Maximum Likelihood DOA Estimation in Distributed Wireless Sensor Network Using Adaptive Particle Swarm Optimization

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ABSTRACT

Source direction of arrival (DOA) estimation is one of the challenging problem in wireless sensor network. Several methods based on maximum likelihood (ML) criteria has been established in literature. Generally, to obtain the exact ML (EML) solutions, the DOAs must be estimated by optimizing a complicated nonlinear multimodal function over a high-dimensional problem space. An adaptive particle swarm optimization (APSO) based solution is proposed here to compute the ML functions and explore the potential of superior performances over traditional PSO algorithm. Simulation results confirms that the APSO-ML estimator is significantly giving better performance at lower SNR compared to conventional method like MUSIC in various scenarios at less computational costs.

Categories and Subject Descriptors

H.4 [**Wireless Sensor Network Application**]: Source localization; D.2.8 [**Array sinal processing**]: [performance measures]

General Terms

Theory

Keywords

MUSIC, Direction-of-arrival, Maximum Likelihood Estimation, PSO, APSO.

1. INTRODUCTION

A wireless sensor network consists of groups of sensors or nodes using wireless links to perform distributed sensing tasks by coordinating among themselves. These sensors are deployed for carrying out specialized tasks like surveillance and security, environmental monitoring, transport, precision agriculture, manufacturing and inventory tracking and health care [1]. The principal advantage is their ability to be deployed in almost all kinds of terrain with hostile environment where it might not be possible or difficult to use conventional wired networks. DOA estimation is an important problem in WSN to estimate the source location which is an well-known problem in the fields of radar, sonar, radio-astronomy, underwater surveillance and seismology etc. One of the simplest versions of this problem is the estimation of the directions-of-arrival (DOAs) of narrow-band sources where the sources are located in the far field of the sensor array [16].

Many high resolution suboptimal techniques have been proposed and analyzed, such as multiple signal classification (MUSIC) [12], the minimum variance method of Capon, estimation of signal parameters via rotational invariance technique (ESPRIT) and more [5]. The ML technique is used here because of its superior statistical performance compared to spectral based methods. The ML method is a standard technique in statistical estimation theory. A likelihood function can be formulated easily if we know the observed parametric data [13]. The ML estimate is computed by maximizing the likelihood function or minimizing the negative likelihood function with respect to all unknown parameters, which may include the source DOA angles, the signal covariance, and the noise parameters. Since the ML function is multimodal, so direct optimization is seems to be unrealistic due to large computational burden.

There are different optimization techniques available in literature for optimization of ML function like AP-AML [16], simulated annealing (SA) [15], genetic algorithms (GA) [6] fast EM and SAGE algorithms [2] and a local search technique e.g. Quasi-Newton methods. GA is one of the most powerful and popular global search tools; however, its implementation is somewhat cumbersome due to slow convergence. All these techniques have several limitations because of multidimensional cost function which need extensive computation, good initialization is also crucial for global optimization and we can not guarantee that these local search techniques always have global converge.

The evolutionary algorithms like genetic algorithm [11], particle swarm optimization and simulated annealing [15] can be designed to optimize the ML function. Genetic algorithm [6] and particle swarm optimization [7] had already used as a global optimization technique to estimate the DOA for uniform array. Here the authors are trying to use these evoloutionary technique in distributed sensor network to estimate sources angle of arrival.

The adaptive particle swarm optimization (APSO) algorithm is applied here to ML criterion functions for accurate DOA estimation. As an emerging technology, PSO has attracted a lot of attention in recent years, and has been successfully applied in many fields, such as phased array synthesis [9], electromagnetic optimization [10]. Most of the applications demonstrated that PSO could give competitive or even better results in a faster and cheaper way, compared with other heuristic methods such as GA. Recently adaptive PSO algorithm has been successfully applied in power dispatch problem [8] and shown that it is giving better performance compared to different constrained PSO.

Due to the multimodal, nonlinear, and high-dimensional nature of the parameter space, the problem seems to be a good application area for APSO, by which the excellent performance of ML criteria can be fully explored. Via extensive simulation studies, we demonstrate that with properly chosen parameters, APSO achieves fast and robust global convergence over PSO and MUSIC.

2. DATA MODEL AND MAXIMUM LIKELIHOOD ESTIMATION PROBLEM

Let us consider an array of M WSN nodes are distributed in an arbitrary geometry and received signals form N narrow band far-field signal sources at unknown locations. The output of sensor nodes modeled by standard equation as

$$\mathbf{x}(i) = \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(i) + \mathbf{n}(i), \qquad i = 1, 2, \ldots, L \tag{1}$$

where $\mathbf{s}(i)$ is the unknown vector of signal waveforms, $\mathbf{n}(i)$ is unpredicted noise process, , L denotes the number of data samples (snapshots). The matrix $\mathbf{A}(\boldsymbol{\theta})$ has the following special structure defined as

$$\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1), \ldots \mathbf{a}(\theta_N)] \tag{2}$$

where $\mathbf{a}(\theta)$ is called steering vector and $\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_N]^T$ are the parameters of interest or true DOA's. The exact form of $\mathbf{a}(\theta)$ depends on the position of the nodes in sensor network.

Further, the vectors of signals and noise are assumed to be stationary, temporarily white, zero-mean complex Gaussian random variables with the following second-order moments given by

$$\begin{aligned} E[\mathbf{s}(i)\mathbf{s}(j)^{\mathrm{H}}] &= \mathbf{S}\delta_{ij} & E[\mathbf{s}(i)\mathbf{s}(j)^{\mathrm{T}}] &= 0 \\ E[\mathbf{n}(i)\mathbf{n}(j)^{\mathrm{H}}] &= \sigma^2\mathbf{I}\delta_{ij} & E[\mathbf{n}(i)\mathbf{n}(j)^{\mathrm{T}}] &= 0 \end{aligned} \tag{3}$$

where δ_{ij} is the Kronecker delta, $(\cdot)^{\mathrm{H}}$ denotes complex conjugate transpose, $(\cdot)^{\mathrm{T}}$ denotes transpose, $E(\cdot)$ stands for expectation.

Under the assumptions taken above, the observation process, $\mathbf{x}(i)$, constitutes a stationary, zero-mean Gaussian random process having second-order moments

$$E[\mathbf{x}(i)\mathbf{x}(i)^{\mathrm{H}}] = \mathbf{R} = \mathbf{A}(\boldsymbol{\theta})\mathbf{S}\mathbf{A}^{\mathrm{H}}(\boldsymbol{\theta}) + \sigma^2\mathbf{I} \tag{4}$$

The problem addressed herein is the estimation of $\boldsymbol{\theta}$ along with the parameter in $\mathbf{S}$ and σ^2(noise power) from a batch of L measured data $\mathbf{x}(1), \ldots, \mathbf{x}(L)$.

Under the assumption of additive Gaussian noise and complex Gaussian distributed signals we can have negative log-likelihood function [7] is given as

$$\ell(\boldsymbol{\theta}, \mathbf{S}, \sigma^2) = \log|\mathbf{R}| + \mathrm{tr}\{\mathbf{R}^{-1}\hat{\mathbf{R}}\} \tag{5}$$

where $\hat{\mathbf{R}}$ is the sample covariance matrix and it defined as

$$\hat{\mathbf{R}} = \frac{1}{N}\sum_{i=1}^{N} \mathbf{x}(i)\mathbf{x}(i)^{\mathrm{H}} \tag{6}$$

The ML criterion function can be concentrated with respect to $\mathbf{S}$ and σ^2 by following [3]. The stochastic maximum likelihood (SML) estimates of the signal covariance matrix and the noise power are obtained by inserting the SML estimates of $\boldsymbol{\theta}$ in the following expressions

$$\hat{\mathbf{S}}(\boldsymbol{\theta}) = \mathbf{A}^{\dagger}(\boldsymbol{\theta})\left(\hat{\mathbf{R}} - \hat{\sigma}^2\mathbf{I}\right)\mathbf{A}^{\dagger\mathrm{H}}(\boldsymbol{\theta}) \tag{7a}$$

$$\hat{\sigma}^2(\boldsymbol{\theta}) = \frac{1}{M-N}\mathrm{Tr}\{\mathbf{P}_{\mathbf{A}}^{\perp}(\boldsymbol{\theta})\hat{\mathbf{R}}\} \tag{7b}$$

where $\mathbf{A}^{\dagger}$ is the psedo-inverse of $\mathbf{A}$ and $\mathbf{P}_{\mathbf{A}}^{\perp}$ is the orthogonal projection onto the null space of $\mathbf{A}^{\mathrm{H}}$ and are defined as

$$\mathbf{A}^{\dagger} = \left(\mathbf{A}^{\mathrm{H}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{H}} \tag{8a}$$

$$\mathbf{P}_{\mathbf{A}} = \mathbf{A}\mathbf{A}^{\dagger} \tag{8b}$$

$$\mathbf{P}_{\mathbf{A}}^{\perp} = \mathbf{I} - \mathbf{P}_{\mathbf{A}} \tag{8c}$$

Therefore the concentrated form of the EML function now can be obtained bu using (7) in (5) as

$$f_{EML}(\boldsymbol{\theta}) = \log|\mathbf{A}(\boldsymbol{\theta})\hat{\mathbf{S}}(\boldsymbol{\theta})\mathbf{A}^{\mathrm{H}}(\boldsymbol{\theta}) + \hat{\sigma}^2(\boldsymbol{\theta})\mathbf{I}| \tag{9}$$

Stoica and Nehorai [14] proved that for uncorrelated sources, the statistical performances of CML and EML are similar; while for highly correlated or coherent sources, the performance of UML is significantly superior.

3. ADAPTIVE PARTICLE SWARM OPTIMIZATION (APSO)

Particle swarm optimization (PSO) is a population based stochastic optimization technique developed by Dr. Eberhart and Dr. Kennedy in 1995 [4]. The swarm is typically modeled by particles in multidimensional space that have a position and a velocity. Consider a D-dimensional problem space and a swarm consisting of P particles. The position of the ith particle is a D-dimensional vector $x_i = [x_{i1}, x_{i2}, \ldots, x_{iD}]$. The velocity of this particle is represented as $v_i = [v_{i1}, v_{i2}, \ldots, v_{iD}]$. These particles fly through hyperspace (i.e., $\mathbb{R}^D$) and have two essential reasoning capabilities: their memory of their own best position and knowledge of the global or their neighborhood's best. The best previous position of the ith particle, which gives the best fitness value, is denoted as $p_i = [p_{i1}, p_{i2}, \ldots, p_{iD}]$ and the best position found by any particle in the swarm is represented by $p_g = [p_{g1}, p_{g2}, \ldots, p_{gD}]$. In a minimization optimization problem, problems are formulated so that best simply means the position with the smallest objective value. Members of a swarm communicate good positions to each other and adjust their own position and velocity based on these good positions. So a particle has the following information to make a suitable change in its position and velocity:

At every iteration, the velocity and the position of each particle are updated according to the following equations:

$$\mathbf{v}_j^{n+1} = w^n \mathbf{v}_j^n + c_1 \mathbf{r}_1^n \odot (\mathbf{p}_j^n - \mathbf{x}_j^n) + c_2 \mathbf{r}_2^n \odot (\mathbf{p}_g^n - \mathbf{x}_j^n) \quad (10)$$

$$\mathbf{x}_j^{n+1} = \mathbf{x}_j^n + \mathbf{v}_j^{n+1} \quad (11)$$

where $\odot$ denotes element-wise product, $j = 1, 2, \ldots, P$, and $n = 1, 2, \ldots,$ indicates the iterations, w is a parameter called the inertia weight, c_1 and c_2 are positive constants referred to as *cognitive* and *social* parameters respectively, $\mathbf{r}_1$ and $\mathbf{r}_2$ are D-dimensional vectors consisting of independent random numbers uniformly distributed between 0 and 1, which are used to stochastically vary the relative pull of $\mathbf{p}_i$ and $\mathbf{p}_g$ in order to simulate the unpredictable component of natural swarm behavior. The inertial weight w is considered critical for the convergence behavior of PSO. A larger w facilitates searching new area and global exploration while a smaller w tends to facilitate local exploitation in the current search area. In this study, w is selected to decrease during the optimization process. In order to increase the search ability, the algorithm should be redefined in the manner that the movement of the swarm should be controlled by the objective function. In [8], the particle position is adjusted such that the highly fitted particle (best particle) moves slowly when compared to the lowly fitted particle. This can be achieved by selecting different weight w_i values between w_{min} and w_{max} for each particle in a particular iteration is according to their rank, as in the following form:

$$w_i = w_{min} + \frac{(w_{max} - w_{min}) \times \text{Rank}_i}{\text{Total Population}} \quad (12)$$

So, from 12 it can be seen that the best particle takes the first rank, and the inertia weight for that particle is set to the minimum value while that for the lowest fitted particle takes the maximum inertia weight, which makes that particle move with a high velocity.

4. APSO-EML DOA ESTIMATION

Here we describe the formulation of the APSO algorithm for EML optimization to estimate source DOA's. At first initialize a population of particles in the search space with random positions and random velocities constrained between 0 and π in each dimension [7]. The N-dimensional position vector of the jth particle takes the form $x_j = [\theta_1, \ldots, \theta_N]$, where θ represents the DOAs. A particle position vector is converted to a candidate solution vector in the problem space through a suitable mapping. The score of the mapped vector evaluated by a likelihood function f_{EML} which is given in (9), is regarded as the fitness of the corresponding particle.

To evaluate the likelihood function f_{EML} required the the data from all the nodes for K number of snapshots. During the evolution of algorithm, in every iteration each particle update their velocity and position, then evaluate the global best. The manipulation of a particles velocity according to (10) is regarded as the central element of the entire optimization. Since there was no actual mechanism for controlling the velocity of a particle, it is necessary to define a maximum velocity to avoid the danger of swarm explosion and divergence. The velocity limit can be applied along each dimension at every node as

$$v_j^n = \begin{cases} V_{\text{MAX}}, & \text{if } v_j^n > V_{\text{MAX}} \\ V_{\text{MIN}}, & \text{if } v_j^n < V_{\text{MIN}} \end{cases} \quad (13)$$

In this work, we keep the limitation of V_{MAX} is set to the half value of the dynamic range, i.e., $V_{\text{MAX}} = 0.5$. The new particle position is calculated using (10).

The optimization iteration will be terminated if the specified maximum iteration number is reached. The final global best position p_g is taken as the ML estimates of source DOA.

5. SIMULATION RESULTS AND DISCUSSIONS

Here we present a numerical example to demonstrate the performance of APSO based DOA estimation using (9) against PSO and MUSIC which is the best known and well investigated algorithm. The performances of those methods are compared in two ways: (a) the DOA estimation root-mean-squared error (RMSE), which is calculated as

$$\text{RMSE} = \sqrt{\frac{1}{N N_{\text{run}}} \sum_{l=1}^{N_{\text{run}}} \sum_{n=1}^{N} \left(\hat{\theta}_n(l) - \theta_n\right)^2} \quad (14)$$

where N is the number of sources, $\hat{\theta}_n(l)$ is the estimate of the nth DOA achieved in the lth run, θ_n is the true DOA of the nth source; and (b) the ability to resolve closely spaced sources known as probability of resolution (PR). By definition, two sources are said to be resolved in a given run if both $|\hat{\theta}_1 - \theta_1|$ and $|\hat{\theta}_2 - \theta_2|$ are smaller than $|\theta_1 - \theta_2|/2$. Let us assume that two equal-power, uncorrelated signals impinge on distributed wireless sensor network with sixteen sensors from 150° and 158°. The SNR varies from -20 dB to 30 dB with the step size of 1 dB taken for simulation. The function f_{EML} is optimized using PSO and APSO algorithms for 20 snapshots in case of PSO and APSO, but 1000 snapshots are taken for MUSIC. The Fig. 1 gives the the DOA estimation RMSE values obtained using PSO-EML, APSO-EML, and MUSIC as a function of SNR. Fig. 2 shows the resolution probabilities (RP) for the same methods. Two sources are considered to be resolved in an experiment if both DOA estimation errors are less than the half of their angular separation.

As can be seen from Fig. 1 and Fig. 2, APSO-EML yields significantly superior performance over PSO-EML as a whole, by demonstrating lower DOA estimation RMSE and higher resolution probabilities. The accurate DOA estimates are observed because 1) EML criterion functions are statistically optimal although computation-extensive, and 2) the designed APSO is a robust and reliable global optimization algorithm. MUSIC, on the other hand, produces less accurate estimates.

6. CONCLUSION

In this paper, a modified PSO known as APSO is proposed to estimate the source DOA using ML function. With

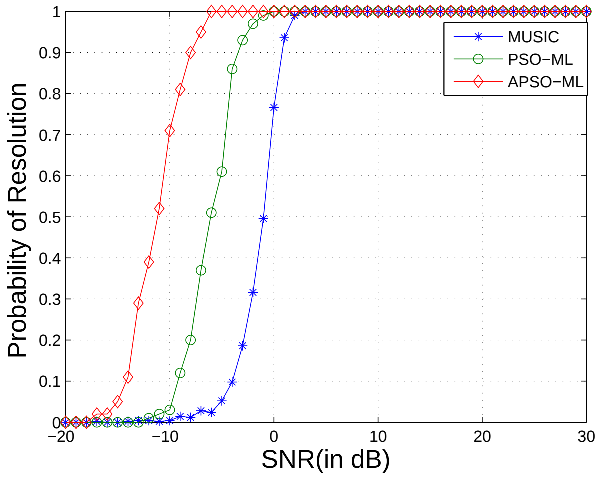

Figure 1: DOA estimation RMSE values of PSO-EML, APSO-EML, and MUSIC versus SNR. Two uncorrelated sources impinge on WSN with 16 sensors at 150° and 158°.

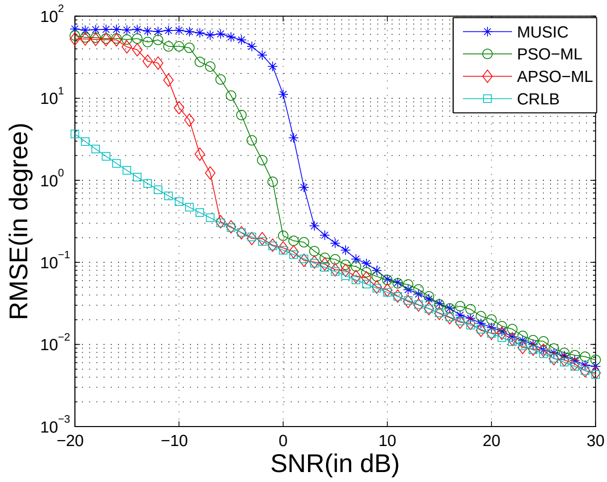

Figure 2: Resolution probabilities of PSO-EML, APSO-EML and MUSIC versus SNR. Two uncorrelated sources impinge on WSN with 16 sensors at 150° and 158°.

newly introduced features matching scheme and intelligent initialization, carefully selected evolution operators and fine-tuned parameters, the APSO-ML estimator achieves fast convergence. Simulation results demonstrate that APSO-ML asymptotically attain statistical CRLB at lower SNR than other estimators like MUSIC.

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