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Thermodynamic optimization of regenerators

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This paper presents a second law of thermodynamics analysis technique for regenerators in single blow operation. Unlike earlier analyses, this technique accounts for the gas and matrix temperature variation along the temporal and spatial coordinates. The results of this study indicate that: 1, well defined optima exist with regard to charging time and number of heat transfer units for maximizing the useful work stored; and 2, whereas in earlier analyses a finite optimum charging time results as N_{tu} approaches infinity, in the present analysis a square wave of specific irreversibility propagates in the medium. To obtain an appropriate analytical expression, this analysis is approximated to suit a low N_{tu} regenerator.

Keywords: thermodynamics; regenerators; optimization

Nomenclature

A	Heat transfer area of matrix (m^2)
C_p	Specific heat of fluid (J kg^{-1} K^{-1})
C_s	Specific heat of solid/matrix (J kg^{-1} K^{-1})
E	Exergy content of fluid ($=W_{min}$) (J)
f	Friction factor
F_o, F_1, G_o, G_1	Special functions given in Appendix
G	Mass velocity of fluid (kg m^{-2} s^{-1})
h	Heat transfer coefficient (W m^{-2} K^{-1})
I	Irreversibility (J)
$\dot{I}$	Irreversibility rate (W)
L	Length of solid/matrix bed (m)
$\dot{m}$	Mass flow rate of fluid (kg s^{-1})
m_f	Mass of fluid contained in bed (kg)
M_s	Total mass of solid (kg)
N_f	Dimensionless friction coefficient $\{=[G/(2\rho P_o)^{1/2}][(R/C_p)(f/N_{st})]^{1/2}\}$
N_{st}	Stanton number
N_{tu}	Number of transfer units $[=hA/(\dot{m}C_p)]$
ΔP	Fluid pressure drop (N m^{-2})
P_o	Ambient pressure (N m^{-2})
R	Ideal gas constant (J kg^{-1} K^{-1})
s	Specific entropy (J kg^{-1} K^{-1})
S	Entropy generation (J K^{-1})
$\dot{S}$	Entropy generation rate (W K^{-1})
t	Time coordinate (s)
t_D	Dimensionless time $[=\dot{m}C_p t/(M_s C_s)]$
T	Temperature (K)
T_o	Ambient temperature (K)
u	Unit step function
v_D	Dimensionless fluid velocity $[=M_s C_s \gamma/(m_f C_p)]$
x	Distance coordinate (m)

Greek letters

γ	Ratio of specific heats for fluid
η	$hAt/(M_s C_s)$ $(=N_{tu}\cdot\theta)$
ϕ	Dimensionless optimum storage capacity $[=(T_{m,opt}-T_o)/(T_{g,i}-T_o)]$
ρ	Fluid density (kg m^{-3})
ξ	$hAx/(\dot{m}C_p L)$ $(=N_{tu}\cdot x_D)$
τ	Dimensionless fluid inlet temperature $[=(T_o-T_{g,i})/T_o]$
θ	Dimensionless charging time $[=t_D-(x_D/v_D)\approx t_D$, neglecting fluid hold-up$]$

Subscripts

a	Average
cv	Control volume
D	Dimensionless
g	Fluid (gas)
i	Inlet
m	Solid or matrix
o	Outlet
opt	Optimum
ΔP	Contribution of pressure drop
sp	Specific
sur	surroundings
ΔT	Contribution of finite temperature difference

The design and operation of energy recovery systems require all possible sources of energy, including waste heat, to be efficiently and economically utilized to satisfy the surplus supply–demand characteristic of the system. Thus, the surplus in the case of a thermal system appears as stored heat when the supply of heat is greater than the demand. Of the many possible ways of storing heat for later use, sensible heat storage systems are attractive because of their relatively low cost, compactness and simplicity. In many sensible heat storage systems, metallic porous beds of high specific heat are utilized for efficient and economic energy storage. Regenerators are one such class of storage units which constitute the single most important component in many thermal systems, including cryogenic systems. In cryogenic refrigerators, the regenerator is at the heart of the engine, which may be based on Stirling, Gifford–McMahon or similar cycles. Apart from this, thermal regnerators are also used in blast furnaces, gas turbines, nuclear reactors and solar energy systems, to cite but a few examples.

In classical methods of utilizing the first law of thermodynamics, the storage systems are assessed in terms of the heat storage capability of the units. This takes no account of the wastage of useful energy during the storing process. Consequently, this approach yields a substantive design for the storage units, but not necessarily the thermodynamically optimum one. In most convective heat transfer processes, the unavailable thermodynamic energy (or anergy) is characterized by two factors: fluid friction and heat transfer across a finite temperature difference. These two interrelated phenomena are the manifestations of thermodynamic irreversibility, and investigation of a process from this single standpoint is based on the second law of thermodynamics in addition to the first. This analysis has led to the maximum storage of useful work by keeping the entropy generation rate at a minimum.

Several authors[1–3] have investigated fluid flow and heat transfer by considering minimum entropy generation to yield optimum design parameters. In their pioneering studies, Bejan[4,5] and Sahoo[6] applied entropy generation techniques to the analysis of sensible heat storage systems. In all these analyses, the authors assumed that there was no spatial temperature variation along the matrix. Such an analysis is termed lumped parameter modelling. In practice, the matrix is considered to be a distributed model where there is a temperature variation along the temporal and spatial coordinates in the matrix. Thus, the purpose of this paper is to investigate refrigeration storage units using a distributed matrix model and to develop optimum design parameters for minimum entropy production due to heat transfer across a finite temperature difference in the presence of fluid friction. In cryogenic and refrigeration applications, since the penalty in terms of cost for wastage of useful energy is large, such analysis can be very useful.

Temperature response of storage units

Typical operation of a refrigeration storage system is shown schematically in *Figure 1*. The system consists of a matrix bed of zero and infinite thermal conductivity along the bed axis and perpendicular to it, respectively.

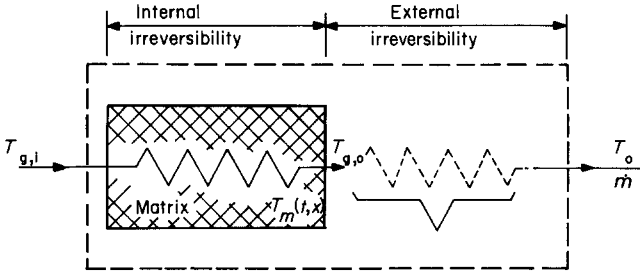

Figure 1 Refrigeration storage system

A stream of cold gas is blown from one side of the bed with the bed at ambient temperature initially. The gas is finally discharged into the environment at the other end of the matrix bed. Gradually, the matrix and exhaust gas temperatures decrease, approaching the cold gas inlet temperature. As the matrix thermal conductivity along the fluid flow direction is zero, the matrix temperature is a function of both distance and time, while the gas outlet temperature at the other end of the matrix is a function of time only.

Using the first law of thermodynamics, the energy balance applied to the system gives the following differential equations (in dimensionless parameters)

$$\frac{\partial T_g}{\partial x_D} = N_{tu}(T_m - T_g) \tag{1}$$

$$\frac{\partial T_m}{\partial \theta} = N_{tu}(T_g - T_m) \tag{2}$$

In most practical situations, the fluid hold-up given by x_D/v_D can be neglected in comparison with the blow period, t_D (Reference 7). Hence the dimensionless charging time, θ, reduces to t_D. The boundary conditions for Equations (1) and (2) are

$$\left.\begin{aligned} T_g(x_D = 0,\ \theta) &= T_{g,i} \\ T_m(x_D,\ \theta = 0) &= T_o \end{aligned}\right| \tag{3}$$

The solution of Equations (1) and (2), along with its boundary conditions expressed by Equation (3), is given explicitly by Montakhab[8]. However the solution can also be presented by special functions[9] which are of interest in the present investigation. These are

$$\frac{T_g(\eta,\ \xi) - T_{g,i}}{T_o - T_{g,i}} = G_o(\eta,\ \xi) \tag{4}$$

$$\frac{T_m(\eta,\ \xi) - T_{g,i}}{T_o - T_{g,i}} = 1 - G_o(\xi,\ \eta) \tag{5}$$

where

$$\eta = N_{tu} \cdot \theta \quad \text{and} \quad \xi = N_{tu} \cdot x_D \tag{6}$$

A short list of the properties of these special functions[9] is given as an appendix.

Entropy generation

The rate of irreversibility can be expressed in terms of entropy generation rate by

$$\dot{I} = T_o \overset{\circ}{S}_{gen} = T_o\left[\frac{dS_{cv}}{dt} + \frac{dS_{sur}}{dt}\right] \tag{7}$$

For a steady state and steady flow process involving a single stream, the above equation can be written as

$$\dot{I} = T_o\left[\frac{dS_{cv}}{dt} + \dot{m}(s_o - s_i)\right] \tag{8}$$

The refrigeration storage process, as shown schematically in *Figure 1*, is associated with three sources of irreversibility. Of these three, two are caused by heat transfer across a finite temperature difference: 1, heat transfer occurs between the cold gas stream and the matrix; and 2, the cold gas stream exhausted from the matrix is heated due to irreversible mixing with ambient air. The third source of irreversibility is the friction due to the flow of fluid through the matrix. Hence the total irreversibility rate due to these three factors can be expressed as

$$\dot{I} = T_o\left[\frac{d}{dt}\left(\frac{M_s C_s}{N_{tu}}\int_0^{N_{tu}} \ln\frac{T_m}{T_o}\,d\xi\right) + \dot{m}C_p \ln\frac{T_o}{T_{g,i}} + \dot{m}C_p\frac{T_{g,o} - T_o}{T_o} + \dot{m}R\ln\left(1 + \frac{\Delta P}{P_o}\right)\right] \tag{9}$$

If the axial temperature variation of the matrix over its length is not too large then the first term under the integral sign can be approximated as

$$\int_0^{N_{tu}} \ln\frac{T_m}{T_o}\,d\xi \approx \left[\ln\frac{T_{m,a}}{T_o}\right]N_{tu} \tag{10}$$

where $T_{m,a}$ is the average matrix temperature over the bed length. The bed length is designated non-dimensionally as N_{tu}. Using the temperature profile Equation (5) the resultant temperature ratio is given by

$$\frac{T_{m,a}}{T_o} = \frac{1}{N_{tu}}\int_0^{N_{tu}} \frac{T_m}{T_o}\,d\xi = 1 - \tau + \frac{\tau F_1(\eta, N_{tu})}{N_{tu}} \tag{11}$$

Also using these relations the irreversibility rate given by Equation (9) can be rewritten as

$$\dot{I} = T_o\left\{\frac{d}{dt}\left[M_s C_s \ln\left(1 - \tau + \frac{\tau F_1(\eta, N_{tu})}{N_{tu}}\right)\right] + \dot{m}C_p\ln\frac{T_o}{T_{g,i}} + \dot{m}C_p\frac{T_{g,o} - T_o}{T_o} + \dot{m}R\ln\left(1 + \frac{\Delta P}{P_o}\right)\right\} \tag{12}$$

In the operation of a regenerator the frictional pressure drop, ΔP, over the length of the matrix bed is usually small compared to the ambient pressure, P_o, so that the last term of Equation (12) can be approximated by

$$\ln\left(1 + \frac{\Delta P}{P_o}\right) \approx \frac{\Delta P}{P_o} \tag{13}$$

Again, the frictional pressure drop for the matrix can be expressed[10] as

$$\frac{\Delta P}{P_o} = \left(\frac{f}{N_{st}}\cdot\frac{G^2}{2\rho P_o}\right)N_{tu} \tag{14}$$

where f, N_{st} and G are, respectively, the friction factor, Stanton number and mass velocity for the gas side. Since

$$I = \int_0^{\tau} \dot{I}\,dt = \frac{M_s C_s}{\dot{m}C_p}\int_0^{\theta} \dot{I}\,d\theta$$

irreversibility, I, can be estimated by the integration of Equation (12), using the temperature profile Equations (4) and (5). The resultant equations express the energy destroyed during the refrigeration storage process due to the finite temperature difference and pressure drop. They are given by

$$I_{\Delta T} = M_s C_s T_o\left\{\ln\left[1 - \tau + \frac{\tau F_1(\eta, N_{tu})}{N_{tu}}\right] + \tau - \frac{\tau F_1(\eta, N_{tu})}{N_{tu}} - \tau\theta - \theta\ln(1 - \tau)\right\} \tag{15}$$

and

$$I_{\Delta P} = M_s C_s T_o \theta\left(\frac{R}{C_p}\cdot\frac{f}{N_{st}}\cdot\frac{G^2}{2\rho P_o}\right)\cdot N_{tu} \tag{16}$$

while

$$I = I_{\Delta T} + I_{\Delta P} \tag{17}$$

where τ is a dimensionless temperature variable expressed as

$$\tau = \frac{T_o - T_{gi}}{T_o} \tag{18}$$

The exergy content of the cold gas at low temperature, $T_{g,i}$, is equal to the minimum work, W_{min}, required for a Carnot refrigerator to produce it from the ambient temperature, T_o. The net exergy needed is the algebraic sum of exergies associated with the flowing mass and resident mass in the regenerator. The derivation is given by

$$E = -\left(\dot{m}C_p t - \frac{x}{L} m_f \frac{C_p}{\gamma}\right) T_o [\tau + \ln(1-\tau)]$$

$$= -M_s C_s \left(\frac{\dot{m}C_p t}{M_s C_s} - \frac{x m_f C_p}{L M_s C_s \gamma}\right) T_o [\tau + \ln(1-\tau)]$$

$$= -M_s C_s \left(t_D - \frac{x_D}{v_D}\right) T_o [\tau + \ln(1-\tau)]$$

$$= -M_s C_s \theta T_o [\tau + \ln(1-\tau)] \quad (19)$$

However, with negligible fluid residence, charging time, θ, can be equated to t_D. An informative expression can be obtained by considering the ratio of the exergy destroyed to the total exergy content of the cold gas. The resultant dimensionless exergy is designated specific irreversibility, I_{sp}. This quantity is the same as the number of irreversibility units introduced by Bejan[5]. The specific irreversibility may be written explicitly from Equations (15), (16), (17) and (19) as

$$\frac{I}{W_{min}} = I_{sp} = (I_{sp})_{\Delta T} + (I_{sp})_{\Delta P} \quad (20)$$

$$(I_{sp})_{\Delta T} = 1 - \{ [\tau - \tau F_1(\eta, N_{tu})/N_{tu} + \ln[1 - \tau + \tau F_1(\eta, N_{tu})/N_{tu}\}/[\theta[\tau + \ln(1-\tau]]\} \quad (21)$$

$$(I_{sp})_{\Delta P} = -\frac{R}{C_p}\left(\frac{f}{N_{st}} \cdot \frac{G^2}{2\rho P_o}\right) N_{tu}/[\tau + \ln(1-\tau)] \quad (22)$$

Equation (21) is defined for finite values of τ and N_{tu}. But the equation can be modified depending on the limiting values of these variables by using the properties of the special functions[9] given in the Appendix.

Case 1: $N_{tu} \to \infty$ and $0 < \tau \leq 1$

As N_{tu} approaches infinity the product hA must also approach infinity since the specific heat and mass flow rate (C_p and $\dot{m}$) are always finite for the cooling gas. So at any cross-section of the matrix bed, since the level of heat transfer is a product of hA and the temperature difference ($T_m - T_g$) between the cold gas and matrix, it is obvious that to have a finite heat transfer value, ($T_m - T_g$) must approach zero. In other words, the matrix temperature must approach the gas temperature. Under these circumstances the system Equations (1) and (2) are transformed into

$$\frac{\partial T}{\partial x_D} = -\frac{\partial T}{\partial \theta} \quad (23)$$

while the boundary conditions are transformed into

$$\left.\begin{aligned} T(x_D, \theta = 0) &= T_o \\ T(x_D = 0, \theta) &= T_{g,i} \end{aligned}\right| \quad (24)$$

Solving Equation (23) with the boundary conditions described by Equation (24) by using Laplace transforms results in

$$\frac{T(x_D, \theta) - T_{g,i}}{T_o - T_{g,i}} = 1 - u(\theta - x_D) \quad (25)$$

where u represents a unit step function. Using Equation (25) it can be shown from Equation (9) that

$$\left.\begin{aligned} (I_{sp})_{\Delta T} &= 0 \quad \text{for} \quad \theta < x_D \\ &\text{and} \\ (I_{sp})_{\Delta T} &= 1 \quad \text{for} \quad \theta \geq x_D \end{aligned}\right\} \quad (26)$$

These expressions clearly indicate that a square wave is propagating with a dimensionless speed of unity.

Case 2: $\tau \to 0$ and $\infty > N_{tu} > 0$

In this case $(I_{sp})_{\Delta T}$ can be evaluated in the limit as $\tau \to 0$, the result being

$$(I_{sp})_{\Delta T} = 1 - \frac{[1 - F_1(\eta, N_{tu})/N_{tu}]^2}{\theta} \quad (27)$$

Optimization of charging time

The amount of cold gas used is determined by the charging time of the matrix. So the time θ plays a major role in determining the loss of useful work due to heat transfer across a finite temperature difference. Examination of Equations (21) and (27) shows that the specific irreversibility, $(I_{sp})_{\Delta T}$, is unity at the two extreme values of θ, that is in the $\theta \to 0$ and $\theta \to \infty$ limits. Hence there exists an optimum value of θ when specific irreversibility reaches its minimum. Specific irreversibility due to temperature difference, $(I_{sp})_{\Delta T}$, is plotted in *Figures 2–4* as a function of θ for discrete values of τ and N_{tu}. The total specific irreversibility is the summation of individual specific irreversibilities due to the temperature and pressure drop. Since the pressure difference contribution to the specific irreversibility is independent of charging time, the above figures portray the optimum charging time for systems in the presence of a pressure difference also. However, it is evident from Equation (21) that as $\tau \to 1$ (that is $T_{g,i} \to 0$ K), $(I_{sp})_{\Delta T} \to 1$, which can be attributed to the infinite exergy content of the inlet cold gas if its temperature approaches absolute zero in the limit, while the stored exergy in the matrix is a finite quantity.

The locus of the optimum charging time, θ_{opt}, can be estimated explicitly by minimizing the specific irreversibility given by Equation (21). Basically two types of conditions arise for the evaluation of these minima

$$\tau\theta_{opt} G_o \frac{\tau - \tau F_1(\eta', N_{tu})/N_{tu}}{1 - \tau + \tau F_1(\eta', N_{tu})/N_{tu}} - [\tau F_1(\eta', N_{tu})/N_{tu} - \tau] + \ln[1 - \tau + \tau F_1(\eta', N_{tu})/N_{tu}] = 0 \quad \text{for} \quad \tau \neq 0 \quad (28)$$

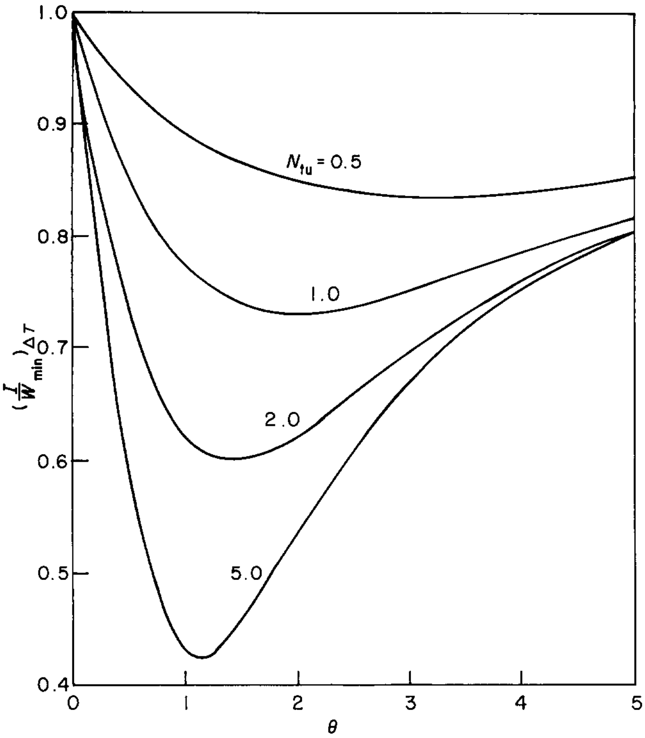

Figure 2 Fraction of exergy destroyed by heat transfer across finite temperature difference ($\tau \rightarrow 0$)

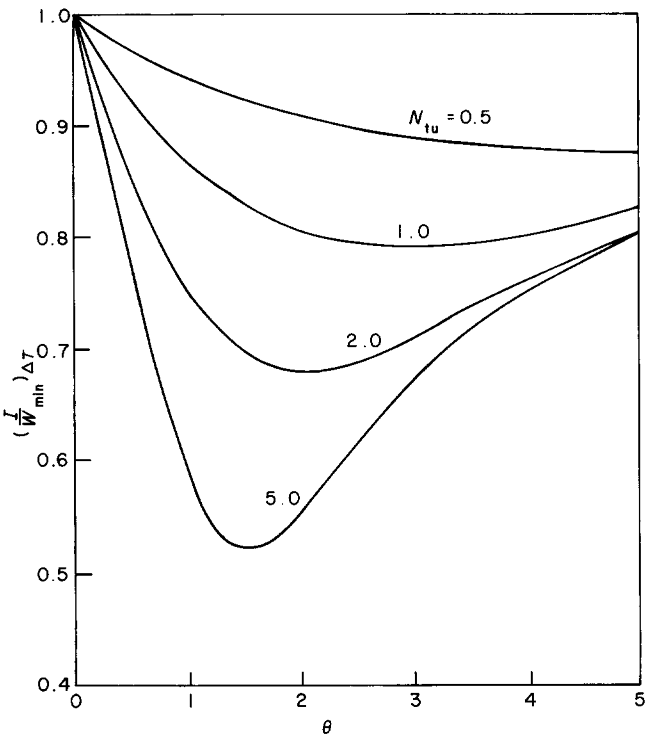

Figure 3 Fraction of exergy destroyed by heat transfer across finite temperature difference ($\tau \rightarrow 0$)

and

$$2\theta_{opt} G_o - [1 - F_1(\eta', N_{tu})/N_{tu}] = 0 \quad \text{for} \quad \tau \rightarrow 0 \quad (29)$$

where

$$\eta' = \theta_{opt} \cdot N_{tu} \quad \text{and} \quad G_o = G_o(\eta', N_{tu}) \quad (30)$$

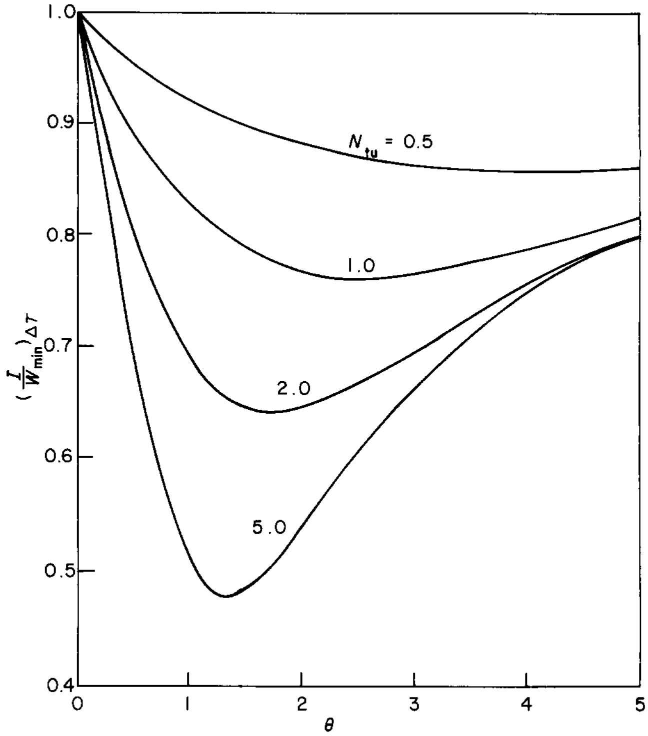

Figure 4 Fraction of exergy destroyed by heat transfer across finite temperature difference ($\tau = 0.5$)

The optimum charging time, θ_{opt}, expressed by Equations (28) and (29) can be calculated numerically using the Mueller's iteration method[11]. The results of these equations are shown in *Figure 5*. It is observed from the plots that the optimum charging time increases with increasing τ, the lowest optimum charging time being in the $\tau \rightarrow 0$ limit (that is $T_{g,i} \rightarrow T_o$). In this case θ_{opt} is finite when the temperature difference for heat transfer is negligibly small, i.e. the gas temperature from the

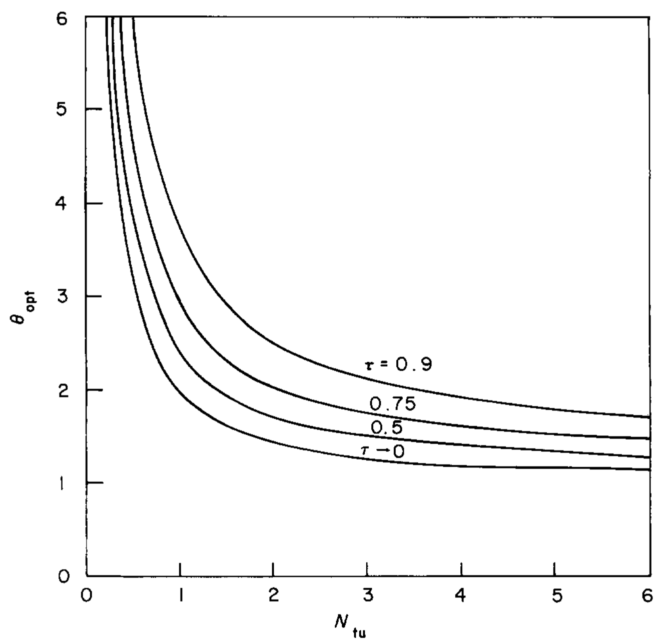

Figure 5 Optimum charging time at minimum specific irreversibility

source approaches the ambient temperature. The variation of θ_{opt} with N_{tu} for various values of τ depicts a rectangular hyperbola where in the $N_{tu} \to 0$ limit, $\theta_{opt} \to \infty$ and in the $N_{tu} \to \infty$ limit, $\theta_{opt} \to 0$.

Optimization of number of transfer units

The most important parameter associated with any heat exchanger is its N_{tu} value. This parameter not only determines the physical dimensions of a heat exchanger but it is also a strong function of heat exchanger effectiveness. In the expression described by Equation (21), the dissipation of useful work due to heat transfer across a finite temperature difference always decreases with increase in N_{tu}. At the same time dissipation of useful work due to drop in pressure increases with N_{tu}, as described by Equation (22).

Thus the combination of Equations (21) and (22) represents the total irreversibility of the system. To optimize N_{tu} the total irreversibility is differentiated with respect to N_{tu} at the corresponding optimum charging time. The result can be expressed as

$$N_f^2 = \frac{\tau F_1(\eta, N_{tu})/N_{tu} - \tau}{[1 - \tau + |\tau F_1(\eta, N_{tu})/N_{tu}]\theta} \times \left\{ \frac{\tau[F_o(\eta, N_{tu}) - \theta G_o(\eta, N_{tu})]}{N_{tu}} - \frac{\tau F_1(\eta, N_{tu})}{N_{tu}^2} \right\} \tag{31}$$

where

$$N_f^2 = \left(\frac{R}{C_p}\right)\left(\frac{f}{N_{st}} \cdot \left| \frac{G^2}{2\rho P_o} \right.\right) \tag{32}$$

Equation (31) can be solved numerically for the optimum number of transfer units. In the solution procedure, Equation (28) is first solved to obtain the optimum charging time at the corresponding N_{tu} for a discrete value of τ. If this pair of θ_{opt} and N_{tu} values satisfy Equation (31) for a discrete value of N_f, the resultant value of N_{tu} is the optimum N_{tu}. This result is plotted in *Figure 6*, which shows that the optimum number of transfer units decreases as the mass velocity increases for the same value of τ. So, for a regenerator bed the mass velocity is fixed to achieve most efficient exergy storage.

Optimization of storage capacity

If the cold recovery process is continued up to a time equal to the optimum charging time of the system then the matrix temperature is cooled to an optimum temperature, $T_{m,opt}$. This corresponds to the optimum exergy stored in the refrigeration unit. However, if cooling of the matrix is prolonged to give the gas inlet temperature $T_{g,i}$, maximum energy is stored in the matrix at the expense of increased exergy waste from the cold source. Thus the fraction ϕ, which determines optimum refrigeration corresponding to the maximum storage of useful work, is shown in *Figure 7*. This figure represents the values of no risk storage of useful work in a regenerator. Deviation from this quantity of stored work runs the risk of destroying useful work during the storage process.

Concluding remarks

By analysing the irreversibility associated with sensible heat in the refrigeration storage process, it is possible to show important trade-offs in optimizing regenerators for single blow operations. These trade-offs are similar to the earlier analyses posed by Bejan[5] and Krane[3].

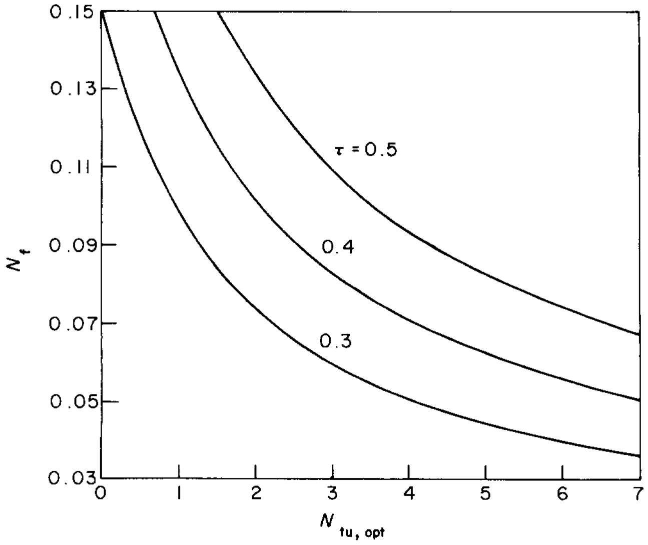

Figure 6 Optimum N_{tu} at minimum specific irreversibility

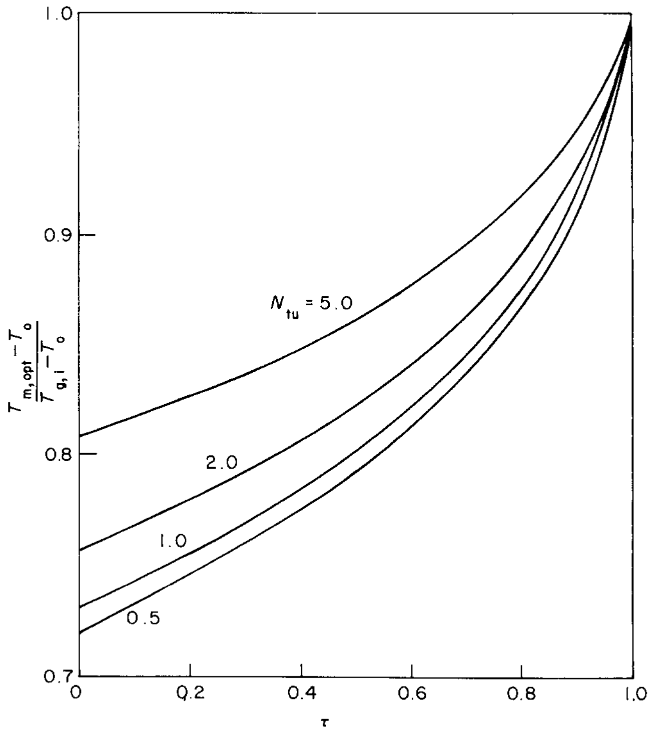

Figure 7 Optimum storage capacity of the matrix

Unlike the earlier analyses which were applied to a huge liquid pool, the present model has a more practical application in cryogenic industry. Due to the involvement of special functions and its mathematical complexity, the model is approximated by an average temperature [given by Equation (11)] to suit low N_{tu} regenerators. Though this is one of the limitations of the present analysis, it greatly simplifies the computation involved and yields a semi-analytical solution which is closer to the real situation than solutions given by existing models[3,5,6].

The model can be extended to a bed which does not have a uniform initial bed temperature. In this case, the initial temperature function may be expanded as a polynomial with regard to the distance coordinate. The first term of the polynomial is a constant and its temperature response is given by Equations (4) and (5). The response due to the successive terms of the polynomial is evaluated by the integration of the preceding terms. For a linear system the integral of the response is the response for the integral of the subsequent term of the polynomial. Hence the resultant temperature response can be utilized to compute the optimum parameters.

Considering the irreversibility analysis again shows that the real purpose of a thermal energy storage system is not to store energy, but to store thermodynamic availability or exergy. Designing a regenerator for single blow operations based on optimum charging time, θ_{opt}, and optimum transfer units, $N_{tu,opt}$, maximizes the fraction of availability or exergy or useful work stored.

It is important to note that a wide variety of other considerations exists for implementation of this analysis. These considerations are mainly economic and practical in nature. The present work, however, addresses only the thermodynamic aspect of designing storage systems, ignoring the other factors.

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Appendix

The function G_o is known as the Anzelius–Schumann function[9] and is defined as

$$G_o(x, y) = e^{-(x+y)} \sum_{r=0}^{\infty} \frac{y^{r+1}}{(r+1)!} \sum_{p=0}^{r} \frac{x^p}{p!}$$

Similar functions, G_1 and F_1, appearing in the text are given by

$$G_1(x, y) = e^{-(x+y)} \sum_{r=0}^{\infty} \frac{y^{r+2}}{(r+2)!} \sum_{p=0}^{r} (r-p+1) \frac{x^p}{p!}$$

and

$$F_1(x, y) = G_o(x, y) + G_1(x, y)$$

The partial derivatives of the function F_1 with respect to its arguments are given by

$$\frac{\partial F_1(x, y)}{\partial x} = -G_o(x, y)$$

$$\frac{\partial F_1(x, y)}{\partial y} = 1 - G_o(y, x)$$

and one of the integrations may be given as

$$\int_0^x G_o(x, y)\mathrm{d}x = -F_1(x, y) + F_1(0, y)$$