# Sidelobe Reduction of LFM Signal Using Convolutional Windows

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***Abstract*— Pulse compression technique is used to enhance radar performance in terms of more efficient use of high power transmitters and increasing the system resolving capability. The linear frequency modulated (LFM) waveform is widely used in radar because it can be generated easily and is Doppler tolerant. But the matched filter output of this signal contains range sidelobes. To reduce these sidelobes different types of windows are used as the weighing function at the receiver. In this paper convolutional windows are applied as weighing function for radar pulse compression which are more insensitive to Doppler shift as compared to conventional windows. It is observed that the radar pulse compression technique using convolutional window as weighing function has higher peak to sidelobe ratio (PSR) at higher Doppler shifts.**

***Keywords*— Pulse compression, LFM, Doppler shift, Convolutional windows, PSR.**

## I. Introduction

In modern radar pulse compression technique is used to achieve long-range detection and good resolution simultaneously. A long pulse of carrier wave is transmitted to get the required energy for long range target detection. A wide bandwidth which is associated with short pulse is obtained by modulating the carrier. In the receiver a matched filter is used to compress the energy into short pulse to get required range resolution. From Fourier transform it is known that a signal with bandwidth B cannot have duration shorter than 1/B; i.e. its time bandwidth product cannot be less than unity. The range resolution of a radar signal is inversely related to bandwidth.

In most of the practical radar systems linear frequency modulated (LFM) waveform is extensively used. The matched filter output of a point target for an arbitrary pulse design is the autocorrelation function (ACF) which forms a Fourier transform pair with the energy spectrum of the signal. For rectangular amplitude weighing, the energy spectrum of an LFM can be approximated as sin(x)/x or sinc(x) shaped ACF. So a compressed LFM signal at the receiver will produce a series of sidelobes surrounding the mainlobe and the first sidelobe occurs at a level of -13 dB compared to the peak of the mainlobe. Most of the cases these sidelobes are undesirable. The conventional method used to suppress these ambiguous sidelobes by modelling the rectangular shape of the chirp spectrum using amplitude weighing. The range side-lobes can be suppressed to the required level by using a suitable window function.

Range sidelobes are inherent part of the pulse compression mechanism and these are occurring due to abrupt rise in the signal spectrum. In radar systems, weighing technique in time or in the frequency domain is mostly used to reduce these range sidelobes with broadening in the mainlobe. Time domain weighing is preferred to frequency domain weighing, because it produces lower sidelobe compression output [1 2 3]. Although weighing when used both on transmitter and receiver provides better results, weighing on receive is preferred because weighing on transmit leads to a power loss since the available transmit power cannot be fully utilizes. Shennawy *et. al.* [4] have used an external Hamming window as weighing function in frequency domain to suppress the range sidelobe from a time-bandwidth product of 50 up to the value of 720. Using the weighing technique the dynamic range of pulse compression system increased. Hamming weighing is used to suppress the range sidelobes for rectangular LFM pulses with time-bandwidth product less than 170 and it is observed from the results that Hamming weighing in time domain produces lower largest range sidelobe as compared to Hamming weighing in frequency domain [5].

The rest of the paper is organised as follows. A brief description about LFM signal described in section-II. The procedure for obtaining the convolutional window is presented in section III. Simulation results are presented by taking various windows as weighing function given in section IV. Finally in section V the conclusions of the investigation are provided.

## II. LFM Signal

An LFM pulse having rectangular envelope mathematically described as

$$\mathrm{S}(t) = \exp\left[ j2\pi \left( f_0 t + \frac{B}{2T} \right) \right] \qquad |t| \le \frac{T}{2} \qquad (1)$$

where $f_0$ =centre frequency.

$B$ =Bandwidth

$T$ = Duration.

The output of the receiver matched filter or compression filter is a pulse with $\frac{\sin(x)}{x}$ envelope as shown in Fig.1.

Due to Doppler shift, when the radar waveform reflects from a moving target changes the radar waveform. Objects with larger velocities experience detection range degradation due to Doppler shift. The reflected pulse is mathematically represented as multiplying the transmitted code with $\exp\left[j2\pi\frac{f_d}{B}t\right]$ and passed through a receiver filter whose impulse response is matched to transmitted expanded pulse. Here $f_d$ is the Doppler shift. So the Doppler shifted reflected pulse is no longer matched to the receiver filter hence signal to noise ratio (SNR) loss occurs.

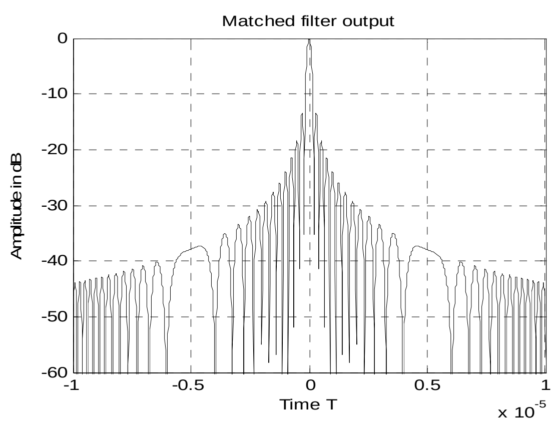


Fig.1 Matched filter output of LFM signal

## III. Convolutional Windows

Windows are time domain weighing functions that are used to reduce Gibbs oscillations caused by the truncation of a Fourier series. They are employed in a variety of traditional applications including power spectral estimation, beam forming and digital filter design. Window functions are generally categorized as fixed or adjustable. Fixed windows have window length as the parameter which alters the mainlobe width. Adjustable windows have two parameters, namely, the window length and a parameter that alters the relative sidelobe amplitude. Classical windows are used to detection of harmonic signals in the presence of nearby strong harmonic interference [6]. Geckinli and Yavuz [7] have introduced some novel windows and compare their frequency domain properties.

Convolutional windows are derived by convolving the window with itself. Reljin *et. al*. [8] have discussed a class of windows that are generated by the time convolution of classical windows to obtain both flat top high sidelobe attenuation. These windows are suitable for harmonic amplitude evaluation in nonsynchronous sampling case. The convolutional windows from second to eighth order for rectangular window are derived in [9]. These windows applied for high accuracy harmonic analysis and parameter estimation of periodic signals. Phase difference algorithm based on Nuttal self-convolutional window is used to eliminate the measurement errors of dielectric loss factor [10]. Dielectric loss factor is caused by non-synchronised sampling and non-integral periodic truncation conditions. A self convolution Hanning window used to complex signal harmonics parameter estimation is presented in [11]. The convolutional window based phase correction algorithm suppresses the impact of fundamental frequency fluctuation and white noise on harmonic estimation.

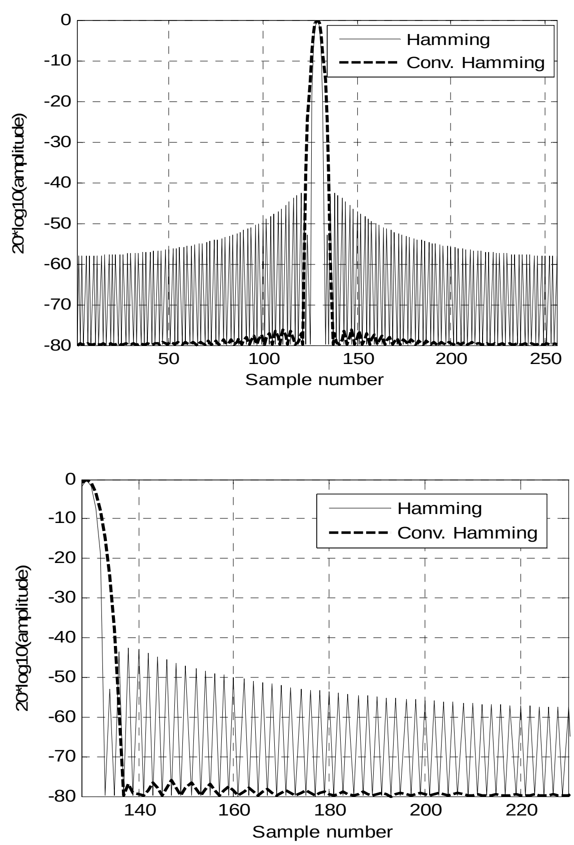


Fig.2 Frequency response curve (a)Hamming And Convolutional Hamming window (b)Zoomed version

The windows used in this paper are

(1) Hamming window

$$w(t)=\begin{cases}0.54+0.46\cos\left(\frac{2\pi\,t}{T}\right) & \left|\frac{t}{T}\right|\le\frac{1}{2}\\ 0 & elsewhere\end{cases}$$

(2) Hanning Window

$$w(t)=\begin{cases}\frac{1}{2}+\frac{1}{2}\cos\left(\frac{2\pi\,t}{T}\right) & \left|\frac{t}{T}\right|\le\frac{1}{2}\\ 0 & elsewhere\end{cases}$$

(3) Kaiser window

$$w(t)=\begin{cases}\dfrac{I_0\left[\beta\sqrt{1-\left(2\frac{t}{T}\right)^2}\right]}{I_0(\beta)} & \left|\frac{t}{T}\right|\le\frac{1}{2}\\ 0 & elsewhere\end{cases}$$

(4) Chebysev window

$$w(t)=\begin{cases}\dfrac{I_1\left[\beta\sqrt{1-\left(2\frac{t}{T}\right)^2}\right]}{I_1(\beta)\sqrt{1-\left(2\frac{t}{T}\right)^2}}+\dfrac{1}{\beta I_1(\beta)}\delta(1-2|t|) & \left|\frac{t}{T}\right|\le\frac{1}{2}\\ 0 & elsewhere\end{cases}$$

The parameter $\beta$ can be determined from the sidelobe attenuation $R$,

$$\beta=cosh^{-1}(R)$$

Frequency response curve of Hamming window and convolutional Hamming window is presented in Fig.2. From Fig.2 it is observed that the sidelobes of convolutional Hamming window is lower than the Hamming window but the mainlobe is widened.

## IV. Simulation Result

Here LFM signal having duration, centre frequency and bandwidth of 10µs, 30MHz, and 5MHz respectively is used for simulation study. Due to Doppler shift the sidelobes which are nearer to mainlobe are mostly affected. The weighing function which can suppress the near in sidelobes are more Doppler tolerant. Different weighing functions which are described in section III is used as weighing function. A window is convolved with itself to get the convolutional window. The output of matched filter for different Doppler shift using Hamming and convolutional Hamming window is depicted in Fig. 3. It is observed from Fig. 3 that the convolutional Hamming window has lowered the near in sidelobes, which are mostly affected by the Doppler shift, as compare to Hamming window. At higher Doppler shift the

TABLE I
Comparison of PSR for different Doppler shift

| Doppler shift $\left(\frac{f_d}{B}\right)$ | PSR using Hamming window in dB | PSR using convolutional hamming window in dB |
|---|---|---|
| 0.01 | 36.2 | 34.46 |
| 0.05 | 32.8 | 33.67 |
| 0.1 | 28.6 | 32.5 |
| 0.15 | 25.2 | 31 |
| 0.2 | 22.2 | 29 |
| Doppler shift $\left(\frac{f_d}{B}\right)$ | PSR using Hanning window in dB | PSR using convolutional Hanning window in dB |
| 0.01 | 31.68 | 33.67 |
| 0.05 | 30.32 | 33 |
| 0.1 | 27.62 | 31.79 |
| 0.15 | 24.47 | 30.44 |
| 0.2 | 22 | 28.7 |
| Doppler shift $\left(\frac{f_d}{B}\right)$ | PSR using Kaiser window in dB | PSR using convolutional Kaiser window in dB |
| 0.01 | 36.6 | 34.2 |
| 0.05 | 33 | 33.4 |
| 0.1 | 28.7 | 32.26 |
| 0.15 | 25.2 | 30.86 |
| 0.2 | 22.3 | 29 |
| Doppler shift $\left(\frac{f_d}{B}\right)$ | PSR using Chebysev window in dB | PSR using convolutional Kaiser window in dB |
| 0.01 | 36 | 34.3 |
| 0.05 | 34.89 | 33.5 |
| 0.1 | 29.89 | 32.5 |
| 0.15 | 25.86 | 31 |
| 0.2 | 26.84 | 29 |

convolutional windows are giving better results in terms of PSR than the conventional windows. The PSR values under different Doppler shifts using various windows are presented in Table-I. From the table it is observed that at lower Doppler shift the conventional windows give better PSR values as compared to corresponding convolutional windows. On the other hand at higher Doppler shifts the convolutional windows give better PSR values.

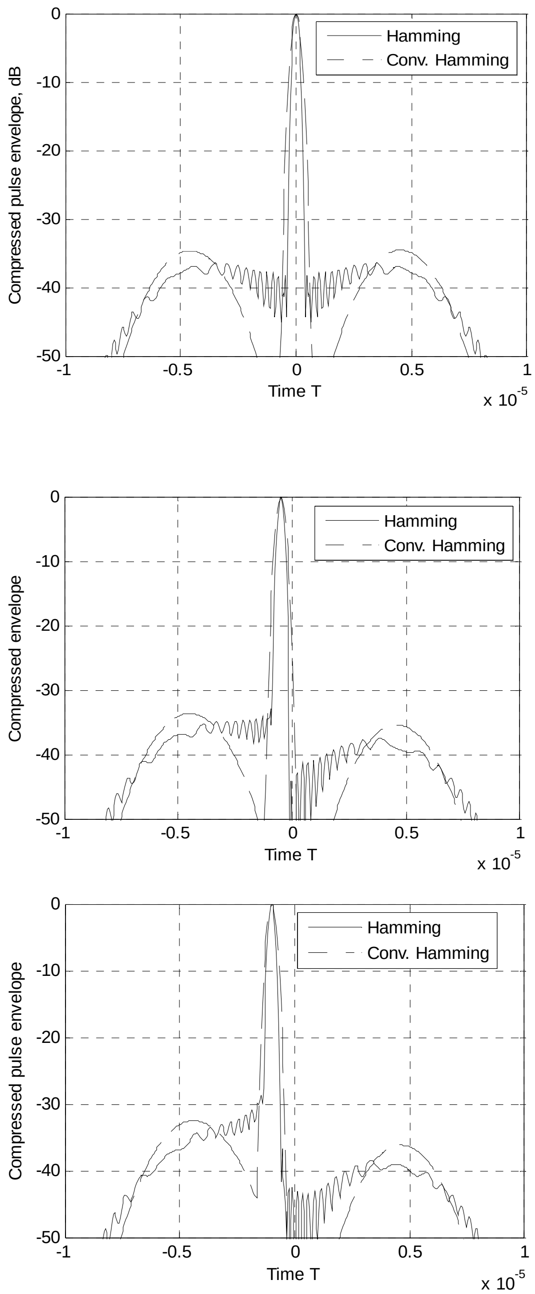


Fig.3 Effect on sidelobes due to Doppler shift(a)without Doppler shift and (b)0.05 Doppler shift (c) 0.1 Doppler shift

## V. CONCLUSIONS

In this paper convolutional windows are applied for radar pulse compression and compared with the performance of conventional windows. From the simulation results it is evident that variation of PSR values in case of convolutional windows is less as compared to that of conventional windows and also PSR of convolutional windows is greater at higher Doppler shifts. However in case of convolutional windows the sidelobes have been lowered but mainlobe width is increased.

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