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## **Performance Analysis of OCV based Non Coherent MA Chaotic Communication System with Adaptive Multi User Receivers**

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# Performance Analysis of OCV based Non Coherent MA Chaotic Communication System with Adaptive Multi User Receivers

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**Abstract**—In this paper, a non coherent multiple access (MA) chaotic communication system using Orthogonal Chaotic Vectors (OCV) as spreading sequences with adaptive multi user receivers are tested and analysed. Here we consider two types of adaptive receivers 1) LMS based detector 2) CSE based detector. Theoretical equations for BER are also derived for both cases considering awgn channel. The simulation results are compared with that of theoretical results.

**Keywords**-Orthogonal Chaotic Vector (OCV); Least Mean Square(LMS);Chaotic Sequence Estimator(CSE); CSE-MMSE;Multiple-Access(MA), awgn.

## I. INTRODUCTION

From past two decades digital communication using chaotic signals is of active research area [1]-[5]. Chaotic signals are of interest in secure communications because of its random nature and sensitive dependence to initial condition of chaotic systems. Therefore chaotic signals can be used as an alternative for PN sequences for spread spectrum communication. Therefore, chaos based communication system has an inherent capability for multiple access. Many coherent and non-coherent chaos based multiple access communication systems has been proposed [6]-[7]. Due to the lack of practical implementation of coherent communication schemes, non-coherent systems find advantage over coherent systems. A non coherent multiple access communication system based on differential chaos shift keying (DCSK) was first proposed by Kolumbán *et. al.* [8]. Later in [9], an improved non coherent multiple access scheme for chaos based communication scheme using two types of adaptive receivers (1. Adaptive Traversal Filter (ATF) based receiver 2. Inverse and Average (IA) receiver) is discussed. Coulon *et. al.* [10], extended and generalized the work in [9], for both synchronous and asynchronous transmission and proposed four different adaptive receivers (1. Linear MMSE detectors 2. LMS detectors 3. Chaotic Sequence Estimator (CSE) 4. CSE – MMSE).

The quasi orthogonal characteristics of chaotic vector lead to residual cross correlation value which leads to multiple access interference (MAI) in multiple access environments. The effect of MAI due to quasi orthogonal characteristics of chaotic vectors can be reduced by using CSE-MMSE detector. In this paper we test and analyze the performance of the adaptive multiple access receivers presented in [10] i.e., LMS detector and CSE based detector when orthogonal chaotic vectors (OCV) are used as spreading sequences. The use of OCV results in zero MAI and thus eliminates the additional processing of the received signal required for MAI reduction.

This paper is organized as follows. Section II gives the brief description of the transmitter structure and the generation of the OCV. In Section III we analyze the performance of the LMS and CSE based detectors for awgn case and also analytical expressions for BER are derived and compared with that of simulated BER's. Finally, in Sections IV and V we have presented discussions and conclusion respectively.

## II. TRANSMITTER STRUCTURE

Transmitter structure of the  $i^{th}$  user is as shown in fig. 1 which is similar to the transmitter structure presented in [9]. The transmitter consists of an orthogonal chaotic vector generator, a group of delay blocks, switch and a multiplier. Data of each user is transmitted by modulating the orthogonal chaotic vector in the form of frames with the frame format for all users being identical. Each frame can be divided in to two parts 1)  $T_b$  number of slots for training and 2)  $L$  number of slots for data.

If  $x(k)^{(i)}$  is the chaotic carrier for  $i^{th}$  user and defining the number of chaotic samples used to transmit single binary bit as  $\beta$ . Gram-Schmidt ortho-normalization process is applied on chaotic vectors to obtain orthogonal chaotic vectors. Gram-Schmidt ortho-normalization process [11] for  $N_u$  number of sequences is given by

$$\hat{x}(k)^{(p)} = \frac{x(k)^{(p)} - \sum_{q=1}^{p-1} \left[ \sum_{k=1}^{\beta} x(k)^{(p)} \hat{x}(k)^{(q)} \right] \hat{x}(k)^{(q)}}{\sqrt{\sum_{k=1}^{\beta} \left[ x(k)^{(p)} - \sum_{q=1}^{p-1} \left[ \sum_{k=1}^{\beta} x(k)^{(p)} \hat{x}(k)^{(q)} \right] \hat{x}(k)^{(q)} \right]^2}} \quad (1)$$

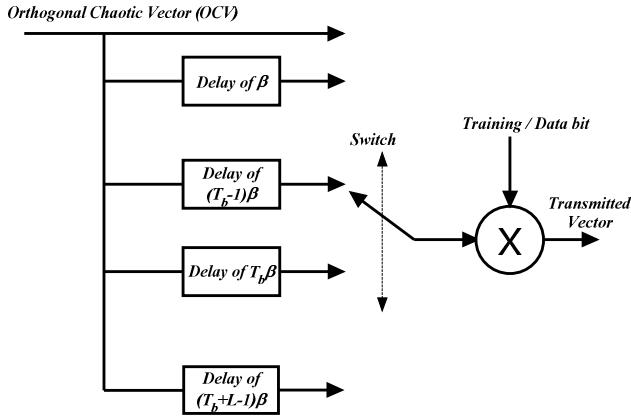


Figure 1. Transmitter structure of the  $i^{th}$  user

Where  $p = 2, 3, \dots, N_u$ , For  $p = 1$

$$\hat{x}(k)^{(1)} = \frac{x(k)^{(1)}}{\sqrt{\sum_{k=1}^{\beta} [x(k)^{(1)}]^2}} \quad (2)$$

### III. PERFORMANCE ANALYSIS

Assuming that the signal is corrupted only due to awgn, received signal  $s_l(k)$  can be represented as

$$s_l(k) = \begin{cases} \sum_{i=1}^{N_u} T_{i,l} \hat{x}_i(k) + \xi_l(k) & l = 1 \text{ to } T_b \\ \sum_{i=1}^{N_u} d_{i,l} \hat{x}_i(k) + \xi_l(k) & l = T_b + 1 \text{ to } T_b + L \end{cases} \quad (3)$$

Where,  $\xi_l(k)$  represents the additive white Gaussian noise in the  $l^{th}$  slot with zero mean and variance  $N_o/2$ .  $T_{i,l}$  and  $d_{i,l}$  are the  $l^{th}$  training and data bits of  $i^{th}$  user respectively. If  $\tilde{x}_j(k)$  is the estimated chaotic vector for  $j^{th}$  user. Then, the correlator output,  $z_{n,j}$  for the  $(n+1)^{th}$  data bit of  $j^{th}$  user is given by,

$$z_{n,j} = \sum_{k=(T_b+n)\beta+1}^{(T_b+n+1)\beta} \left\{ \left[ \sum_{i=1}^{N_u} d_{i,n} \hat{x}_i(k) + \xi_n(k) \right] \left[ \tilde{x}_j(k) \right] \right\} \quad (4)$$

Where,  $n = 0, 1, 2, 3, \dots, (L-1)$

#### A. LMS Detector:

In this type of detector with the use of training sequence and LMS algorithm the orthogonal chaotic vectors of each user are estimated iteratively. The LMS update equations for  $j^{th}$  user are given by,

$$e_{l,j} = \left( \sum_{k=m\beta+1}^{(m+1)\beta} \tilde{x}_{l-1,j}(k) s_l(k) \right) - T_{l,j} \quad (5)$$

Where,  $m = 0, 1, 2, \dots, (T_b - 1)$

$$\tilde{x}_{l,j}(k) = \tilde{x}_{l-1,j}(k) - \mu' e_{l,j} s_l(k) \quad (6)$$

$$\mu' = \frac{\mu}{\gamma + \sum_{k=m\beta+1}^{(m+1)\beta} s_l(k)^2} \quad (7)$$

Where,  $m = 0, 1, 2, \dots, (T_b - 1)$ ,

$\mu$  is the step size and  $\gamma$  is the small offset value.  $\tilde{x}_{l,j}(k)$  is the estimated chaotic vector of  $j^{th}$  user after  $l$  iterations which is initially set to null vector. Assuming that for a sufficiently large number of training data the estimation error is negligible, then

$$\tilde{x}_j(k) \approx \hat{x}_j(k) \quad (8)$$

Therefore, the correlator output is given by,

$$z_{n,j} = \sum_{k=(T_b+n)\beta+1}^{(T_b+n+1)\beta} \left\{ \left[ \sum_{i=1}^{N_u} d_{i,n} \hat{x}_i(k) + \xi_n(k) \right] \left[ \tilde{x}_j(k) \right] \right\} \quad (9)$$

Assuming that the correlator output has Gaussian distribution and considering the fact that the chaotic vectors used for each user is orthogonal to each other. Bit error rate of  $j^{th}$  user is given by,

$$BER^{(j)} = 0.5 \operatorname{erfc} \left( \sqrt{\frac{\hat{E}_b}{N_o}} \right) \quad (10)$$

Where,  $\hat{E}_b$  denotes the energy per bit in the demodulation process

In fig. 2 the plots of BER v/s Eb/No curves for the non coherent multiple access system using OCV as spreading sequences with LMS based detector are compared for different lengths of training sequence. BER curve for single user DCSK, single user antipodal CSK (ACSK) system with coherent receiver and BER obtained from eq. 10 are also included for comparison. As the length of training sequence is increased the BER curve converges towards the theoretical lower bound, which is given in eq. (10).

In fig. 3 we study the BER performance as a function of number of user's ( $N_u$ ) for different lengths of training sequence. We see that as the number of user's increases the performance of the system using non-OCV deteriorates

whereas when using OCV, BER remains unaffected by number of users provided the length of the training sequence is large.

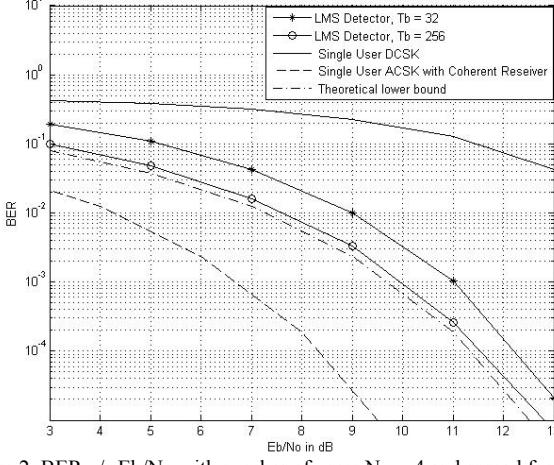


Figure 2. BER v/s Eb/No with number of users  $N_u = 4$  and spread factor  $2\beta = 200$

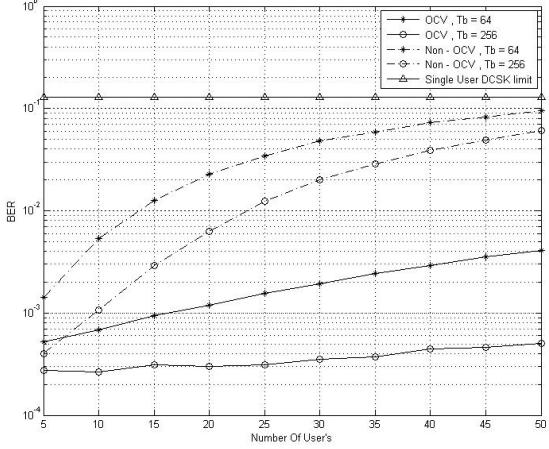


Figure 3. BER v/s Number of users ( $N_u$ ), with spread factor  $2\beta = 200$  and  $Eb/No = 11$  dB

The dependency of BER with spread factor ( $2\beta$ ) is studied in fig. 4. From fig. 4 we see that the BER performance of the system using OCV is much better than the system using non-OCV for smaller values of spread factor. As the spread factor is increased(above 1000) the BER performance for both OCV and non-OCV has identical BER's.

#### B. CSE Detector:

In this type of detector the estimated orthogonal chaotic vector  $\tilde{x}_j(k)$  is given by,

$$\tilde{x}_j(k) = \frac{1}{T_b} \left\{ \sum_{l=1}^{T_b} T_{l,j} [T_{l,j} \hat{x}_j(k) + \xi_l(k)] \right\} \quad (11)$$

$$\tilde{x}_j(k) = \hat{x}_j(k) + \frac{1}{T_b} \xi_j(k) \quad (12)$$

Where,

$\xi_j(k) = \sum_{l=1}^{T_b} T_{l,j} \xi_l(k)$  is the noise component with zero mean and variance  $No/2$ .

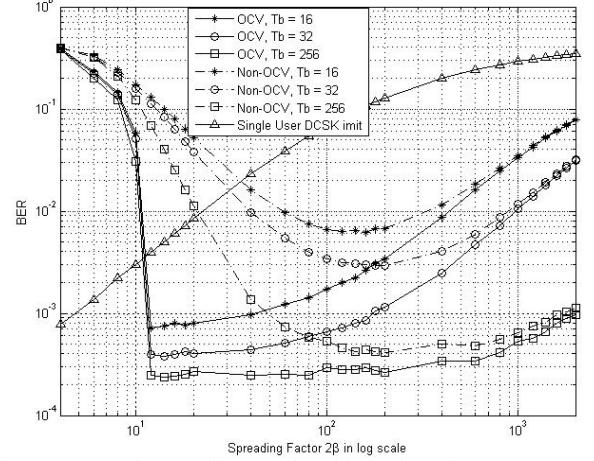


Figure 4. BER v/s Spread factor ( $2\beta$ ), with number of users  $N_u = 5$  and  $Eb/No = 11$  dB

Then the correlator output is given by,

$$z_{n,j} = \sum_{k=(T_b+n)\beta+1}^{(T_b+n+1)\beta} d_{n,l} \hat{x}_i(k)^2 + \sum_{i=1, i \neq j}^{N_u} \sum_{k=(T_b+n)\beta+1}^{(T_b+n+1)\beta} d_{n,l} \hat{x}_i(k) \hat{x}_j(k) + \sum_{k=(T_b+n)\beta+1}^{(T_b+n+1)\beta} \xi_n(k) \hat{x}_j(k) + \frac{1}{T_b} \sum_{i=1}^{N_u} \sum_{k=(T_b+n)\beta+1}^{(T_b+n+1)\beta} d_{n,l} \xi_j(k) \hat{x}_i(k) + \frac{1}{T_b} \sum_{i=1}^{N_u} \sum_{k=(T_b+n)\beta+1}^{(T_b+n+1)\beta} d_{n,l} \xi_j(k) \xi_n(k) \quad (13)$$

Assuming that the correlator output has Gaussian distribution then the BER of  $j^{th}$  user is given by,

$$BER^{(j)} = 0.5 \operatorname{erfc} \left( \frac{E[z_{n,j} | d_{n,j} = +1]}{\sqrt{2 \operatorname{var}[z_{n,j} | d_{n,j} = +1]}} \right) \quad (14)$$

Applying central limit theorem to eq. (13) we get,

$$E[z_{n,j} | d_{n,j} = +1] = \beta E[\hat{x}_i(k)^2] = \hat{E}_b \quad (15)$$

Where,  $\hat{E}_b$  denotes the energy per bit in the demodulation process.

$$\begin{aligned} \operatorname{var}[z_{n,j} | d_{n,j} = +1] &= \beta \frac{N_o}{2} E[\hat{x}_j(k)^2] + \\ &\quad \frac{N_u \beta N_o}{2 T_b} E[\hat{x}_i(k)^2] + \frac{\beta}{2} \left( \frac{N_o}{2} \right)^2 \end{aligned} \quad (16)$$

Using eq. (15) and (16) in eq. (14)

$$BER^{(j)} = 0.5 \operatorname{erfc} \left( \left[ \left( \frac{\hat{E}_b}{N_o} \right)^{-1} \left[ 1 + \frac{N_u}{T_b} + \frac{\beta}{2T_b} \left( \frac{\hat{E}_b}{N_o} \right)^{-1} \right] \right]^{-\frac{1}{2}} \right) \quad (17)$$

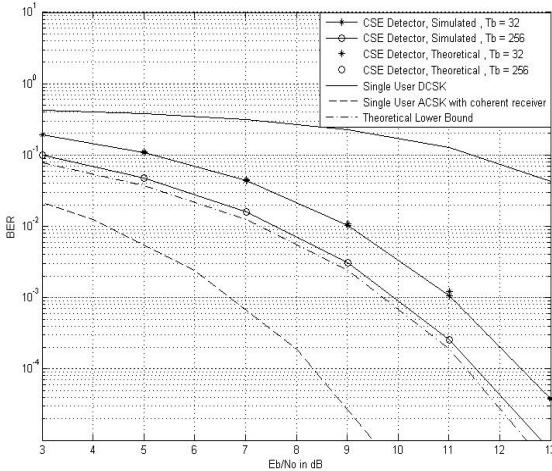


Figure 5. BER v/s Eb/No with number of users  $N_u = 4$  and spread factor  $2\beta=200$

Equation (17) converges to equation (10) when the length of the training sequence ( $T_b$ ) is very large. Therefore theoretical lower bound for the system with CSE based receiver is also equal to equation (10).

In fig. 5 the plots of BER v/s Eb/No curve for the non coherent multiple access system using orthogonal chaotic vectors as spreading sequence for different length of training sequence with CSE based detector are compared with that of single user DCSK, single user antipodal CSK (ACSK) system with coherent receiver and with theoretical BER derived in eq. (17). We see that the simulated and theoretical BER's are in exact match.

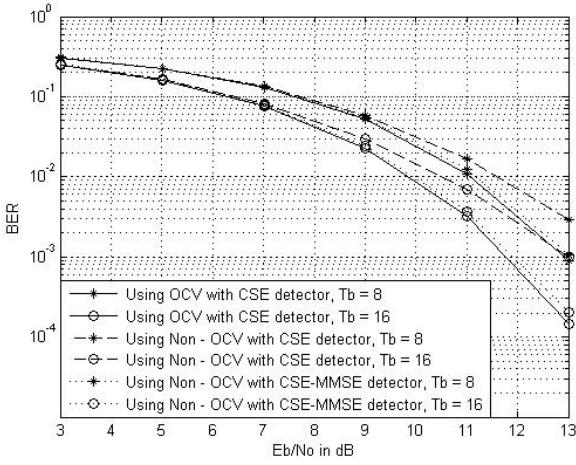


Figure 6. BER v/s Eb/No with number of users  $N_u = 4$  and spread factor  $2\beta = 200$

In fig. 6 BER plots for the non coherent multiple access system using OCV with CSE based detector and non coherent multiple access system using non – OCV with CSE and CSE-MSE based detector are compared. From BER

plots we can observe that the use of OCV will results in a better performance for higher Eb/No values where as for lower values of Eb/No the performance is almost identical. It is also observed from the figure that the performance of the system using OCV with CSE based detector is slightly better than that of using non-OCV with CSE-MMSE detector.

Fig. 7 gives the comparison of BER as a function of number of user's. We see that as the number of user's increases the BER of the system using OCV is constant provided the training sequence used is sufficiently large.

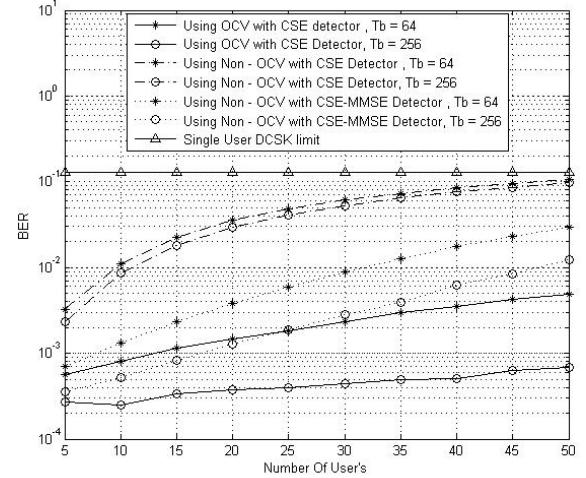


Figure 7. BER v/s Number of users ( $N_u$ ), with spread factor  $2\beta = 200$  and  $Eb/No = 11$  dB

The plots of BER as a function of spread factor ( $2\beta$ ) is shown in fig. 8. We see that the BER performance of the system using OCV is much better for small values(less than 1000) of spread factor than the system using non-OCV with CSE and CSE-MMSE based detectors.

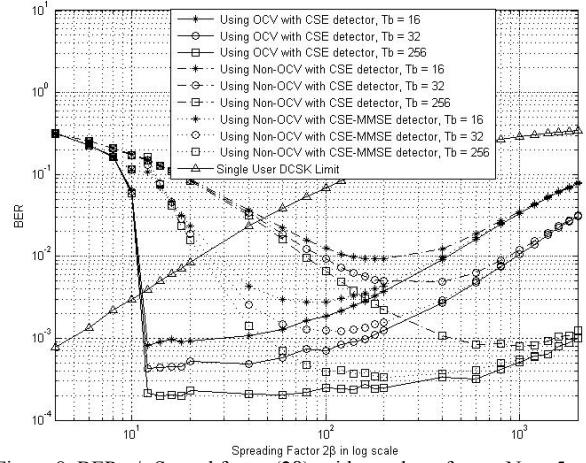


Figure 8. BER v/s Spread factor ( $2\beta$ ), with number of users  $N_u = 5$  and  $Eb/No = 11$  dB

## IV. DISCUSSION

### A. Training Sequence:

The convergence time of the adaptive sequence estimators depends on the choice of training sequence. By using orthogonal training sequence for each user will reduce

the convergence time for LMS based detector. The equation for BER for system using CSE based detector is derived by considering that the training sequence for each user are orthogonal. If the non orthogonal training sequences are used then the BER performance converges to equation (10) when length of the training sequence is very large. The advantage of using orthogonal codes as training sequences is lost if the transmission is asynchronous.

In this paper, all simulations are done by using Walsh-hadamard codes as training sequence.

#### B. Bit Energy to Noise ratio:

In equations (10) and (18) the BER's are expressed in terms of  $\hat{E}_b$ , which is the energy per bit in demodulation process. If  $E_b^{(j)}$  is the average bit energy of the  $j^{\text{th}}$  user. Then  $E_b^{(j)}$  and  $\hat{E}_b$  are related by,  $E_b^{(j)} = 2\hat{E}_b$ .

All BER plots in this paper are plotted with respect to average bit energy  $E_b$  to noise ratio  $E_b/N_o$  and it is assumed that the energy per bit of all user's are equal. If  $\hat{E}_b$  is used instead of  $E_b$  then the BER plots will be same but with -3 dB shift.

#### C. Spread Factor:

Each slot in the transmit frame consists of  $\beta$  number of chaotic samples which are modulated by either training bit or data bit. Assuming the length of training sequence  $T_b$  is equal to the length of data sequence in each frame  $L$ . Under this assumption it requires  $2\beta$  samples to transmit one bit of information. Thus the average spreading factor per bit is equal to  $2\beta$ .

#### D. Bit Error Rate

In the derivation of BER equation for LMS based detector we have not considered the estimation error which becomes negligible when very long training sequence is used for training. Therefore, eq. (10) provides only the theoretical lower bound for LMS based detectors as well as for CSE based detectors when orthogonal chaotic vectors are used as spreading sequences.

#### E. Synchronous and asynchronous transmission

The transmission is said to be as synchronous if the transmission of data from all users is done at same instant else the transmission is said to be as asynchronous.

Orthogonal Chaotic Vectors(OCV) can't be used when the transmission is asynchronous. Since, under the asynchronous transmission orthogonal property is lost and thus the performance degrades and for the generation of OCV requires the knowledge about the chaotic vectors used by all other users which leads to additional processing of data. Therefore, OCV based non-coherent MA chaotic communication system finds advantage under synchronous transmission (for example, Broadcast transmission). For

asynchronous transmission non-OCV with CSE-MMSE receiver can be used.

## V. CONCLUSION

In this paper, the performance of non coherent system using orthogonal chaotic vectors with LMS and CSE based detectors have been tested and analyzed. Theoretical equations for BER are derived. BER for the LMS based detector is derived ignoring the estimation error, which becomes negligible when the length of training sequence is very large. For non coherent MA system using orthogonal chaotic vector with CSE based detector has an excellent BER performance than that of system using non orthogonal chaotic vectors with CSE and CSE-MMSE based detector. From simulation results (Fig. 3,4,7&8) and also from derived BER equation (eq. 10 & 17) we observe that when sufficiently long training sequence is used the BER for the non coherent multiple access system using OCV as spreading sequence is least affected by spread factor and number of users.

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