(s) = k_{p,ii} + \frac{k_{i,ii}}{s} \qquad (10)$$

The closed-loop forward transfer function was defined as

$$g_{c,ii}(s)g_{R,ii}(s) = \frac{k_{i,ii}}{s} e^{-\theta_{Riis}} \qquad (11)$$

The individual elements could be derived as

$$K = \begin{pmatrix} 2.2200 & -2.9400 & 0.0170 & -0.6400 \\ -2.3300 & 3.4600 & -0.5100 & 1.6800 \\ -1.0600 & 3.5110 & 4.4100 & -5.3800 \\ -5.7300 & 4.3200 & -1.2500 & 4.7000 \end{pmatrix}$$

$$\Lambda = \begin{pmatrix} 3.4568 & -0.9689 & 0.0518 & -1.5397 \\ -3.3193 & 2.6946 & -0.7307 & 2.3554 \\ -0.1169 & 0.1175 & 2.3053 & -1.3059 \\ 0.9794 & -0.8432 & -0.6264 & 1.4902 \end{pmatrix}$$

$$\Phi = \begin{pmatrix} 1.4979 & 0.0220 & 0.0371 & -0.5570 \\ -1.0785 & 1.3103 & -0.3012 & 1.0695 \\ -0.0066 & -0.0021 & 1.6285 & -0.6199 \\ 0.5872 & -0.3303 & -0.3644 & 1.1075 \end{pmatrix}$$

$$\Gamma = \begin{pmatrix} 0.4246 & -0.0205 & 0.7395 & 0.3710 \\ 0.3207 & 0.4733 & 0.4286 & 0.4656 \\ 0.0546 & -0.0159 & 0.7195 & 0.4867 \\ 0.6148 & 0.4016 & 0.5965 & 0.7493 \end{pmatrix}$$

According to the RGA-NI-RNGA criterion, the input-output Pairs are selected as 1-1/2-2/3-3/4-4,
The ETF parameters were

$$\hat{K} = \begin{pmatrix} 0.6292 & 2.7308 & 0.3387 & 0.4262 \\ 0.6929 & 1.2498 & 0.7257 & 0.7314 \\ 8.8079 & 26.9076 & 1.9483 & 4.2242 \\ -5.9989 & -5.2532 & 2.0461 & 3.2339 \end{pmatrix}$$

$$\hat{T} = \begin{pmatrix} 25.8986 & -0.9701 & 28.5451 & 21.5158 \\ 22.4504 & 15.1454 & 27.4331 & 26.0717 \\ 1.8566 & -0.3810 & 11.6556 & 8.2742 \\ 35.6571 & 22.0884 & 31.3770 & 39.7103 \end{pmatrix}$$

$$\hat{L} = \begin{pmatrix} 1.0614 & -0.0010 & 0.1479 & 7.4192 \\ 1.6036 & 0.4780 & 3.2148 & 0.9311 \\ 1.2013 & -0.2064 & 0.7267 & 0.2434 \\ 1.5369 & 0.0040 & 1.6703 & 0.8616 \end{pmatrix}$$

Which gives,

$$\hat{G}^T(s) = \begin{pmatrix} \frac{25.8986s+1}{0.64}e^{1.0614s} & \frac{22.4505s+1}{0.6929}e^{1.6036s} & \frac{1.8566s+1}{8.8079}e^{1.2013s} & \frac{35.6571s+1}{-5.9989}e^{1.5369s} \\ \frac{-0.9701s+1}{2.7308}e^{-0.0010s} & \frac{15.1454s+1}{1.2498}e^{0.4780s} & \frac{-0.381s+1}{26.9076}e^{-0.2064s} & \frac{22.0884ss+1}{-5.2532}e^{0.004s} \\ \frac{28.5451s+1}{0.7257}e^{0.1479s} & \frac{27.4331s+1}{0.7257}e^{3.2148s} & \frac{11.6535s+1}{1.9483}e^{0.7267s} & \frac{31.3770s+1}{2.0461}e^{1.6703.s} \\ \frac{21.5158s+1}{0.4262}e^{7.4192s} & \frac{26.6717s+1}{0.7314}e^{0.9331s} & \frac{8.2742s+1}{4.2242}e^{0.2434s} & \frac{39.7163s+1}{3.2339}e^{0.8616s} \end{pmatrix}$$

Using the normalized decoupling control system design rules, the decoupled forward transfer function was selected as

$$G_R(s) = \begin{pmatrix} \frac{1}{28.5451s+1}e^{-7.4192s} & & & \\ & \frac{1}{27.4331s+1}e^{-3.2148s} & & \\ & & \frac{1}{11.6556s+1}e^{-1.2013s} & \\ & & & \frac{1}{39.7163s+1}e^{-1.6703s} \end{pmatrix}$$

Which gives a stable, causal and proper decoupler

$$G_I(s) = \hat{G}^T(s)G_R(s) \qquad \textbf{....}(12)$$

$$G_I(s)=\begin{pmatrix} \frac{41.16s+1.5893}{28.5451s+1}e^{-6.3578s} & -\frac{32.40063s+1.4432}{27.4331s+1}e^{-2.0135s} & \frac{0.2107s+0.1135}{11.6556s+1} & -\frac{5.9439s+0.1667}{31.716s+1}e^{-0.1394s} \\ -\frac{0.3552s-0.3661}{28.5451s+1}e^{-7.4202s} & \frac{12.11825s+0.8001}{27.4331s+1}e^{-2.7368s} & -\frac{0.0141s-0.03716}{63.84s+1}e^{-1.4077s} & -\frac{4.2046s+0.1903}{58.49s+1}e^{-1.6663s} \\ \frac{3.9334s+1.3779}{28.5451s+1}e^{-7.2713s} & -\frac{27.4331s+1.3779}{27.4331s+1} & \frac{5.9362s+0.5132}{11.6556s+1}e^{-0.4746s} & \frac{15.335s+0.4887}{39.716s+1} \\ \frac{50.489s+2.3463}{28.5451s+1} & -\frac{30.9976s+1.3672}{27.4331s+1}e^{-2.2837s} & \frac{1.9587s-0.2367}{11.6556s+1}e^{-0.9579s} & \frac{12.2816s-0.3092}{39.716s+1}e^{-0.8087s} \end{pmatrix}$$

The modified ETFs are determined as

$$g_{I_{11}}=\frac{41.16s+1.5893}{28.5451s+1}e^{-6.3578s}$$

$$g_{I_{22}}=\frac{12.11825s+0.8001}{27.4331s+1}e^{-2.7368s}$$

$$g_{I_{33}}=\frac{5.9362s+0.5132}{11.6556s+1}e^{-0.4746s}$$

$$g_{I_{44}}=\frac{12.2816s-0.3092}{39.716s+1}e^{-0.8087s}$$

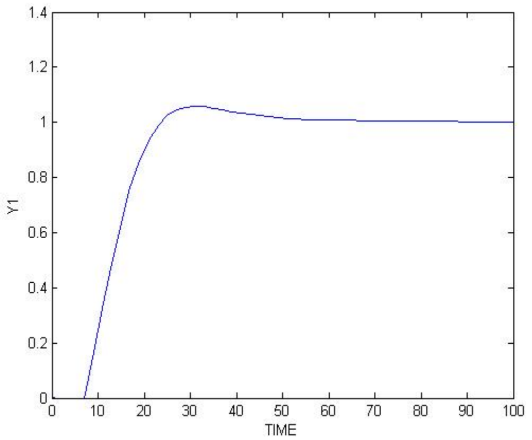
To apply the SIMC method consequently, the PID controllers are obtained as

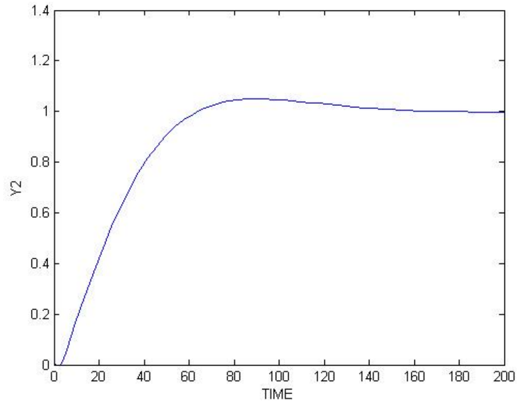
$$g_{c_{11}}(s)=0.02185+\frac{2.54853}{s}$$

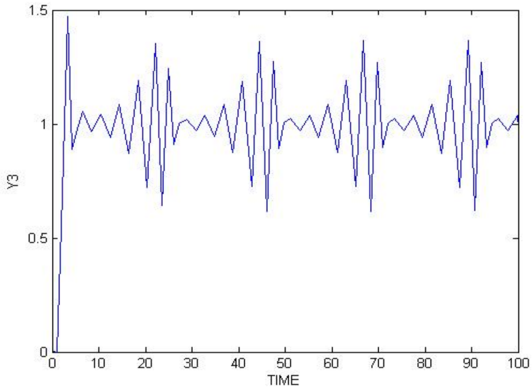
$$g_{c,22}(s)=0.727145+\frac{2.33354}{s}-7.10071s$$

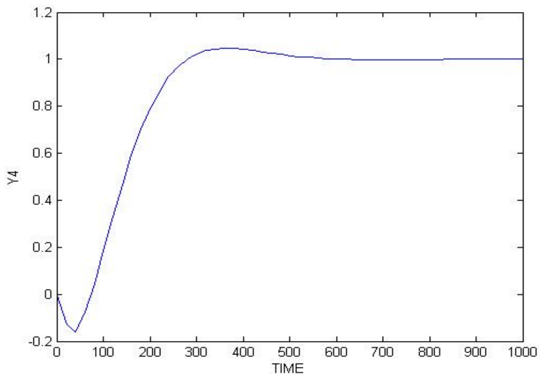
$$g_{c,33}(s)=2.2204+\frac{8.8422}{s}$$

$$g_{c,44}(s)=2.2204e^{16}+\frac{9.3969e^{15}}{s}$$









(Fig.2 Output responses of Unit step inputs are y1, y2, y3, and y4)

IV.CONCLUSION

An approach was proposed for decentralized PID control design of MIMO processes with integrators and/or differentiators. Based on the RGA-NI-RNGA criterion, the input-output pairing was determined. Then ETFs were derived for the selected input-output pairs using the RGA and RNGA information. To maintain integrity, the ETFs were modified for controller tuning. Since the modified ETFs had properly taken account of the loop interactions, the MIMO process was perceived to be decomposed into a set of independent SISO processes so that the PID controllers were designed independently. The sole advantage of the proposed approach is its simplicity in carrying out a systematic decentralized control design, which can easily be understood and implemented by field engineers. Examples illustrated the design procedures and verified by success of the approach.The output response of the controller are shown in the fig.2.

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