

Effect of temperature-dependent specific heat of the working fluid on the performance of cryogenic regenerators

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The governing differential equations of a regenerator operating with a fluid with temperature-dependent specific heat have been formulated in terms of characteristic reduced parameters. A numerical solution of these equations is presented for several combinations of fluid-flow parameters using normal and parahydrogen as working fluid. It is observed that a constant specific heat model with the harmonic mean reduced length is adequate except at large values of reduced length or period and when the specific heat variation exceeds a factor of two over the temperature range. The matrix and gas exit temperature profiles, however, show a significant difference between the two models, which may be critical in some applications.

Keywords: cryogenics; regenerator; temperature; specific heat

Nomenclature

A	Heat transfer area per unit volume (m^2/m^3)
$A_{1,2,3}$	Intermediate variables defined in appendix
$B_{1,2}$	Intermediate variables defined in appendix
$C_{1,2,3}$	Intermediate variables defined in appendix
C	Specific heat ($\text{J kg}^{-1} \text{K}^{-1}$)
\bar{C}_p	Characteristic specific heat (C_p) of the fluid
E	Enthalpy (J kg^{-1})
G	Fluid mass velocity ($\text{kg m}^{-2} \text{s}^{-1}$)
h	Heat transfer coefficient ($\text{W m}^{-2} \text{K}^{-1}$)
H	$E/C_{p, \text{hm}}$ (K)
i, j	grid indices
K_1-K_5	Intermediate variables defined in appendix
L	Length of the regenerator (m)
N	Number of grid divisions in distance or time coordinate
P	Total cycle period (s)
R	Intermediate variable
t	Time coordinate (s)
T	Temperature (K)
y	Distance coordinate (m)

α	$\Delta T_D \Lambda_{\text{hm}}/2$
β	$\Delta T_D \Pi/2$
$\delta_{1,2,3}$	Constants used in convergence criteria
ε	Effectiveness
Λ	Reduced length = $hAL/G_g C_p$
$\bar{\Lambda}$	Characteristic reduced length
Π	Reduced period = $hAP/2\rho_m C_m$
ρ	Density (kg m^{-3})
θ	Integer part of temperature, T (K)

Subscripts

c	Cold
D	Dimensionless
eq	Equivalent
E	Enthalpy
g	Working fluid
h	Hot
hm	Harmonic mean
i	Inlet
m	Matrix
o	Outlet
T	Temperature

A regenerator essentially consists of a porous medium called the matrix, through which the hot and cold fluids flow alternately. The exchange of energy between the two fluid streams takes place by transfer of heat from the hot fluid to the matrix and its subsequent transfer to the cold fluid. That is why regenerators are often termed as 'storage-type heat exchangers' in contrast with recuperators or 'transfer-type heat exchangers'.

Regenerative heat exchangers have been used in hot-air engines, Cowper stoves in steel making, gas turbines and air separation systems. In recent years, they have also found extensive use in small cryogenic refrigerators based on Stirling, Gifford-McMahon, and similar cycles, in which they constitute the single most important component. The classical design procedure for regenerators is given by Hausen¹. The effectiveness is expressed graphically in terms of two dimensionless parameters: reduced length, Λ , and reduced period, Π , defined as

$$\Lambda = \frac{hAL}{G_g C_p} \quad \text{and} \quad \Pi = \frac{hAP}{2\rho_m C_m}$$

where

h = heat transfer coefficient

A = heat transfer area per unit volume

L = length of regenerator

P = total period of a cycle

G_g = fluid mass velocity

ρ_m = density of the matrix

C_m, C_p = specific heats of matrix and fluid respectively

In many regenerator applications, the working fluid undergoes a large temperature change from inlet to exit. In such cases due consideration must be given to the fact that fluid properties, such as viscosity, thermal conductivity and specific heat, may vary considerably along the bed. Among the working gases used at cryogenic temperatures, hydrogen shows significant variation of specific heat with temperature. As hydrogen is receiving increasing attention as a future automotive fuel and (in many countries where helium is not easily available) as a working fluid in Stirling and other regenerative cryorefrigerators, it is time to develop a design procedure for regenerators with temperature-dependent specific heat of the fluid. The heat capacity of the matrix is also temperature dependent², often more strongly than that of the fluid. In this Paper, however, it is assumed to be constant to highlight the effects of variable fluid specific heat.

Since the fluid specific heat, C_p , is a function of temperature, it is not possible to define a unique dimensionless length, Λ . It is possible, however, to define a characteristic dimensionless length, $\bar{\Lambda} = hAL/G_g \bar{C}_p$ to characterize a regenerator, \bar{C}_p being a characteristic specific heat.

In this Paper, the governing differential equations have been formulated in terms of the characteristic dimensionless parameters $\bar{\Lambda}$ and Π . The resulting equations are a pair of coupled hyperbolic partial differential equations³. They have been reduced to finite difference form and solved numerically. Temperature profiles and effectiveness values for a wide range of flow parameters have been presented

and compared with calculations based on constant specific heat.

Governing equations

In deriving the governing differential equations for the regenerator, the following assumptions are made:

- 1 the fluid flow through the regenerator is parallel and uniform throughout any cross-section;
- 2 the thermal conductivity of the matrix is zero in the direction of fluid flow and infinite perpendicular to it. Therefore, the regenerator may be characterized by the temperature profile along the flow axis, the temperature being uniform over any cross-section;
- 3 the convective heat transfer coefficient is constant throughout the regenerator;
- 4 the thermal properties of the fluid and matrix materials, except fluid specific heat, are constant;
- 5 fluid hold-up and pressure cycling have no effect on the performance of the regenerator;
- 6 no phase change of the working fluid takes place within the regenerator;
- 7 the boundaries are adiabatic and there is no heat exchange with the surroundings;
- 8 the regenerator is in balanced operation, i.e. $G_{g,h} = G_{g,c}$ and $P_h = P_c = P/2$;
- 9 regular periodic conditions have been established for all matrix elements.

On the basis of these idealizations, the energy conservation relations over a differential element of the regenerator may be expressed as

$$G_g \frac{\partial E_g}{\partial y} = hA(T_m - T_g) \quad (1a)$$

and

$$\rho_m C_m \frac{\partial T_m}{\partial t} = hA(T_g - T_m) \quad (1b)$$

where

E_g = enthalpy of the fluid

T_m, T_g = temperatures of matrix and fluid, respectively

y = distance coordinate

t = time coordinate

The associated boundary and reversal conditions are:

$$\begin{aligned} T_g(y=0, t) &= T_{hi} & \text{for } 0 < t \leq P/2 \\ T_g(y=L, t) &= T_{ci} & \text{for } P/2 < t \leq P \\ T_m(y, t+P) &= T_m(y, t) & \text{for } 0 < y \leq L \end{aligned} \quad (2)$$

where T_{hi} and T_{ci} are the inlet temperatures of the hot and cold fluids, respectively.

Defining non-dimensional length and time coordinates

$$y_D = y/L \quad \text{and} \quad t_D = 2t/P$$

the governing equations reduce to

$$\frac{\partial H}{\partial y_D} = \bar{\Lambda}(T_m - T_g) \quad (3a)$$

$$\frac{\partial T_m}{\partial t_D} = \Pi(T_g - T_m) \quad (3b)$$

The dimensionless parameters, $\bar{\Lambda}$ and Π , are defined earlier and the variable H , which has the dimension of temperature, is defined as E/\bar{C}_p . Equations (3a) and (3b) are identical to those in constant specific heat problems except that the variable H substitutes for T_g in Equation (3a). The boundary and reversal conditions now reduce to

$$\begin{aligned} T_g(y_D = 0, t_D) &= T_{hi} & \text{for } 0 < t_D \leq 1 \\ T_g(y_D = 1, t_D) &= T_{ci} & \text{for } 1 < t_D \leq 2 \\ T_m(y_D, t_D + 2) &= T_m(y_D, t_D) & \text{for } 0 < y_D \leq 1 \end{aligned} \quad (4)$$

Harmonic mean reduced length

In the previous section, the governing equations have been developed in terms of a characteristic reduced length, $\bar{\Lambda}$, defined on the basis of a characteristic specific heat, \bar{C}_p . For maximum convenience, $\bar{\Lambda}$ should be chosen in such a way that the effectiveness computed on the basis of a constant specific heat, \bar{C}_p , is close to that by exact calculation.

The reduced length for an arbitrary differential element of the regenerator is given as

$$d\Lambda = \frac{hA \, dy}{G_g C_p}$$

This expression may be integrated over the length of the regenerator to give the total equivalent reduced length.

$$\Lambda_{eq} = \int_0^L \frac{hA \, dy}{G_g C_p} \quad (5)$$

In the integrand in Equation (5) the variable C_p is a function of temperature T_g and, hence, of position y and time t . Since the exact temperature profile cannot be known, a linear temperature profile with temperatures equal to T_{hi} and T_{ci} at the ends may be taken as the first approximation. Then,

$$\frac{dT_g}{dy} = - \frac{T_{hi} - T_{ci}}{L}$$

Substituting in Equation (5)

$$\Lambda_{eq} = \frac{hA}{G_g} \int_{T_{hi}}^{T_{ci}} \left(\frac{dT_g}{dy} \right)^{-1} \frac{dT_g}{C_p(T_g)}$$

$$\begin{aligned} &= \frac{hA}{G_g} \frac{L}{T_{hi} - T_{ci}} \int_{T_{ci}}^{T_{hi}} \frac{dT_g}{C_p(T_g)} \\ &= \frac{hAL}{G_g C_{p,hm}} = \Lambda_{hm} \end{aligned} \quad (6)$$

$C_{p,hm}$ is the harmonic mean specific heat of the fluid over the temperature range $T_{ci} - T_{hi}$ and is defined as

$$C_{p,hm} = \left[\frac{1}{T_{hi} - T_{ci}} \int_{T_{ci}}^{T_{hi}} \frac{dT_g}{C_p(T_g)} \right]^{-1} \quad (7)$$

Thus, with the assumption of a linear temperature profile the equivalent reduced length of the regenerator is equal to its harmonic mean reduced length. This assumption is never strictly valid because (a) a temperature-dependent specific heat precludes a linear temperature profile; and (b) at finite effectiveness the average end temperatures will not be equal to T_{hi} and T_{ci} .

Although Λ_{hm} will not give the true effectiveness of a regenerator, it is still the best initial estimate and will be used throughout this Paper to characterize a regenerator.

Numerical solution

Numerical scheme

By replacing the characteristic reduced length, $\bar{\Lambda}$, by the harmonic mean reduced length, Λ_{hm} , the governing equations can be written as

$$\frac{\partial H}{\partial y_D} = \Lambda_{hm}(T_m - T_g) \quad (8a)$$

and

$$\frac{\partial T_m}{\partial t_D} = \Pi(T_g - T_m) \quad (8b)$$

The pair of Equations (8a) and (8b) need to be expressed in a suitable numerical scheme for computer solution. We follow the trapezoidal method⁴ to express the partial differential equations in finite difference form.

Referring to the grid array shown in *Figure 1*, the following relations may be written

$$H(i+1, j+1) = H(i, j+1) +$$

$$\frac{\Delta y_D}{2} \left\{ \frac{\partial H}{\partial y_D}(i+1, j+1) + \frac{\partial H}{\partial y_D}(i, j+1) \right\} \quad (9a)$$

$$T_m(i+1, j+1) = T_m(i+1, j) +$$

$$\frac{\Delta t_D}{2} \left\{ \frac{\partial T_m}{\partial t_D}(i+1, j+1) + \frac{\partial T_m}{\partial t_D}(i+1, j) \right\} \quad (9b)$$

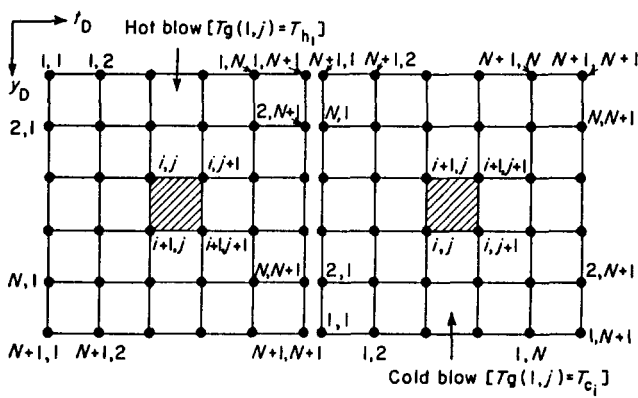


Figure 1 Grid array for numerical solution

On eliminating the partial derivatives between Equations (8) and (9)

$$H(i+1, j+1) - H(i, j+1) = \alpha \{ T_m(i+1, j+1) + T_m(i, j+1) - T_g(i+1, j+1) - T_g(i, j+1) \} \quad (10a)$$

and

$$T_m(i+1, j+1) - T_m(i+1, j) = \beta \{ T_g(i+1, j+1) + T_g(i+1, j) - T_m(i+1, j+1) - T_m(i+1, j) \} \quad (10b)$$

where

$$\alpha = \frac{\Delta y_D}{2} \Lambda_{hm} \text{ and } \beta = \frac{\Delta t_D}{2} \Pi$$

Equations (10a) and (10b) are the basic finite difference equations to be solved. This involves the determination of $H(i, j)$ from the corresponding $T_g(i, j)$ and *vice versa* at every point. These interpolations within the innermost loop of the computer program require excessive computer time. To achieve quick interpolation, the following technique is adopted.

The values of specific heat, C_p , are tabulated in computer memory at every integer value of temperature. Then, one can read the specific heat, C_p , at any integral temperature, Θ as the Θ^{th} element of this array. The value of C_p at any fractional temperature may be calculated by linear interpolation between the two neighbouring integral temperatures. From the basic relations between enthalpy, E , temperature, T_g , and specific heat, C_p , one can write

$$E(T_g) = E(\Theta_g) + \int_{\Theta_g}^{T_g} C_p(T_g) dT_g$$

where Θ_g is the integer part of the temperature T_g . Since the interval $T_g - \Theta_g < 1$ K, one can approximate $C_p(T_g)$ by a mean value over the interval Θ_g and $\Theta_g + 1$, and write

$$E(T_g) = E(\Theta_g) + \frac{C_p(\Theta_g) + C_p(\Theta_g + 1)}{2} (T_g - \Theta_g) \quad (11)$$

Similarly, $H(T_g)$, which is equal to $E(T_g)/C_{p, hm}$, can be expressed in terms of the integer part of temperature T_g as

$$H(T_g) = H(\Theta_g) + \frac{C_p(\Theta_g) + C_p(\Theta_g + 1)}{2 C_{p, hm}} (T_g - \Theta_g) \quad (12)$$

By defining a quantity $R(\Theta_g)$ as

$$R(\Theta_g(i, j)) = [C_p(\Theta_g(i, j)) + C_p(\Theta_g(i, j)+1)] / 2 C_{p, hm} \quad (13)$$

Equation (12) can be written as

$$H(i, j) = H(\Theta_g(i, j)) + R(\Theta_g(i, j)) [T_g(i, j) - \Theta_g(i, j)] \quad (14)$$

where $H(i, j)$ is the same as $H(T_g(i, j))$. By using Equation (14) and the parameters B , C and K defined in the Appendix, Equation (10) may be expressed in the form

$$T_g(i+1, j+1) = K_1 T_g(i, j+1) + K_2 T_m(i, j+1) + K_3 T_g(i+1, j) + K_4 T_m(i+1, j) + K_5 \quad (15a)$$

and

$$T_m(i+1, j+1) = B_1 T_m(i+1, j) + B_2 \{ T_g(i+1, j+1) + T_g(i+1, j) \} \quad (15b)$$

Equations (15a) and (15b) are valid for all grid points except the first column and the first row (*Figure 1*) in each half cycle, i.e. $i = 1$ or $j = 1$.

The initial boundary conditions are obtained partly from Equations (4) and partly by application of Equations (10) and (14).

Boundary relations ($i = 1$).

$$T_g(1, j) = T_{h_i}; 1 < j < N+1 \text{ during hot blow period} \quad (16a)$$

$$T_g(1, j) = T_{c_i}; 1 < j < N+1 \text{ during the cold blow period} \quad (16b)$$

$$T_m(1, j+1) = B_1 T_m(1, j) + B_2 \{ T_g(1, j+1) + T_g(1, j) \} \quad (17)$$

where N is the number of grid divisions in both distance and time coordinates.

Initial and reversal relations ($j = 1$).

$$T_m(i, 1) = T_m(N+2-i, N+1) \text{ of previous half cycle.}$$

$$1 \leq i \leq N+1 \quad (18)$$

$$T_g(i+1, 1) = A_1 T_g(i, 1) + A_2 \{ T_m(i+1, 1) + T_m(i, 1) \} + A_3 \quad (19)$$

The equations derived above for the case of temperature-dependent specific heat are equally applicable to the constant specific heat problem. In the latter case the parameter, R is set equal to unity. The resulting equations are identical to those derived by Willmott.

Solution procedure

To start the solution process, an initial matrix temperature profile has to be assumed. In our case, this profile is provided by the solution of the constant specific heat problem for the same value of Λ_{hm} and Π . The initial matrix temperature profile for the constant specific heat problem, on the other hand, is chosen by the following method.

An initial estimate of the overall efficiency of the regenerator is made from Tipler's formula⁵

$$\epsilon_T = \frac{\Lambda_{hm}}{\Pi} \tanh \left(\frac{\Pi}{\Lambda_{hm} + 2} \right)$$

The matrix temperature profile at the start of the hot blow period (which is the same as that at the end of the cold blow period) is assumed⁶ to be a straight line between the end temperatures, $T_{hi} - 1.25(1 - \epsilon_T)$ ($T_{hi} - T_{ci}$) and $T_{ci} + .25(1 - \epsilon_T)(T_{hi} - T_{ci})$

$$T_m(i, 1) = T_{hi} - \left\{ 1.25(1 - \epsilon_T) + \frac{i-1}{N} \left(1 - 1.5(1 - \epsilon_T) \right) \right\} (T_{hi} - T_{ci}) \quad (20)$$

The matrix temperature profile for $j = 1$ being known, the gas temperature profile is computed using Equation (19). Now Equation (15) and boundary conditions (16) and (17) are used to compute the entire grid array for the hot blow period. On completion of the hot blow period, the flow reverses. The initial matrix temperature profile is determined by using Equation (18) and the entire grid array is computed in the same way as for the hot blow period. The whole process is repeated till a steady cyclic condition is established.

In the case of temperature dependent specific heat, equal division of the geometrical length does not result in equal division of the reduced length. Assuming the temperature profile of the constant specific heat problem, the reduced length at the i^{th} grid location is computed as

$$\Lambda(i) = \sum_{k=2}^i \frac{\Lambda_{hm}}{N} \frac{C_{p,hm}}{C_p(\Theta_g(k, 1))} \quad (21)$$

The temperature at $\Lambda(i)$ is again determined from the constant specific heat results and this is taken as the starting temperature, $T_m(i, 1)$, at the grid point $(i, 1)$. The above procedure considerably reduces the iterations necessary for convergence and results in saving of computer time.

The initial matrix temperature profile being known, the other grid points are computed by using Equations (15)–(17). The process is repeated until certain convergence criteria are satisfied.

Convergence criteria and effectiveness

Following Willmott⁴, the mean exit temperature/enthalpy of the gas stream over a half cycle is calculated using the Gregory formula as

$$\begin{aligned} \bar{\Phi} = & \frac{1}{N} \left\{ \frac{1}{2} \Phi_{N+1,1} + \sum_{j=2}^N \Phi_{N+1,j} + \frac{1}{2} \Phi_{N+1,N+1} \right. \\ & - \frac{1}{12} \left(\nabla \Phi_{N+1,N+1} - \Delta \Phi_{N+1,1} \right) \\ & - \frac{1}{24} \left(\nabla^2 \Phi_{N+1,N+1} + \Delta^2 \Phi_{N+1,1} \right) \\ & \left. - \frac{19}{720} \left(\nabla^3 \Phi_{N+1,N+1} - \Delta^3 \Phi_{N+1,1} \right) \right\} \quad (22) \end{aligned}$$

where Φ refers to the variables E or T_g . Δ and ∇ refer to forward and backward differences, respectively. The effectiveness is computed separately for hot and cold blow periods

$$\epsilon_{E,h} = \frac{E_{hi} - \bar{E}_{ho}}{E_{hi} - E_{ci}} \quad \text{and} \quad \epsilon_{E,c} = \frac{\bar{E}_{co} - E_{ci}}{E_{hi} - E_{ci}}$$

Two types of convergence criteria have been used: (a) *Energy balance criterion*⁷ – the net energy gain of the regenerator over one complete cycle has to be zero, i.e. $\epsilon_{E,h} = \epsilon_{E,c}$. Hence, $|1 - \epsilon_{E,h}/\epsilon_{E,c}| \leq \delta_1$ is used as a criterion of convergence; (b) *steady state criterion* – to insure that steady cyclic conditions are established, the condition

$$\left| \left(\frac{\epsilon_{E,h} + \epsilon_{E,c}}{2} \right)_f - \left(\frac{\epsilon_{E,h} + \epsilon_{E,c}}{2} \right)_{f-1} \right| \leq \delta_2$$

is used as a convergence criterion. The subscript f refers to the f^{th} cycle.

Criterion (b) can be alternatively expressed in terms of the average exit temperatures as

$$|(\bar{T}_{gho})_f - (\bar{T}_{gho})_{f-1}| \leq \delta_3$$

and

$$|(\bar{T}_{gco})_f - (\bar{T}_{gco})_{f-1}| \leq \delta_3$$

In this Paper, δ_1 and δ_3 have been taken as 0.0001 and 0.001 K, respectively.

The number of grid divisions along the length coordinate and that in time coordinate over a half cycle is taken to be the same in this Paper. Consequently, the truncation error associated with each grid point is⁸ $0[(\Delta r_D)^3]$. The additional error introduced by discretization of specific heat data is not significant. Initially, a coarse grid is adopted and the equations are solved until the convergence criteria are satisfied. The resulting matrix temperature profile serves as the initial condition for the next grid rise, which is taken as half the previous one. The new grid points are filled by linear interpolation between neighbouring points. The equations are solved and the efficiency determined for at least three grid sizes, the difference of the efficiencies computed with the two finest grids being < 0.001 . The results with the three finest grid sizes are extrapolated to zero grid size by the relation³

$$\epsilon = \frac{\frac{N_1 - N_2}{N_3} \epsilon_3 + \frac{N_2 - N_3}{N_1} \epsilon_1 + \frac{N_3 - N_1}{N_2} \epsilon_2}{\frac{N_1 - N_2}{N_3} + \frac{N_2 - N_3}{N_1} + \frac{N_3 - N_1}{N_2}} \quad (23)$$

where ϵ_1 , ϵ_2 , ϵ_3 are the efficiencies corresponding to the number of grid divisions N_1 , N_2 , and N_3 .

Results

The numerical scheme was programmed into a Burroughs B6700 computer and the efficiencies computed for selected values of temperature range, pressure, characteristic reduced length, Λ , and reduced period, Π . The detailed computer program has been given elsewhere⁶. The specific heat data on normal hydrogen and parahydrogen have been taken from Reference 9 and are shown in Figures 2 and 3. Five combinations of working fluid, temperature range and pressure were selected. They have been given in Table 1.

Table 1 Combinations of fluid parameters used in computation

	Working fluid	Temperature range (K)	Pressure (MPa)	Computed $C_{p,hm}$ ($\text{KJ kg}^{-1} \text{K}^{-1}$)
I	Parahydrogen	21-77	0.1	10.85
II	Parahydrogen	28-77	0.5	11.78
III	Parahydrogen	21-77	2.0	14.93
IV	Normal hydrogen	77-300	0.1	12.92
V	Normal hydrogen	77-300	2.0	13.20

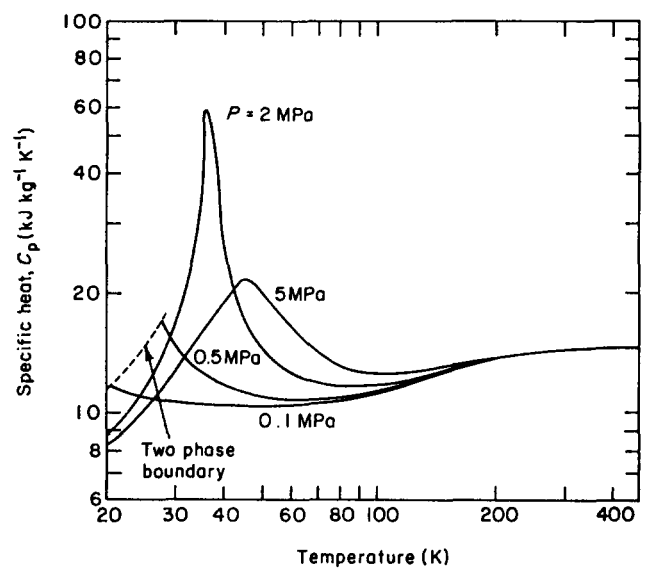


Figure 2 Specific heat, C_p of normal hydrogen at constant pressure⁹

Effectiveness at $\Lambda_{hm} = 10, 20, 50, 100$ and 200 , and $\Pi/\Lambda_{hm} = 0.1, 0.2, 0.4, 0.6, 0.8$ and 1.0 at each of the five combinations of Table 1 are presented in Table 2.

It may be observed from Table 2 that in four out of the five fluid flow parameter combinations the variable specific heat results are close to those with constant specific heat. The results, however, are quite different in the case of combination III: parahydrogen at 2 MPa and between 21 and 77 K. This case has been

Table 2 Computed effectiveness (%) shown against Λ_{hm} , Π and fluid flow parameters

Λ_{hm}	Π/Λ_{hm}	Constant C_p	Fluid flow parameter combination (Table 1)				
			I	II	III	IV	V
10	0.1	83.22	83.34	83.37	75.52	82.91	83.03
	0.2	82.89	83.02	83.04	74.83	82.56	82.69
	0.4	81.70	81.85	81.88	72.40	81.31	81.46
	0.6	79.84	80.05	80.10	69.06	79.35	79.54
	0.8	77.24	77.52	77.62	65.23	76.57	76.82
	1.0	73.76	74.13	74.28	61.13	72.91	73.22
20	0.1	90.78	90.82	90.79	85.70	90.62	90.68
	0.2	90.44	90.48	90.44	84.68	90.27	90.34
	0.4	89.35	89.39	89.33	81.30	89.13	89.22
	0.6	87.67	87.75	87.69	76.82	87.33	87.47
	0.8	85.10	85.29	85.27	71.93	84.53	84.75
	1.0	81.14	81.49	81.55	66.81	80.22	80.56
50	0.1	96.04	96.04	96.00	93.64	95.97	96.00
	0.2	95.79	95.79	95.73	92.58	95.73	95.76
	0.4	95.07	95.05	94.94	88.60	94.99	95.02
	0.6	93.91	93.89	93.73	83.08	93.76	93.83
	0.8	91.93	91.98	91.80	77.37	91.51	91.68
	1.0	87.93	88.25	88.15	71.48	86.85	87.24
100	0.1	97.95	97.95	97.92	96.68	97.92	97.93
	0.2	97.79	97.78	97.74	95.87	97.76	97.78
	0.4	97.33	97.30	97.20	91.62	97.30	97.32
	0.6	96.56	96.51	96.32	85.45	96.49	96.52
	0.8	95.13	95.10	94.83	79.54	94.83	94.96
	1.0	91.41	91.80	91.41	73.54	90.16	90.61
200	0.1	98.95	98.94	98.92	98.29	98.94	98.94
	0.2	98.86	98.96	98.83	97.73	98.84	98.85
	0.4	98.61	98.58	98.50	93.23	98.60	98.60
	0.6	98.13	98.07	97.90	86.79	98.10	98.12
	0.8	97.21	97.14	96.79	80.91	96.94	97.11
	1.0	93.91	94.16	93.66	75.12	92.41	92.94

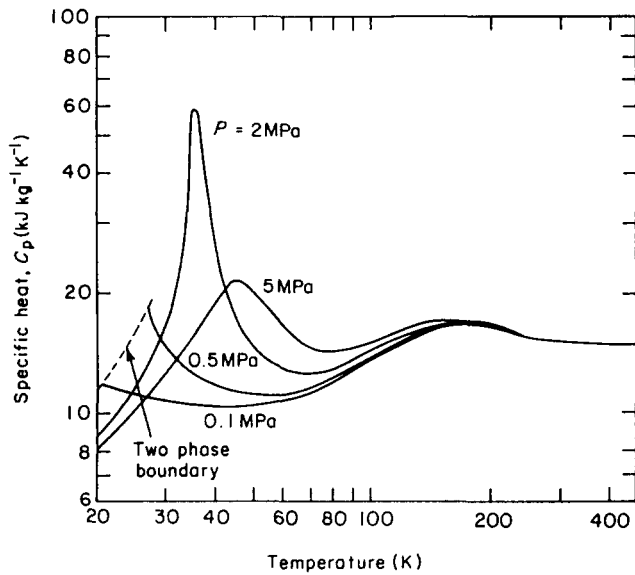


Figure 3 Specific heat, C_p , of parahydrogen at constant pressure⁹

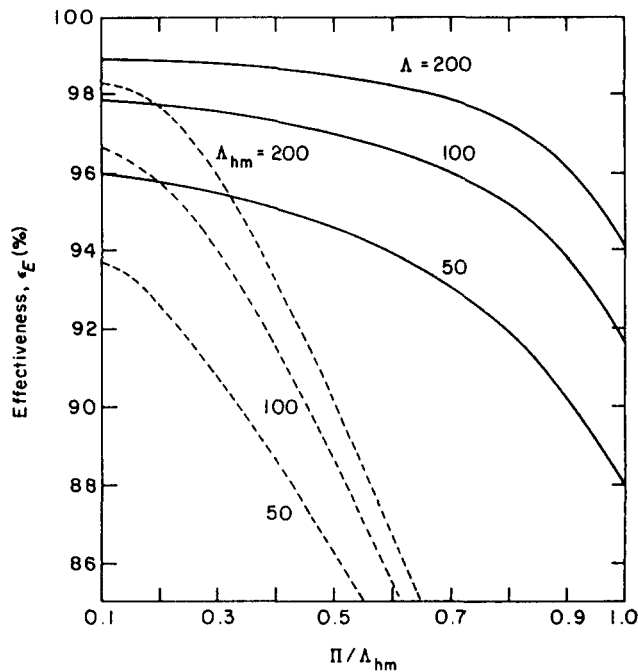


Figure 4 Efficiency of regenerator with variable specific heat of the working fluid compared with constant specific heat calculation. Parahydrogen, 77–21 K, 2.0 MPa, ———, Constant specific heat; - - -, variable specific heat

illustrated in *Figure 4*. In cryogenic practice, most regenerators operate with high efficiency. Hence, it is more appropriate to compare the inefficiency. The marginal change in inefficiency $(1 - \epsilon_E)$ between variable and constant specific heat calculations have been shown for fluid flow parameters III and V in *Figures 5* and *6*, respectively. It may be observed that the difference between the two calculations is very high in the case of combination III, but is significant for combination V only at high Λ_{hm} and Π . *Figures 2* and *3* show that in the case of combination III, the specific heat varies by a factor of nine over the temperature range of interest, but in all other cases it is limited to a factor of two. Hence, we may conclude that if the specific heat variation within the temperature range of interest is limited to a factor of two, a constant specific

heat calculation with $\Lambda = \Lambda_{hm}$ will give accurate results. But at higher variation or at high values of Λ_{hm} or Π , an exact computation is necessary.

Although the overall effectiveness is correctly predicted by the constant specific heat model (with $\Lambda = \Lambda_{hm}$), in most cases the actual temperature profile in the regenerator shows significant variation. This has been illustrated in *Figures 7* and *8* for two specific examples. The deviation of the temperature profile from that with constant specific heat is high for the case where the specific heat itself varies over a wide range.

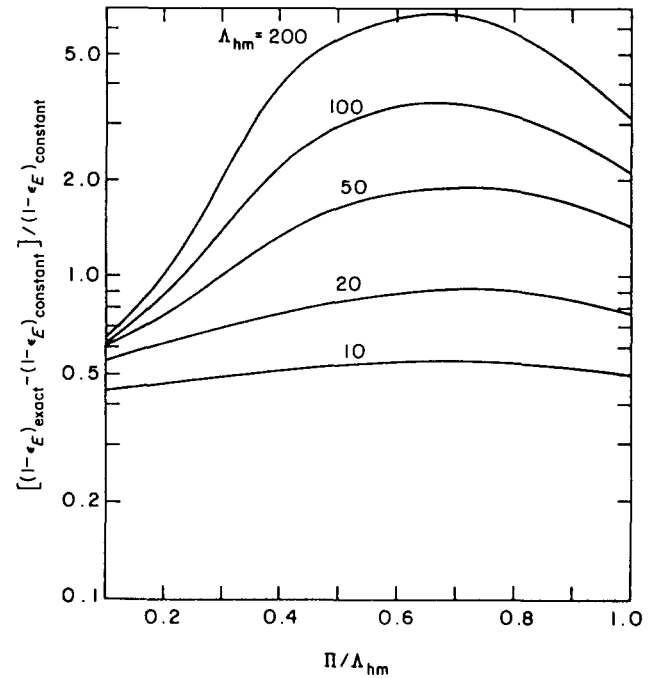


Figure 5 The difference in inefficiency between exact and constant specific heat calculations shown against Λ_{hm} and Π/Λ_{hm} . Parahydrogen, 77–21 K, 2.0 MPa

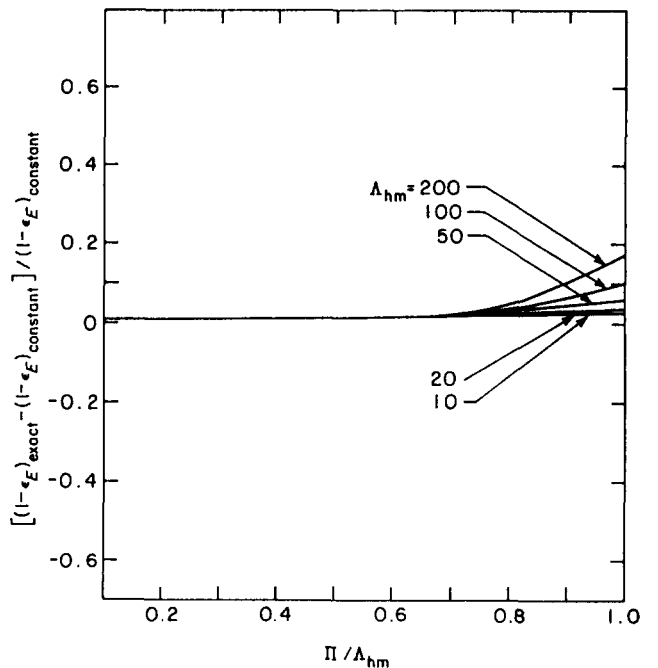


Figure 6 The difference in inefficiency between exact and constant specific heat calculations shown against Λ_{hm} and Π/Λ_{hm} . Normal hydrogen 300–77 K, 2.0 MPa

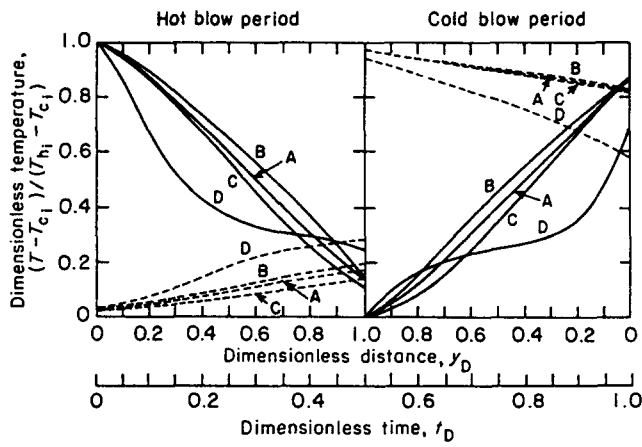


Figure 7 Matrix and gas exit temperature profiles under steady cyclic conditions. $\Lambda_{hm} = 20$, $\Pi = 4$. A: Constant specific heat; B: Normal hydrogen, 300–77 K, 2.0 MPa; C: Parahydrogen, 77–28 K, 0.5 MPa; D: Parahydrogen, 77–21 K, 2.0 MPa; —: Matrix temperature *versus* dimensionless length at the end of hot and cold periods; - - - -: Gas exit temperature *versus* dimensionless time

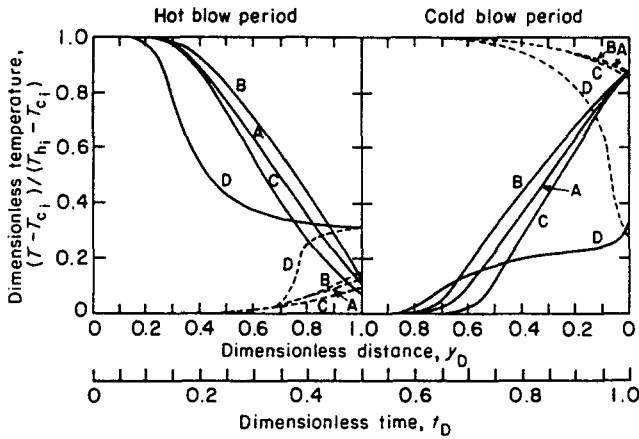


Figure 8 Matrix and gas exit temperature profiles under steady cyclic conditions. $\Lambda_{hm} = 100$, $\Pi = 40$. A: Constant specific heat; B: normal hydrogen, 300–77 K, 2.0 MPa; C: parahydrogen, 77–28 K, 0.5 MPa; D: parahydrogen, 77–21 K, 2.0 MPa. —: Matrix temperature *versus* dimensionless length at the end of hot and cold blow periods; - - - -: gas exit temperature *versus* dimensionless time

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Appendix

Definition of coefficients used in the finite difference equations

$$A_1 = \frac{R(\Theta_g(i, j)) - \alpha}{R(\Theta_g(i+1, j)) + \alpha}$$

$$A_2 = \frac{\alpha}{R(\Theta_g(i+1, j)) + \alpha}$$

$$A_3 = \frac{\{H(\Theta_g(i, j)) - \Theta_g(i, j) \cdot R(\Theta_g(i, j)) - H(\Theta_g(i+1, j))\}}{R(\Theta_g(i+1, j)) + \alpha} + \frac{\Theta_g(i+1, j) \cdot R(\Theta_g(i+1, j))}{R(\Theta_g(i+1, j)) + \alpha}$$

$$B_1 = \frac{1 - \beta}{1 + \beta}$$

$$B_2 = \frac{\beta}{1 + \beta}$$

$$C_1 = \frac{R(\Theta_g(i, j+1)) - \alpha}{R(\Theta_g(i+1, j+1)) + \alpha}$$

$$C_2 = \frac{\alpha}{R(\Theta_g(i+1, j+1)) + \alpha}$$

$$C_3 = \frac{\{H(\Theta_g(i, j+1)) - \Theta_g(i, j+1) \cdot R(\Theta_g(i, j+1))\}}{R(\Theta_g(i+1, j+1)) + \alpha}$$

$$- \frac{H(\Theta_g(i+1, j+1)) + \Theta_g(i+1, j+1) \cdot R(\Theta_g(i+1, j+1))}{R(\Theta_g(i+1, j+1)) + \alpha}$$

$$K_1 = \frac{C_1}{1 - C_2 B_2}$$

$$K_2 = \frac{C_2}{1 - C_2 B_2}$$

$$K_3 = \frac{C_2 B_2}{1 - C_2 B_2}$$

$$K_4 = \frac{C_2 B_1}{1 - C_2 B_2}$$

$$K_5 = \frac{C_3}{1 - C_2 B_2}$$