# An Active and Reactive Power Analysis of Solid Oxide Fuel Cell

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Abstract—This paper presents an active and reactive power analysis of Solid Oxide Fuel Cell (SOFC) model. This model is developed from electrochemical and thermal equations. It accounts for temperature-dynamics, ohmic loss, concentration loss, activation loss and output-voltage loss. The relaxation time is strongly related to the transient temperature distribution of the solid oxide fuel cell structure. The proposed model contains the hydrogen, oxygen and water block separately to increase the active power. The analytical detail of active and reactive power output of a stand-alone SOFC power plant is controlled by fuzzy logic. This model is validated through Matlab program under transient behavior. A comparative assignment of PI and Fuzzy – Logic controller is carried out for SOFC power plant model

## Keywords- SOFC, Fuzzy Controller, FCPP

### I. INTRODUCTION

Many researchers have done the modeling of Molten Carbonate Fuel Cell. Looking in 2004's, Francisco Jurada gives a model of solid oxide fuel cell without considering the temperature effect. But he does not consider the concentration loss of the solid oxide fuel cell. This concept of molten carbonate fuel cell is exhaust to concentration loss and comparison is given in [1] and [2]. Another very useful modeling is given in the literature in 2004's which was suggested by Kourosh Sedghisigarchi, [3] and [4]. This papers considers all the subsystems of fuel cell, that includes hydrogen block, oxygen block, water block, activation block, concentration block and temperature block. Looking in 1995's, [5] Elmar Achenbach has described the thermal equation to model the temperature block. In temperature block, the relaxation time is closely related to the transient temperature distribution of the solid oxide fuel cell structure. The internal cell resistances are strongly temperature dependent. Thus, the relaxation time depends on the thermal properties, size and configuration of the cell, and operating conditions. In [6], SOFC model is given without considering thermal unit. In the present investigation; these structures are modified by the modeling of thermal unit. A first comprehensive nonlinear dynamic model of solid oxide fuel cell that can be used for dynamic and transient stability studies is developed by K. Sedghisigarchi and Ali Feliachi in 2004 [3]. The model is based on electrochemical and thermal equations, accounts for temperature dynamics and voltage losses. The output voltage response of a stand-alone fuel-cell

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plant to a step change, a fuel flow step change, and fast load variations are simulated to illustrate the dynamic behavior of SOFC for fast and slow perturbations.

This paper presents a SOFC model and designs the control strategies and the controllers for the AC voltage control via the DC/AC inverter and the active/reactive power control of the DC/AC inverter. Both PI and Fuzzy logic control scheme are employed separately for the design of the controllers and a comparative assessment is conducted.

#### II. PROPOSED SOFC DYNAMIC MODEL

The typical model of SOFC is shown in Fig.1. The proposed dynamic model is based on chemical and physical principles. This stack-model is defined on the following assumptions. *Model assumptions* 

Model assumptions

- The gases are ideal and temperature is stable at all times.
- The channels that transport gases along the electrodes have a fixed volume, but their lengths are small, so that it is only necessary to define one single pressure value in their interior.
- The exhaust of each channel is via a single orifice. The ratio of pressures between the interior and exterior of the channel is large enough to consider that the orifice is chocked.
- > The Nernst equation is applied.

The SOFC model consists of a) Electrochemical modelcomponent material balance equations. b) Thermal model-Energy balance equations. c) Nernst voltage equation d) Reformer model. e) Power conditioning unit model.

## A. Electrochemical Model

The change in concentration of each species that appears in the chemical reaction can be written in terms of input and output flow rates and exit molarities  $(x_i)$  due to the following chemical reaction

$$\frac{PV}{RT}\frac{d}{dt}x_{i} = N_{i}^{\ in} - N_{i}^{\ 0} + N_{i}^{\ r}$$
(1)

Where, V-compartment volume  $(m^3)$ ;  $N_{i,}^{in}$ ,  $N_i^0$  are molar flow rates (mol/s) of  $i^{th}$  reactant at the cell input and output (exit), respectively.  $N_i^r$  is reaction rate (mol/s) of the  $i^{th}$  reactant. The cell utilization factor (u) is defined through the input and output hydrogen flow rates  $(N_{\mu})$  as follows

$$u = \frac{N_{H_2}{}^{in} - N_{H_2}{}^0}{N_{H_2}{}^{in}}$$
(2)

Molar flow of any gas through the valve is proportional to its partial pressure inside the channel. The molar expression is given by

$$\frac{N_{H_2}}{x_{H_2}} = K_{H_2} \frac{N_{H_2O}}{x_{H_2O}} = K_{H_2O}$$
(3)

where ,  $N_{H_2}$  is hydrogen flow that reacts (k mol/s),  $K_i$  is valve

molar constant,  $x_i$  is molar fraction of species.

Applying the Laplace transformation to the above equation and isolating the hydrogen partial pressure, the following expression is obtained.

$$x_{H_2} = \frac{\frac{1}{K_{H_2}}}{1 + \tau_{H_2s}} \left( N^{in}_{H_2} - 2K_r I \right)$$
(4)

Where,  $\tau_{H2} = \frac{V}{K_{H2}RT}$ , where  $\tau_{H2}$  expressed in seconds, is

the time constant associated with hydrogen flow and is a function of temperature.  $K_r$  is the constant dependent on Faraday's constant and number of electrons (N) in the reaction.

$$K_r = \frac{N}{4F}$$

Where, F is the Faraday's constant.



Fig.1 SOFC Dynamic Model

## B. Thermal modeling:

The Nernst potential and loss mechanism are temperature sensitive; the temperature throughout the cell must be known to accurately determine the cell's electrical performance. During the normal operation, the predominant variation in cell temperature occurs along the length of the cells. Heat transfer occurs by the thermal conduction, convection and radiation. Conduction occurs in the solid cell materials and air free tube. Axial conduction in the solids was neglected due to long conduction path and reduces areas.

Convection occurs between the solid surfaces and the various gas streams. Radiation transfers heat between the cell and air feed tube and also between adjacent cells. There are heat sources due to exothermic reaction of fuel and oxygen. In addition to the various types of heat transfer and the sources, the thermal model also includes the variation of material properties with temperature. The fuel cell power output is closely related to the temperature of the unit cell. The heat storage in the thin fuel unit or oxidant gas layer is neglected. The thin fuel unit or oxidant gas layers are lumped to the cell unit and gas layers are assumed to have the same temperature as the unit cell.

E. Achenbach in [5] has formed a relationship between two parameters  $S_0$  and  $F_0$  as presented below

$$F_0 = 0.72 S_0^{-1.1} \tag{5}$$

Where, 
$$F_0 = \frac{\lambda \Delta t}{\rho C_p h^2_{eff}}$$
 and  $S_0 = \frac{U_j(1-\eta)}{\eta \lambda \Delta T} h_{eff}$ 

By simplifying the above eq (5), we get

$$F_0^{0.909} = 0.72S_0^{-1} \tag{6}$$

$$F_0^{0.909} = 0.72 \left( \frac{Y}{\lambda \Delta T} \right)^{-1}$$
(7)

Where,  $Y = \frac{U_j(1-\eta)}{\eta} h_{eff}$ ,  $U_j$  is the power density (electric

output),  $\eta$  is efficiency of the fuel cell,  $\lambda$  is the thermal conductivity,  $\Delta T$  is change in temperature.

From the above eq (7) we get,

$$\Delta T = \frac{F_0^{0.909} Y}{0.72\lambda}$$
(8)

This is the increase in temperature from the initial condition after a relaxation time t. Assuming the relation between t, dt and temperature T. We can predict the temperature at on next simulation instant as

$$T_0 = T + \left(\frac{T_{in} + \Delta T - T}{t}\right) dt \tag{9}$$

# C. Nerst voltage equation:

Applying Nernst's equation and ohmic law (taking into account ohmic, concentration, and activation losses), the stack output voltage is represented by

$$V_{dc} = V_0 - rI - \eta_{act} - \eta_{con} \tag{10}$$

$$V_0 = N_0 \left( E^0 + \frac{RT^0}{2F} \ln \frac{x_{H_2} x_{O_2}^{-\frac{1}{2}}}{x_{H_2O}} \right)$$
(11)

Where  $V_0$  is open-circuit reversible potential (in volts),  $E^0$  is standard reversible cell potential,  $x_i$  is mole fraction of

species,  $N_0$  is the number of cells in stack,  $\eta_{act}$  is the activation losses in volts,  $\eta_{con}$  is concentration losses in volts. C.1. Concentration loss:

The reactant is consumed at the electrode by electrochemical reaction; there is a loss of potential due to the inability of the surrounding material to maintain the initial concentration of the bulk fluid. That is, a concentration gradient is formed. Several processes may contribute to concentration polarization: slow diffusion in the gas phase in the electrode pores, solution/dissolution of reactants/ products into/out of the electrolytes, or diffusion of reactants/products through the electrolyte to/ from the electrochemical reactions site. At practical current densities, slow transport of reactants/ products to/from the electrochemical reaction site is a major contributor to concentration polarization:

$$\eta_{con} = \frac{RT}{n_a F} \ln \left( 1 - \frac{i}{i_L} \right) \tag{12}$$

Where, i is the stack current,  $i_L$  is the limiting current and

 $n_{a}$  is the number of electrons participating in the reaction.

## C.2. Activation losses:

Activation polarization is present when the rate of an electrochemical reaction at an electrode surface is controlled by sluggish electrode kinetics. In other words, activation polarization is directly related to the rates of electrochemical reactions. There is a close similarity between electrochemical and chemical reactions in that both involve an activation barrier that must be overcome by the reacting species. In the case of an electrochemical reaction with  $\eta_{act} \ge 50\text{-}100\text{mV}$ ,  $\eta_{act}$  is described by the general form of the Tafel equation:

$$\eta_{act} = \frac{RT}{\alpha nF} \log\left(\frac{i}{i_0}\right) \tag{13}$$

Where,  $\alpha$  the electron is the transfer coefficient of the reaction at the electrode and  $i_0$  is the exchange current density. Tafel plots provide a visual understanding of the activation polarization of a fuel cell. They are used to measure the exchange current density and transfer coefficient.

Voltage drop due to activation loss can be expressed by a semi –empirical equation, called the Tafel equation. It is given  $\eta_{act} = a + b \log i$ 

Where, 
$$a = \left(\frac{-2.3RT}{\alpha nF}\right) \log i_0$$
 and  $b = \frac{2.3RT}{\alpha nF}$  are

called Tafel constant and Tafel slope respectively. C.3. Ohmic resistance losses:

Ohmic losses occur because of resistance to the flow of ions in the electrolyte and resistance to flow of electrons through the electrode materials. The dominant ohmic losses, through the electrolyte, are reduced by decreasing the electrode separation and enhancing the ionic conductivity of the electrolyte. Because, both the electrolyte and the fuel cell electrodes obey Ohm's law. This resistance is dependent on the cell temperature and is obtained by

$$r = \alpha \exp\left[\beta \left(\frac{1}{T_0} - \frac{1}{T}\right)\right]$$
(14)

Where, T is the stack temperature and  $\alpha, \beta$  is the constant coefficients.

D. Reformer Model:

In [7], the author introduced a simple model of a reformer that generates hydrogen through reforming methane. The model is a second order transfer function. The mathematical form of the model can be written as follows:

$$\frac{q_{H_2}}{q_{methane}} = \frac{C_v}{\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2)s + 1}$$
(15)

Where,  $q_{methane}$  is the methane flow rate [kmol/s],  $C_v$  is the conversion factor [kmol of hydrogen per kmol of methane],  $\tau_1 \tau_2$  are reformer time constants [s].

To control hydrogen flow according to the output power from the fuel cell, a feedback from the stack current is considered. A proportional integral (PI) controller is used to control the flow rate of methane in the reformer [7]. Oxygen flow is determined using the hydrogen-oxygen flow ratio  $r_{H-O}$ .

# E. Power conditioning Unit Model:

The power conditioning unit is used to convert DC output voltage to AC. As mentioned before, the power conditioning unit includes a DC/DC converter to raise DC output voltage, followed by a DC/AC inverter to convert the DC bus voltage to AC. In this paper, only a simple model of a DC/AC inverter is considered for the following reasons: the dynamic time constant of inverters is of the order of microseconds or milliseconds. The time constants for the reformer and stack are of the order of seconds. Hence, including, the inverter time constant will have negligible effect on the time response accuracy. On the other hand, it complicates the system model. A simple model of the inverter is given in [6], where output voltage and output power are controlled using the inverter modulation index and the phase angle  $\delta$  of the AC

voltage,  $V_{ac}$ . The output voltage and the output power as a function of the modulation index and the phase angle can be written as:

$$V_{ac} = m V_{cell} \angle \delta \tag{16}$$

$$P_{ac} = \frac{mV_{cell}V_s}{X}\sin(\delta) \tag{17}$$

$$Q = \frac{(mV_{cell})^2 - mV_{cell}V_s\cos(\delta)}{X}$$
(18)

where,  $V_{ac}$  is the AC output voltage of the inverter [V], *m* is inverter modulation index,  $\delta$  is phase angle of the AC voltage  $mV_{cell}$  [rad],  $P_{ac}$  is the AC output power from the inverter [W], *Q* is reactive output power from the inverter [VAR],  $V_s$  is the load terminal voltage [V], *X* is reactance of the line connecting the fuel cell to the infinite bus [ $\Omega$ ]. F. Analysis of Active and Reactive power control:

In traditional synchronous generators, the amount of steam input to the turbine controls the power angle, which controls the active power output from the generator. In synchronous machines, the power angle is not measured, but the adjustment of the power angle occurs following changes in steam input and rotor speed. In the FCPP, there is no speed control but the similar relationship between output voltage phase angle and the flow of hydrogen can be adopted as follows. Given that the load voltage ( $Vs \angle 0$ ) is constant and the AC source voltage out of the inverter  $V_{ac}$  is given in (16), the angle  $\delta$  controls the power flow from the fuel cell to the load, as in (17). The phase angle  $\delta$  can be controlled using the input flow of hydrogen. The expression for  $\delta$ , therefore, provides the relationship between the power output as a regulated quantity, and the amount of flow of fuel input. This relationship is described by the following equations:

Assuming a lossless inverter, we get

$$P_{ac} = P_{dc} = V_{cell} I \tag{19}$$

According to the electrochemical relationships, a relationship between the stack current and the molar flow of hydrogen can be written as

$$q_{H_2} = \frac{N_0 I}{2FU} \tag{20}$$

From equations (17), (19), and (20)

$$\sin \delta = \frac{2FUX}{mV_s N_0} q_{H_2} \tag{21}$$

Assuming a small phase angle  $\sin \delta \cong \delta$ ; (21) can be written as

$$\delta = \frac{2FUX}{mV_s N_0} q_{H_2} \tag{22}$$

Equation (22) describes the relationship between output voltage phage angle  $\delta$  and hydrogen flow  $q_{H_2}$ . Equations (17) and (22) indicate that the active power as a function of the voltage phase angle  $\delta$  can be controlled by controlling the amount of hydrogen flow.

## III. RESULT AND ANALYSIS

The active/reactive power control of the DC/AC inverter is implemented by PI or Fuzzy logic control scheme.

PI controller is used to increase the speed of the response and also to eliminate the steady-state error. However these controllers requires precise linear mathematical model of the system, which is difficult to obtain under parameter variations and input flow disturbances. Recently fuzzy logic controller is used in various power electronics applications and fuel cell power plant model. The fuzzy logic controller does not require a mathematical model of the system. Here in this paper, TS fuzzy controller is incorporated because of its simple structure. The 2 X 2 linguistic rules consequent is made variable by means of its parameters. As the rules consequent is variable, the TS fuzzy control scheme can produce an infinite number of gain variation characteristics. In essence, the TS fuzzy controller is capable of offering more and better solutions to a wide variety of non-linear control problems.

Fuel cell is designed for 320 V D.C. output voltages. Fig. 2 shows the output voltage of fuel cell. Temperature of fuel cell

is dependent on active power requirement. Consider that active power is increased from 0.3 p.u. to 0.6 p.u at 1000 sec. and then decreased to 0.3 p.u at 2000sec. Maintaining reactive power constant; the temperature of fuel cell is changed accordingly as shown in Fig. 3. Fig.4 (a) and 4(b) shows the active and reactive power which is dependent on each other. The active power requirement is changed from 0.2 p.u to 0.6 p.u at 200 sec. and again it is changed to 0.4 p.u at 300 sec. From Fig. 4(a), it is seen that fuel cell is capable of providing the active power requirement. As active power demand is changed, the reactive power will also change Fig.4(b) shows the capability of fuel cell to meet the reactive power demand. Let initially rms value of active power is 30 kW and if the demand is that this rms value changes sinusoidally i.e., Pacref changes at 200 sec. Now consider that reference active power changes (P<sub>acref</sub>) shown in Fig.5 (a) and (b).Fig. 5 (a) shows the performance of fuel cell with PI controller and Fig.5(b) shows the performance with TS fuzzy controller. From these figures, it is seen that TS fuzzy controller can meet the power demand perfectly as compared to PI controller. In the similar fashion the reactive power demand is changed and performance of fuel cell with PI controller and TS fuzzy controller is observed in figures 6(a) and 6(b). From these figures, it is seen that TS fuzzy controller can track the required reactive power demand perfectly as compared to PI controller.



Fig. 4 (b) Reactive power



Fig. 5 (a) Active Power with PI controller



Fig. 5 (b) Active powers with Fuzzy controller



Fig. 6 (a) Reactive powers with PI controller



Fig. 6 (b) Reactive power with fuzzy controller

## IV. CONCLUSIONS

The proposed SOFC dynamic model is developed along with the subsystem of concentration block, activation block, water, oxygen and hydrogen block. Thermal block is also incorporated in the SOFC dynamic model. The integrated model includes fuel cell and power conditioning unit. The DC to AC converter is developed which is based on the mathematical equations. To test the proposed model, its active and reactive power outputs are compared with the required load demand. The dynamic behavior of SOFC model is analyzed by using PI or TS fuzzy controller. Here TS fuzzy logic controller is taken because of its simple structure. It is observed that the dynamic performance of fuel cell with TS fuzzy controller is better as compared to PI controller. TS fuzzy controller can track the power demand perfectly as compared to PI controller.

#### REFERENCES

- M.D. Lukas, K.Y Lee. And H.Ghezel-Ayagh, "Development of a stack Simulationmodel for Control Study on direct reforming molten carbonate Fuel Cell Power Plant," *IEEE Trans. Energy Conversion*, Vol.14, pp.1651-1657, Dec.1999.
- [2] Wolfgang Friede, Stephane Rael, and Bernard Davat, "Mathematical Model and characterization of the Transient Behaviour of a PEM Fuel Cell," *IEEE Trans. on Power Electronics*, Vol.19. No5, September 2004
- [3] K. Sedghisigarchi, Ali Feliachi, "Dynamic and Transient Analysis of power distribution systems with fuel cell," - Part I Fuel-cell Dynamic Model *IEEE Transactions on Energy Conversion*, Vol, 19, No.2, June 2004.
- [4] J.Padulles, G.W. Ault, and J. R. McDonald, "An integrated SOFC plant dynamicmodel for power system simulation," J. Power Sources, pp.399-408, Mar.1993.
- [5] E.Achenbach, "Response of a solid fuel cell to load change," J.Power Sources, Vol.57, pp.105-109, 1995.
- [6] M.Y. El-Sharkh, A. Rahman, M.S. Alam, A.A. Sakla, "Analysis of Active and Reactive Power Control of a Stand-Alone PEM Fuel Cell Power Plant," *IEEE Trans. on Power Systems*, Vol.19, No. 4, November 2004
- [7] M.Y. El-Sharkh, A. Rah man, M. S. Alam, "Neural networks-based control of active and reactive power of a stand-alone PEM fuel cell power plant," *J.Power Sources* 135(2004) 88-94.

Fuel Cell Operating Data	
Temperature(T)	900 $^{0}K$
Faraday's constant (F)	96484600 C/kmol
Universal gas constant (R)	8314.47 J/(kmol K)
No load voltage $\left( E_{0} ight)$	1V
Number of cells $(N_0)$	384
$K_r$ constant = $\frac{N_0}{4F}$	0.996*10 <sup>-6</sup> kmol/(sA)
Utilization factor $(U)$	0.8
Hydrogen valve constant $(K_{H_2})$	$4.22*10^{-5}$ kmol/(s
Water valve constant $(K_{H_2O})$	7.716*10 <sup>-6</sup> kmol/( s atm)
Oxygen valve constant $(K_{O_2})$	2.11*10 <sup>-5</sup> kmol/(s atm)
Hydrogen time constant $( au_{H_2})$	3.37 s
Water time constant $(\tau_{H_2O})$	18.418 s
Oxygen time constant $(\tau_{O_2})$	6.74 s
Reformer time constant $( au_1)$	2 s
Reformer time constant $(\tau_2)$	2 s
Conversion factor $(C_v)$	2
Line reactance $(X)$	$0.05\Omega$
PI gain constants $\left(K_{P}^{},K_{I}^{} ight)$	0.1
Base Voltage $(V_r)$	400v
Methane reference signal $(Q_{methref})$	0.000015 kmol/s
Hydrogen-Oxygen flow ratio $(r_{H-O})$	2
Base MVA	100kVA

Fuel Cell Operating Data