Effect of finite thermal conductivity of the separating wall on the performance of counterflow heat exchangers

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published in Cryogenics, 1983, 23(4), P 212-216

Key words: cryogenics, heat exchanger, thermal conductivity, performance

Nomenclature

A, B, G, H, P, Q, r	intermediate variables defined in text
A_c	cross sectional area of the wall
A_w	log mean heat transfer area of the wall
C	heat capacity flow rate, $\dot{m}c_p$
k	thermal conductivity
l	length of the heat exchanger
t	thickness of separating wall
x	nondimensional length; $0 \leqslant x \leqslant 1$

Greek symbols

β_1	N_{tu} of side 1 = h_1A_1/C_1
β_2	N_{tu} of side 2 = h_2A_2/C_2
β_c	Effective N_{tu} due to finite wall conductance = kA_w/tC_1
β^*	intermediate variable = $\beta_1 + \beta_2/\nu + 4\beta_c$
γ	intermediate variables defined in text
ϵ	heat exchanger effectiveness
λ	axial conduction parameter = kA_c/lC_1
ν	C_1/C_2
θ	nondimensional temperature = $T - T_{2,in}/T_{1,in} - T_{2,in}$

Subscripts

1	fluid with smaller heat capacity flow rate, $\dot{m}c_p$
2	fluid with larger heat capacity flow rate, $\dot{m}c_p$
c	wall
e	exit
opt	optimum

The performance deterioration in counterflow heat exchangers due to axial conduction has been investigated by several authors.[1-3] Based on the original formulation by Landau and Hlinka,[1] Kroeger[3] has computed the overall inefficiency of heat exchangers for a wide range of flow parameters. But in none of these papers has the resistance of the separating wall to lateral heat transfer been explicitly considered. Any attempt to reduce axial conduction by using material of low thermal conductivity for the separating wall results in increased resistance to lateral heat flow, thereby reducing the overall thermal efficiency of the heat exchanger.

Since a wide range of materials ranging from copper and aluminium to plastics[4] are being used in heat exchanger design, it is time to analyse the heat transfer process including both axial conduction and lateral resistance by the separating wall. Chowdhury and Sarangi[5,6] have recently derived an expression for the optimum thermal conductivity of the separating wall in a concentric tube heat exchanger on the basis of minimum-entropy-generation principle:

$$\frac{k_{\text{fluid}}}{k_{\text{wall}}} = \frac{2}{Pe}\left[1 + \frac{C_{\text{tube}}}{C_{\text{shell}}}\right] \qquad (1)$$

where Pe is the Peclet Number of the fluid in the (inner) tube and C the heat capacity rate of the fluid (mass flow rate x specific heat). The second law analysis, while predicting the optimum thermal conductivity of the wall, is unable to help in computing the thermal efficiency for a given set of flow parameters. In this paper, we derive an expression for the efficiency of heat exchangers considering

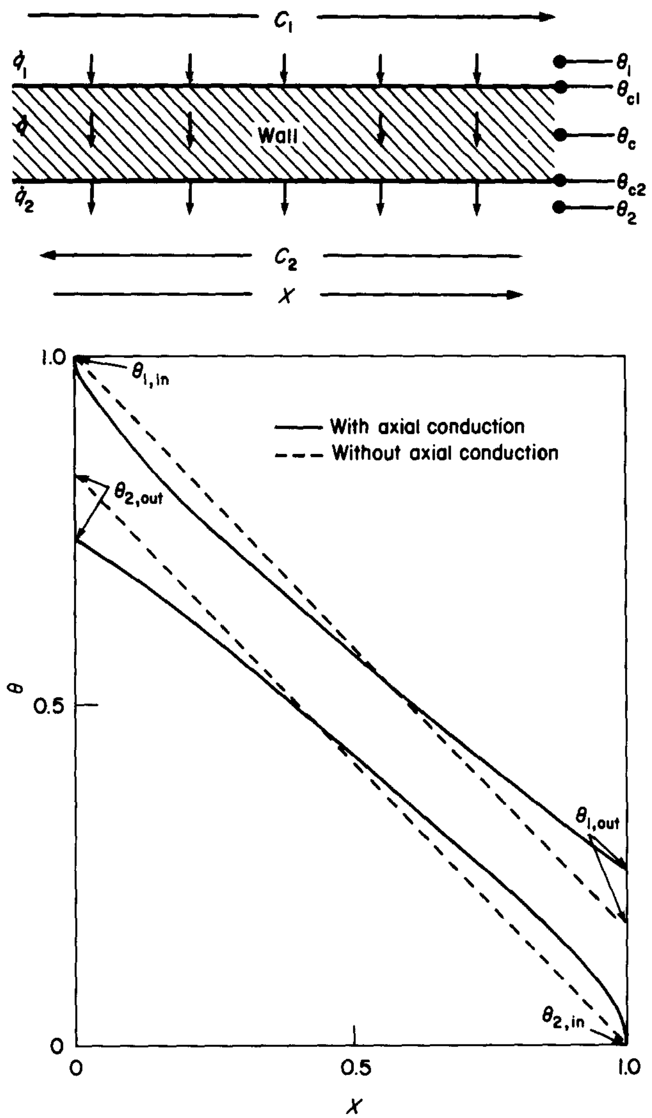

Fig. 1 Longitudinal section of the wall and a schematic temperature profile

both axial conduction and lateral resistance due to the separating wall in terms of relevant nondimensional parameters.

Theory

The following assumptions are usually valid for concentric tube counterflow heat exchangers used in cryogenic systems: The wall thickness is small compared to tube diameter. (Wall thickness is determined by the pressure difference between the two sides); both ends of the heat exchanger are adiabatic; and there is no heat leak from the surroundings. A longitudinal section of a portion of the heat-exchanger wall is shown in Fig. 1, explaining the nondimensional temperatures (θ) and the heat flow rates ($\dot{q}$) at different locations.

The energy conservation equations for the two fluid streams and the axial heat transfer equation may be expressed in dimensionless form as:

$$\left(\frac{\mathrm{d}}{\mathrm{d}x} + \beta_1\right)\theta_1 - \beta_1\theta_{c1} = 0 \qquad (2)$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}x} - \beta_2\right)\theta_2 + \beta_2\theta_{c2} = 0 \qquad (3)$$

$$-\frac{\mathrm{d}\theta_1}{\mathrm{d}x} + \frac{1}{\nu}\frac{\mathrm{d}\theta_2}{\mathrm{d}x} + \lambda\frac{\mathrm{d}^2\theta_c}{\mathrm{d}x^2} = 0 \qquad (4)$$

with boundary conditions:

$$x = 0: \ \theta_1 = 1 \ \text{and} \ \frac{\mathrm{d}\theta_c}{\mathrm{d}x} = 0 \qquad (5)$$

$$x = 1: \ \theta_2 = 0 \ \text{and} \ \frac{\mathrm{d}\theta_c}{\mathrm{d}x} = 0 \qquad (6)$$

The solution of (2) – (6) gives the temperature profiles in the two fluid streams and the wall. The overall inefficiency $(1-\epsilon)$ is then given by the expression

$$1-\epsilon = \frac{\begin{vmatrix} \frac{H_1-G_1}{r_1}\exp(-r_1) & \frac{H_2-G_2}{r_2}\exp(-r_2) & \frac{H_3-G_3}{r_3}\exp(-r_3) \\ \exp(-r_1) & \exp(-r_2) & \exp(-r_3) \\ 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} \frac{H_1\exp(-r_1)-G_1}{r_1} & \frac{H_2\exp(-r_2)-G_2}{r_2} & \frac{H_3\exp(-r_3)-G_3}{r_3} \\ \exp(-r_1) & \exp(-r_2) & \exp(-r_3) \\ 1 & 1 & 1 \end{vmatrix}}$$

where r_j, G_j and H_j are functions of the parameters β_1, β_2, β_3, ν and λ. The solution procedure and definition of r_j, G_j and H_j are given in Appendix 1.

For the special case of balanced flow ($\nu = 1$) condition, some of the terms in (7) become indeterminate. An alternative derivation is given in Appendix 2 resulting in the expression:

$$1-\epsilon = \beta_{0c} + B_{1c}\left(1 - \frac{1}{\beta_1} - \frac{1}{2\beta_c}\right) + \sum_{j=2}^{3} G_j B_{jc}\exp(-r_j) \qquad (8)$$

Conclusion

For tube-in-tube and shell-and-tube heat exchangers ϵ may be expressed in terms of λ and a geometrical parameter l/t as $\beta_c = \lambda(l/t)^2$. Then the efficiency ϵ, and hence the effective N_{tu} are functions of five parameters β_1, β_2, l/t, λ and ν. Fig. 2 (a–c) show the dependence of N_{tu} (effective) on λ, l/t, β_1 and ν for the case of balanced design ($\beta_1 = \beta_2$). Kroeger[3] has shown that variation of β_1/β_2 has a negligible effect on the overall inefficiency of the heat exchanger. The optimum value of λ and hence of k_{wall} agree exactly with (1). The symmetry of the $N_{tu,\,eff}$ vs ln λ curve suggests that a material should be chosen for the wall to keep the ratio k_{wall}/k_{opt} (or k_{opt}/k_{wall}, whichever is greater than unity) the lowest. The computed results also confirm that Kroeger's[3] results can be used to predict efficiency if the lateral resistance of the wall is incorporated into the total N_{tu}. In most practical cases the difference between Kroeger's results and exact calculation is negligible.

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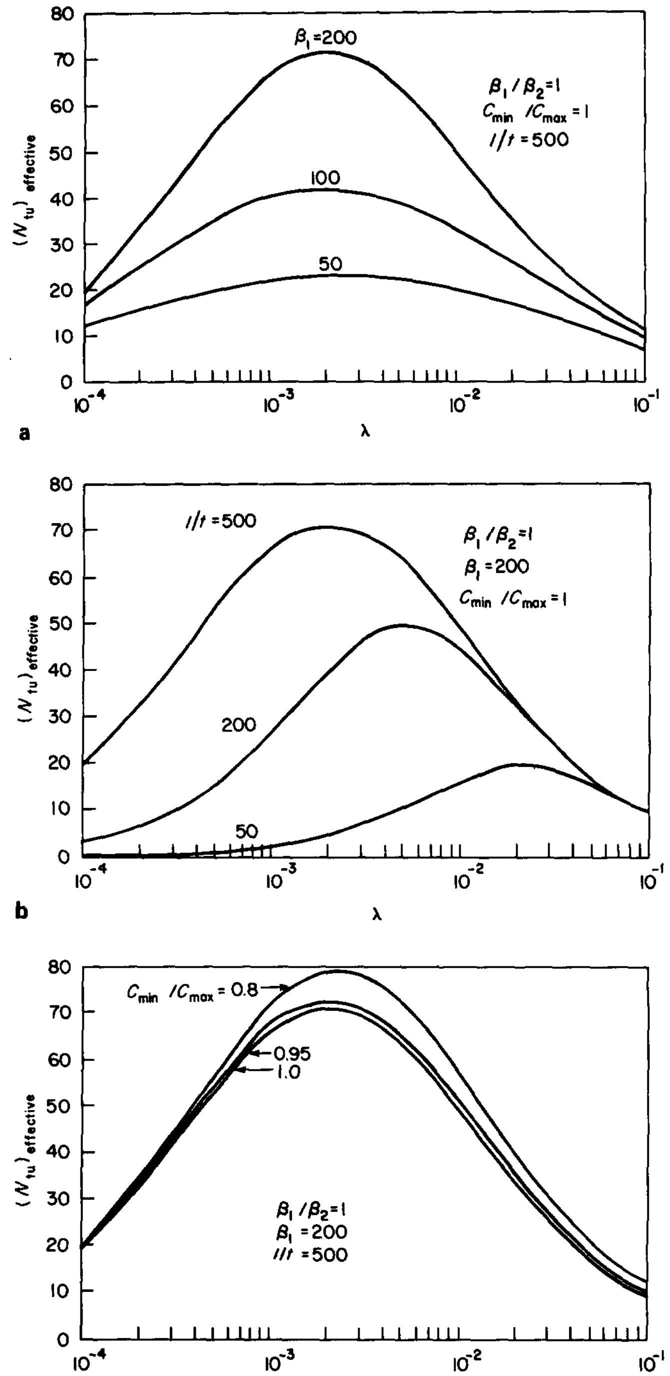

Fig. 2 a – Variation of $(N_{tu})_{effective}$ with β_1 and λ; b – variation of $(N_{tu})_{effective}$ with $1/t$ and λ; c – variation of $(N_{tu})_{effective}$ with ν and λ

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The authors are grateful to the Department of Atomic Energy (India) for a fellowship to KC and to the Department of Science and Technology for financial support.

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Appendix 1

Since axial conduction is small compared to the total lateral heat flow, the following may be assumed.

$$\theta_c = (\theta_{c1} + \theta_{c2})/2$$

and (A1)

$$\dot{q} = (\dot{q}_1 + \dot{q}_2)/2$$

or

$$\beta_1 (\theta_1 - \theta_{c1}) + \frac{\beta_2}{\nu} (\theta_{c2} - \theta_2) = 2\beta_c (\theta_{c1} - \theta_{c2}) \quad \text{(A2)}$$

Solving (A1) and (A2)

$$\theta_{c1} = \gamma_1\theta_1 - \gamma_2\theta_2 + \gamma_{c2}\theta_c$$

and (A3)

$$\theta_{c2} = -\gamma_1\theta_1 + \gamma_2\theta_2 + \gamma_{c1}\theta_c$$

where,

$$\gamma_1 = \beta_1/\beta^*$$
$$\gamma_2 = \beta_2/\nu\beta^*$$
$$\gamma_{c1} = (2\beta_1 + 4\beta_c)/\beta^*$$
$$\gamma_{c2} = (2\beta_2 + 4\nu\beta_c)/\nu\beta^*$$
$$\beta^* = \beta_1 + \beta_2/\nu + 4\beta_c$$

The governing equations (2) – (4), then transform to:

$$\left(\frac{d}{dx} + \beta_1 - \beta_1\gamma_1\right)\theta_1 + \beta_1\gamma_2\theta_2 - \beta_1\gamma_{c2}\theta_c = 0 \quad \text{(A4)}$$

$$\left(\frac{d}{dx} - \beta_2 + \beta_2\gamma_2\right)\theta_2 - \beta_2\gamma_1\theta_1 + \beta_2\gamma_{c1}\theta_c = 0 \quad \text{(A5)}$$

$$-\frac{d\theta_1}{dx} + \frac{d\theta_2}{dx} + \lambda\frac{d^2\theta_c}{dx^2} = 0 \quad \text{(A6)}$$

Following Kroeger,[3] the solution may be sought in the form:

$$\theta_k = A_{0k} + \sum_{j=1}^{3} A_{jk} \exp(-r_j x); \; k = 1, 2 \text{ and } c \tag{A7}$$

where r_j is the non-zero roots of the characteristic equation:

$$\begin{vmatrix} -r+\beta_1(1-\gamma_1) & \beta_1\gamma_2 & -\beta_1\gamma_{c2} \\ -\beta_2\gamma_1 & -r-\beta_2(1-\gamma_2) & \beta_2\gamma_{c1} \\ r & -r & \lambda r^2 \end{vmatrix} = 0 \tag{A8}$$

On substituting (A7) into the modified governing equations (A4) – (A6) and equating co-efficients of $\exp(-r_j x)$,

$$\left.\begin{aligned} A_{01} &= A_{02} = A_{0c} \\ A_{j1} &= G_j A_{jc}; j = 1, 2, 3 \\ \text{and} \\ A_{j2} &= H_j A_{jc}; j = 1, 2, 3 \end{aligned}\right| \tag{A9}$$

where,

$$G_j = \frac{4\beta_1\beta_2\beta_c + 2\beta_1(\beta_2/\nu + 2\beta_c)r_j}{4\beta_1\beta_2\beta_c + \{(1/\nu - 1)\beta_1\beta_2 + 4\beta_c(\beta_1 - \beta_2)\} r_j - \beta^* r_j^2}$$

and

$$H_j = \frac{4\beta_1\beta_2\beta_c - 2\beta_2(\beta_1 + 2\beta_c)r_j}{4\beta_1\beta_2\beta_c + \{(1/\nu - 1)\beta_1\beta_2 + 4\beta_c(\beta_1 - \beta_2)\} r_j - \beta^* r_j^2} \tag{A10}$$

The constants A_{jc} may be determined by substituting (A9) into the boundary conditions (5) and (6).

$$A_{0c} = \frac{1}{P} \sum_{Q} \frac{H_1}{r_1} \exp(-r_1) [\exp(-r_2) - \exp(-r_3)] \tag{A11}$$

and

$$A_{1c} = -\frac{1}{P} \frac{1}{r_1} [\exp(-r_2) - \exp(-r_3)] \tag{A12}$$

where $\sum_Q$ represents a cyclic sum over the subscripts 1, 2 and 3 and A_{2c} and A_{3c} obtained by a cyclic permutation of suffixes in (A12). In (A11) and (A12),

$$P = \sum_{Q} \frac{H_1 \exp(-r_1) - G_1}{r_1} [\exp(-r_2) - \exp(-r_3)] \tag{A13}$$

Inefficiency of the heat exchanger,

$$\begin{aligned} 1 - \epsilon = \theta_{1e} &= [\theta_1]_{x=1} \\ &= A_{01} + \sum_{j=1}^{3} A_{j1} \exp(-r_j) \\ &= A_{0c} + \sum_{j=1}^{3} G_j A_{jc} \exp(-r_j) \\ &= A_{0c} - \frac{1}{P} \sum_{\alpha} \frac{G_1}{r_1} \exp(-r_1) [\exp(-r_2) - \exp(-r_3) \end{aligned}$$

Using (A11),

$$1 - \epsilon = \frac{1}{P} \sum_{Q} \frac{H_1 - G_1}{r_1} \exp(-r_1) [\exp(-r_2) - \exp(-r_3) \tag{A14}$$

Equation (A14) may be expressed in the form of determinants to yield (7).

Appendix 2

In the case of balanced flow ($\nu = 1$), two of the roots of the characteristic equation (A8) are zero and the solution will be of the form:

$$\theta_k = B_{0k} + B_{1k} x + \sum_{j=2}^{3} B_{jk} \exp(-r_j x); k = 1, 2 \text{ and } c \tag{A15}$$

The arbitrary constants B_{jk} ($j = 0, 1, 2, 3$) may be determined by substituting (A15) into governing equations (A4) – (A6) and boundary conditions (5) and (6).

$$B_{01} = B_{0c} - \frac{B_{1c}}{\beta_1}\left(1 + \frac{\beta_1}{2\beta_c}\right)$$

$$B_{02} = B_{0c} + \frac{B_{1c}}{\beta_2}\left(1 + \frac{\beta_2}{2\beta_c}\right)$$

$$B_{11} = B_{12} = B_{1c}$$

$$B_{j1} = G_j B_{jc}; j = 2, 3$$

$$B_{j2} = H_j B_{jc}; j = 2, 3$$

$$B_{0c} = \frac{1}{Q}\left[-\left(\frac{1}{\beta_2} + \frac{1}{2\beta_c} + 1\right) \right.$$

$$r_2 r_3 [\exp(-r_2) - \exp(-r_3)] - H_2 r_3 \exp(-r_2)$$

$$\left. [1 - \exp(-r_3) + H_3 r_2 \exp(-r_3) [1 - \exp(-r_2)] \right]$$

$$B_{1c} = \frac{1}{Q}\left[r_2 r_3 \left[\exp(-r_2) - \exp(-r_3)\right]\right] \quad \text{(A16)}$$

$$B_{2c} = \frac{r_3}{Q}\left[1 - \exp(-r_3)\right]$$

$$B_{3c} = -\frac{r_2}{Q}\left[1 - \exp(-r_2)\right]$$

where,

$$Q = -\left(\frac{1}{\beta_1} + \frac{1}{\beta_2} + \frac{1}{\beta_c} + 1\right) r_2 r_3$$

$$\left[\exp(-r_2) - \exp(-r_3)\right]$$

$$+ \left[G_2 - H_2 \exp(-r_2)\right] r_3 \left[1 - \exp(-r_3)\right]$$

$$- \left[G_3 - H_3 \exp(-r_3)\right] r_2 \left[1 - \exp(-r_2)\right] \quad \text{(A17)}$$

Inefficiency of the heat exchanger:

$$1 - \epsilon = [\theta_1]_{x=1}$$

$$= B_{01} + B_{11} + \sum_{j=2}^{3} B_{j1} \exp(-r_j)$$

$$= B_{0c} + B_{1c}\left(1 - \frac{1}{\beta_1} - \frac{1}{2\beta_c}\right)$$

$$+ \sum_{j=2}^{3} G_j B_{jc} \exp(-r_j) \quad \text{(A18)}$$