Robust Diffusion LMS over Wireless Sensor Network in Impulsive Noise

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Abstract— **Distributed wireless sensor networks have been proposed as a solution to environment sensing, target tracking, data collection and other applications. Energy efficiency, high estimation accuracy, and fast convergence are important goals in distributed estimation algorithms for sensor network. This paper studies the problem of robust adaptive estimation in impulsive noise environment using robust cost function like Wilcoxon norm and saturation nonlinearity. The diffusion cooperative scheme conventionally used in sensor network in which each node have local computing ability and share them with their predefined neighbors, is not robust to impulsive type of noise. In this paper the robust norm is introduced in diffusion cooperative distributed network to estimate the desired parameters in presence of Gaussian contaminated impulsive noise. The simulation study shows that Wilcoxon norm and saturation linearity based diffusion LMS is robust to impulsive noise.**

Keywords- Adaptive networks, contaminated Gaussian distribution, distributed processing, incremental algorithm, diffusion LMS, Wilcoxon norm, error saturation nonlinearity algorithm.

I. INTRODUCTION

In wireless sensor networks(WSN) comprising the nodes are employed to collect data like local temperature, wind speed, humidity, or concentration of some materials etc over a geographic area and are envisioned to make a dramatic impact on a number of applications such as, precision agriculture, disaster relief management, radar, and acoustic source localization. In these applications, each node with its computational power, able to send data to a subset of the network nodes, and tries to estimate the parameter of interest [2, 13]. Therefore, there is a great deal of effort to devising algorithms that are able improve the estimate of the parameters of interest in every node with information exchange between nodes [19, 20]. More precisely, in mathematical terms, each node optimizes a cost function that depends on all information in the network. The main challenges in optimizing such functions are that (i) no node has direct access to all information, and (ii) the network topology can change over time (due to link failures, position changes, and/or reachability problems). (iii) the presence of impulsive noise or outliers *i.e.* when data is contaminated with non-Gaussian noise. The conventional estimation algorithms, which is based on squared error as the cost function, is not robust to outliers in the training signal. Hence presence of such impulsive noise in the training signal severely detoriates the estimation performance. Thus there is need to develop robust estimation algorithm in a distributed scenario to alleviate the effect of outliers.

LMS algorithm [6, 21] has been reported in the literature using Wilcoxon norm which is not distributed in nature. This algorithm is not directly useful in applications which is inherently distributed in nature. In this paper we focus on developing a novel distributed estimation algorithm using Wilcoxon norm and compared the results with other robust algorithm like error saturation nonlinearity which are robust to impulsive noise or outliers.

It is known that when data is contaminated with non-Gaussian noise, the conventional adaptive filters minimizing least square or mean square criterion yield poor performance. This leads to a new domain of research in modern communication systems, where the performance is limited by interference of impulsive nature. In many physical environment the additive noise is modeled as impulsive and is characterized by long-tailed non-Gaussian distribution. The performance of such system is evaluated under the assumption that the Gaussian noise is severally degraded by the non-Gaussian or Gaussian mixture [23] noise due to deviation from normality in the tails [12, 18]. Nonlinear techniques are employed to reduce the effect of impulsive interference on the systems. The effects of saturation type of non-linearity on the least-mean square adaptation for Gaussian inputs and Gaussian noise have been studied [7, 9]. Recent research focus is to develop adaptive algorithm that are robust to impulsive noise or outliers present in the training data. A number of algorithms have been proposed [1, 10, 12, 14] in the literature to reduce the effects of impulsive noise.

The LMS algorithm is a popular adaptive algorithm because of its simplicity [16, 24]. Recently several distributed type of algorithms based on LMS has been suggested and analyzed in the literature. These are not robust against outliers in the training signal as the squared error norm is used as the cost function in deriving the algorithm. On the other hand centralized robust class of least-mean square algorithm with error saturation nonlinearity is of special importance. The error nonlinearity analysis [3, 4] using weighted-energy conservation method for Gaussian data has been dealt in literature. However the theory in [8] provides the basic for extending to Gaussian mixture case. In this paper we exploit both the ideas to develop a new generalized distributed algorithm which is robust to impulsive type of noise. The steady-state analysis of saturation nonlinearity incremental LMS (SNILMS) in presence of contaminated Gaussian impulsive noise is carried out and shown by simulation its robustness over the conventional incremental LMS algorithm (ILMS) is demonstrated.

II. ROBUST DIFFUSION LMS ALGORITHM

Recently the concept of distributed adaptive incremental algorithms has been developed in the literature [5, 11, 15] to increase the energy efficiency of sensor network. One of such schemes is incremental cooperative technique which provides a truly global solution in estimating unknown parameters in WSN. But it is a fact that the gradient based incremental algorithm is not robust to impulsive type of noise present in the training signal. To make the algorithm robust for impulsive noise, here authors have introduced a new class of distributed algorithm based on error saturation nonlinearity LMS.

Let define a sensor network has N number of nodes in which each node has access only to its immediate neighbor node. We assume that the sensors make noisy vector measurements of surrounding environment. Each node k has access to $\{d_k(i), u_{k,i}\}$ data out of a total $\{d_k, u_k\}, data\ set k = 1,2,3,...N$. The symbol $d_k(i)$ represents a scalar measurement and $u_{k,i}$ as a $1\times M$ regression row vector at time i and is given by:

$$\mathbf{u}_{k,i} = [u_k(i), u_k(i-1), \dots u_k(i-M+1)]$$

Let the local estimate of optimum weight $\mathbf{w}^\circ$ is $\Psi_k^{(i)}$ at time i. It is also assume that node k has access to only $\Psi_{k-1}^{(i)}$, which is an estimate of $\mathbf{w}^\circ$ at its immediate neighbor node k-1 in the defined topology. Let the current global estimate of $\mathbf{w}^\circ$ is $\mathbf{w}_{i-1}$ which is the initial condition at i defined as $\Psi_0^{(i)} = \mathbf{w}_{i-1}$ and at the end of one cycle the local estimate vector at node N is assigned as $\mathbf{w}_i$ *i.e.* $\Psi_N^{(i)} = \mathbf{w}_i$.

A. Adaptive Algorithm with Error saturation Nonlinearity

The estimate of an $M\times 1$ unknown vector w° by using row regressor $\mathbf{u}_i$, of length M and output samples $d(i)$ is given as

$$d(i) = \mathbf{u}_i\mathbf{w}^\circ + v(i) \tag{1}$$

Where $v(i)$ is represents the Gaussian contaminated impulsive noise instead of white Gaussian. The weight update equation of well known LMS algorithm is given by [16, 22, 24]

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \mu e(i)\mathbf{u}_i^T$$

The update equation using error saturation nonlinear LMS which is robust to impulsive noise [8] is given as

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \mu\mathbf{u}_i^T f[e(i)] \quad i \ge 0 \tag{2}$$

where $\mathbf{w}_i$ is the estimate of $\mathbf{w}$ at time i and μ is the step size.

The error term at ith time instant is represented as

$$e(i) = d(i) - \mathbf{u}_i\mathbf{w}_{i-1} = \mathbf{u}_i\mathbf{w}^\circ - \mathbf{u}_i\mathbf{w}_{i-1} + v(i) \tag{3}$$

and

$$f(y) = \int_0^y \exp[-u^2/2\sigma_s^2]du = \sqrt{\frac{\pi}{2}} erf\left[\frac{y}{\sqrt{2}\sigma_s}\right] \tag{4}$$

B. Wilcoxon Norm

To define Wilcoxon norm of a vector[17], we need a score function $\varphi : [0,1] \to \mathrm{R}$ which is non decreasing such that

$$\int_0^1 \varphi^2(u)du < \infty.$$

The score associated with the score function φ is defined by

$$a(i) = \varphi\left(\frac{i}{l+1}\right), \qquad i \in \underline{l}$$

where l is a fixed positive integer.

It can be shown that the following function is a pseudonorm on R^l :

$$|v|_W := \sum_{i=1}^{l} a(R(v_i))v_i = \sum_{i=1}^{l} a(i)v_{(i)}$$
$$:= [v_1, v_2, \dots, v_l]^T \in \mathrm{R}^l$$

where $R(v_i)$ denotes the rank of v_i among $v_1,\dots,v_l, v_{(1)} \le \dots \le v_{(l)}$ are the ordered values of $v_1,\dots,v_l, a(i) := \varphi\left(\frac{i}{l+1}\right)$, and $\varphi(u) := \sqrt{12}(u-0.5)$.

Then we call $|v|_W$ defined in (5) is the Wilcoxon Norm of the vector v .

C. Model for Impulsive Noise

The performance analysis of adaptive filters that are available in literature is for the case of white Gaussian noise. But in practical situation the impulsive noise is encountered. The impulsive noise is modeled as two components of the Gaussian mixture[12, 18, 23] and may be given as

$$v(i) = v_g(i) + v_{im}(i) = v_g(i) + b(i)v_w(i) \tag{6}$$

where $v_g(i)$ and $v_w(i)$ are independent zero mean Gaussian noise with variances σ_g^2 and σ_w^2, respectively; $b(i)$ is a switch sequence of ones and zeros and is modeled as an iid Bernoulli random process with occurrence probability $P_r(b(i)=1) = p_r$ and $P_r(b(i)=0) = 1-p_r$. The variance of $v_w(i)$ is chosen to be very large than that of $v_g(i)$ so that when $b(i)=1$, a large impulse is experienced in $v(i)$. The corresponding pdf of $v(i)$ in (3) is given by

$$f_v(x) = \frac{1-p_r}{\sqrt{2\pi}\sigma_g}\exp\left(\frac{-x^2}{2\sigma_g^2}\right) + \frac{p_r}{\sqrt{2\pi}\sigma_T}\exp\left(\frac{-x^2}{2\sigma_T^2}\right) \quad (7)$$

where $\sigma_T^2 = \sigma_g^2 + \sigma_w^2$ and $E[v^2(i)] = \sigma_v^2 = \sigma_g^2 + p_r\sigma_w^2$. It is noted that when $p_r = 0$ or 1, $v(i)$ is a zero-mean Gaussian variate.

D. *Distributed Robust Diffusion LMS Algorithm*

The distributed algorithms defined till now are simple and it provides good performance in WGN. But in reality, the sensor networks are working in environments where impulsive noise is common. This type of noise not occur frequently, but when it occurs the LMS based algorithms fail to perform satisfactorily. So we modify the distributed incremental LMS algorithm by adding non-linearity in the error term which is defined in (3).

This algorithm is similar to Diffusion LMS algorithm in[19], but here the weight update equation after local diffusion is modified accordingly the error modified algorithm given in [7]. The algorithm is given as:

$$\phi_k^{(i-1)} = \sum_{l \in \mathrm{N}_{k,i-1}} c_{kl}\psi_k^{(i-1)}, \qquad \phi_k^{(-1)} = 0$$

$$e_k(i) = d_k(i) - u_{k,i}\phi_k^{(i-1)}$$

$$\psi_k^{(i)} = \phi_k^{(i-1)} + \mu_k x_{k,i}^T g(e_k(i)) \quad (8)$$

where

$$g(y) = \int_0^y exp[-u^2/2\sigma_{sat}^2]du = \sqrt{\frac{\pi}{2}}erf\left[\frac{y}{\sqrt{2}\sigma_{sat}}\right]$$

III. SIMULATION RESULTS

The performance of the error saturation nonlinearity incremental LMS is evaluated through simulation study and the results are compared with those obtained by theoretical investigation. Further to access the performance of the proposed method both theoretical and simulation results of incremental LMS are also plotted. All simulations are carried out using regressor input with shift structure. The desired data are generated according to the model given in (1), and the unknown vector $\mathbf{w}^\circ$ is set to $[1,1,\ldots,1]^T/\sqrt{M}$.

In the same example as in [20] is simulated for facilitating comparison. The network consists of twenty nodes with each regressor size of (1×10) collecting data through a correlated process given by

$$u_k(i) = a_k \cdot u_k(i-1) + b_k \cdot n_k(i), \qquad i > -\infty \quad (9)$$

Here, $a_k \in [0,1)$ is the correlation index and $n_k(i)$ is a spatially independent white Gaussian process with unit variance and $b_k = \sqrt{\sigma_{u,k}^2 \cdot (1-a_k^2)}$. The regressor power profile $\{\sigma_{u,k}^2\} \in (0,1]$. The resulting regressors have Toeplitz covariance with co-relation sequence $r_k(i) = \sigma_{u,k}^2 \cdot (a_k)^{|i|}, i = 0,1,2,\ldots,M-1$. These parameters are chosen randomly, have taken same as in [20] for comparison purpose and are depicted in Figs.1 and 2. The background Gaussian noise with variance $\sigma_{g,k}$ are also generated randomly.

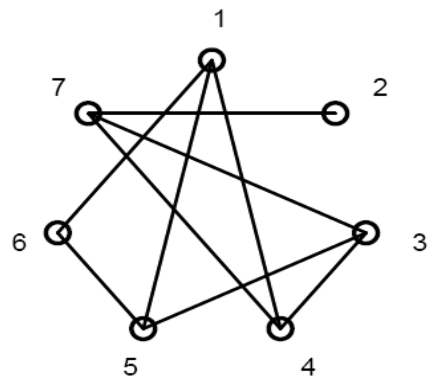
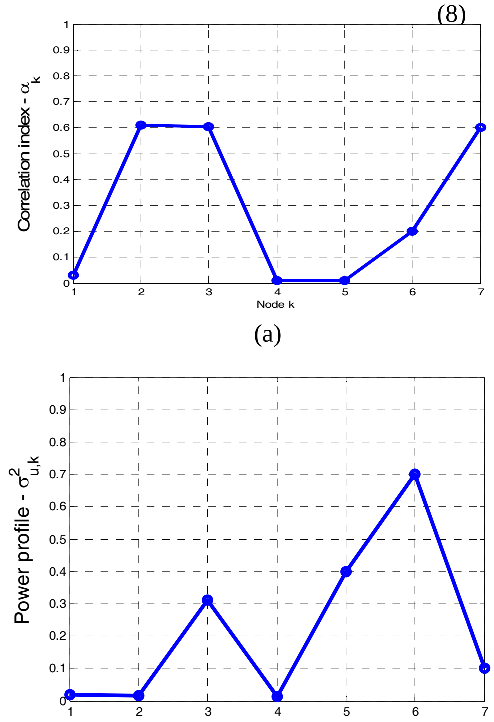
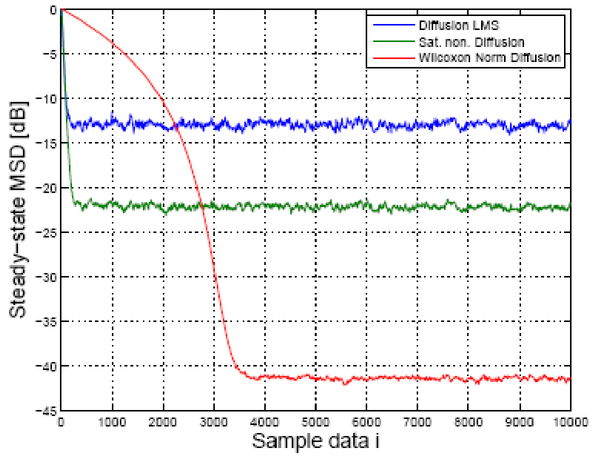


Fig.1 Network topology

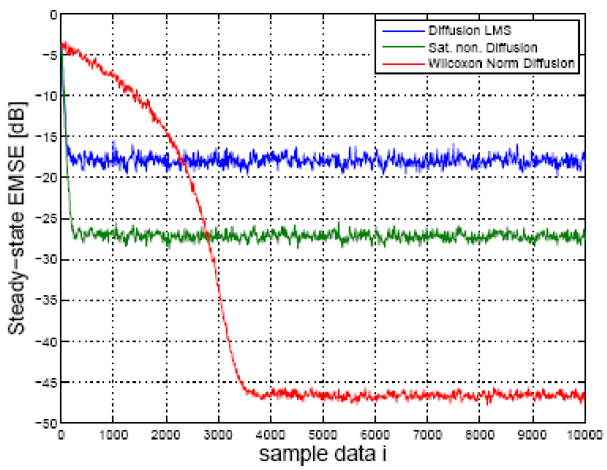


(b)

Figure 2: Network statistics (a) Regressor power profile. (b) Correlation index per node



(a)



(b)

Figure 3: Performance of the algorithm for $p_r = 0.1\ \sigma_s^2 = 0.01, \mu = 0.03, \sigma_g^2 = 0.001, \sigma_w^2 = 10000\sigma_g^2$ and the step size in Wilcoxon norm is $\eta = 0.002$ (a) global mean square deviation (MSD) (b) global excess mean square error (EMSE)

To generate the performance curves, ten independent experiments are performed and the results are averaged. The steady-state curves are generated by running the networks for 3000 iterations. The quantities such as the MSD, EMSE and MSE are obtained by averaging the last 300 samples of the corresponding learning curves.

The robustness of the algorithm in 10% impulsive noise is tested. The variance of impulsive noise is defined as 10^4 times the back ground Gaussian noise variance defined for each node in Fig. 3. Figure 3 clearly show the robustness of algorithm in impulsive noise over incremental LMS. The steady-state values attained by both the MSD and EMSE are around -25dB for MSD and -30dB for EMSE respectively where as the incremental LMS attains poor performances such as 10dB and 5dB for MSD and EMSE respectively.

The main objective of all the adaptive algorithms are to estimate optimum weights. If the noise is stationary, then the mean-square error is close to the background noise. But when the noise is non stationary, then the MSE does not converge. In this situation the estimated weights obtained by using LMS type algorithm diverge from the desired values. It is because the error is used directly in weight update equation of the adaptive algorithm. Therefore the MSD and EMSE do not converge and hence the steady-state values are high in case of distributed incremental LMS.

But in case of the saturation error nonlinearity distributed incremental algorithm, the error is not used directly in the weight update equation. In this case the error is fed through the Gaussian nonlinearity function, where the error is mapped within limit of $[-\sqrt{\frac{\pi}{2}}\sigma_s, \sqrt{\frac{\pi}{2}}\sigma_s]$ which is again limited by the proper of σ_s . The error may be high enough due to impulsive nature, but that mapped to small value with in the defined limit. So that the estimated weights are approaching towards the desired weight due to the presence of error nonlinearity in update equation. This reflects in the steady-state performance of the filter. The MSD and EMSE attain very low values indicating that Ψ_k^∞ is a good estimate for $\mathbf{w}^\circ$. But the error remains unchanged, so that the steady-state MSE does not converge in both the cases.

In case of Wicoxon norm diffusion algorithm, the error vector is weighted. The maximum error is then multipied with less weight to minimize its affect on weight upadate process. The computation in this method is more compare to saturation diffusion algorithm because of every time we have to arrange the error in ascending ordet to find the Wilcoxon norm.

IV. Conclusion

This paper presents the steady-state performance of robust distributed diffusion algorithms under the contaminated Gaussian impulsive noise which exhibits in simulation studies its robustness over the conventional diffusion LMS. One of the key results of this work is to show the robust estimation in impulsive noise environment over wireless sensor network. The proposed algorithms using diffusion distributed scheme need same computation and communication resources as required in case of diffusion LMS.

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